General Relativity II – Effects

{**Abstract:** *In this segment of the "How Fast Is It" video book, we cover the effects of general relativity and how they differ from what Newton's gravity predicts. Our first effect is the orbit of Mercury that precesses more than Newtonian gravity predicts. To understand the non-Euclidian space that Mercury orbits in, we introduce the Schwarzschild metric and compare it to the Minkowski metric for flat space-time. We illustrate the positive curvature around the Sun using concentric circles with shrinking circumferences. We then show how this slight difference in curvature produces additional movement in the precessing perihelion of Mercury's orbit that exactly fits the measured number. Our next effect is the bending of light. We cover Arthur Eddington's famous measurement during a total eclipse of the Sun and show how the amount of starlight bending matched Einstein's calculations better than Newton's. We extend this bending effect to show how Einstein Rings and gravitational lensing work. And we show how this effect tips over light cones and changes world-lines. Our third effect is gravitational time dilation. We show how it works and cover how our GPS uses it. We also cover the Pound-Rebka experiment used the Mossbauer Effect to showed how this time dilation impacts gravitational redshift. We also illustrate how this effect resolves the Twin Paradox we introduced in the Special Relativity segment. Our final implication involves frame-dragging. To understand this effect, we introduce the Kerr Metric that covers rotating energy densities that literally drag space along with them. We use Gravity Probe B to illustrate how it works and how it is measured. We finish with an in depth look at the black hole in the movie Interstellar.*}

Introduction

With GR we now have a theory of gravity quite different than Newton's. But is this a difference without a difference? Or does GR predict different physical phenomena than Newton's theory?

If you've seen the "How small is it" video book on quantum mechanics and the standard model, you may have noticed that much of the theory was developed to explain experimental evidence. In GR, we find that the theory was developed without much experimental evidence.

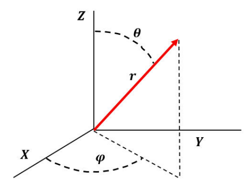
But the arrival of the theory, put experimental physics to work to prove or disprove it. Einstein himself showed that the field equations predict the orbit of Mercury better than Newton's. He also proposed two additional tests: one was the bending of light around the Sun, and the second was gravitational redshift.

We'll go into each of these plus one more on the twisting of space around rotating masses. We'll finish with a close look at how this all comes together around black holes.

*[**Music**: Mozart - Flute Concerto No 2 - Composed in the spring or summer of 1777 as an Oboe concerto.]*

Orbit of Mercury

[When working with GR, it is easiest to use polar coordinates because we are always dealing with the structure of space-time some distance away from a large mass.]



Minkowski metric (flat space)

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$$

In polar coordinates:

$$ds^2 = c^2dt^2 - dr^2 - r^2d\Omega^2$$

Where $\Omega = d\theta^2 + cos\theta^2\, d\varphi^2$

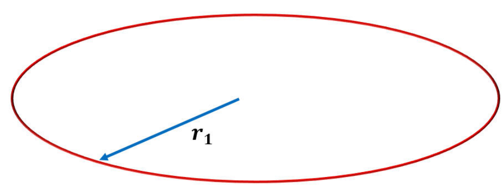
In 1916, the same year that Einstein published his GR paper, Karl Schwarzschild published his exact solution for space around a large non-rotating mass. His metric is now called the Schwarzschild metric and it works quite well for slowly rotating masses like the Earth and Sun and planets in our solar system. We'll use this metric for the first 3 tests.

Schwarzschild metric (curved space)

$$ds^2 = \left(1 - \frac{2MG}{rc^2}\right)c^2\, dt^2 - \frac{1}{\left(1 - \frac{2MG}{rc^2}\right)}\, dr^2 - r^2 d\Omega^2$$

This metric shows that we exist in a gently positively curved world.

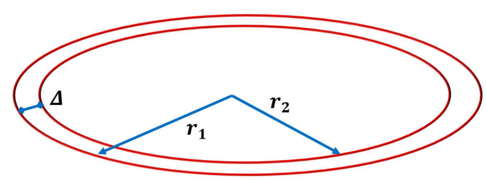
Let's take a look at what our space-time curvature looks like with this metric. If we draw the circumference of the Earth's orbit, we get a length that is 2 π times our distance from the sun.



Flat Space

$C_1 = 2\pi r_1$

If we exited in flat Euclidean space, we would calculate the circumference of an orbit one km closer to the sun and see that the distance between the orbits is one km.

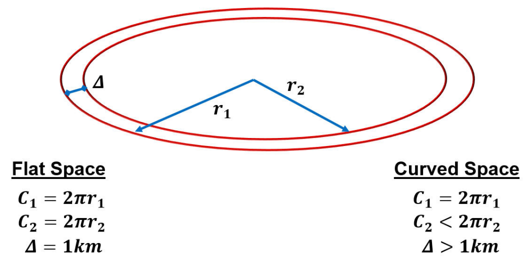


Flat Space

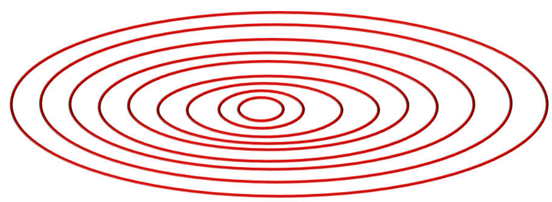
$C_1 = 2\pi r_1$
$C_2 = 2\pi r_2$
$\Delta = 1km$

But because of our positive curvature, if we were to measure the circumference with a radius that is 1 km shorter than the first, we'd find that it is less than 2 π times the shorter radius.

Which means that the distance between the circumferences would be greater than the 1 km difference in the radii! But only a little.

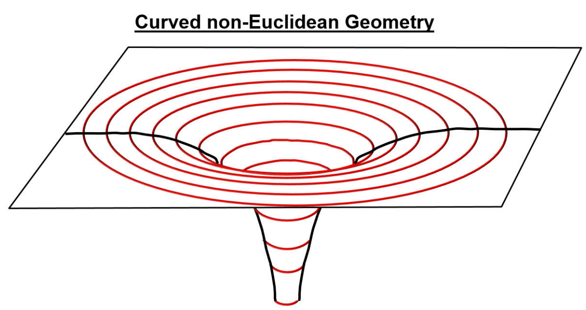


We can repeat this process all the way to the surface of the sun. With each successive radius, the difference between the orbits would increasingly diverge from the Euclidian numbers.



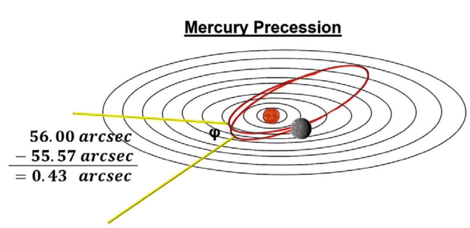
If we were to telescope this picture, you'd see the standard diagrams that are used to help explain GR. But diagrams like this are misleading in two ways. First, they represent an external curvature into another dimension, when in fact, we are talking about intrinsic curvature.

There is no evidence for the existence of a fourth special dimension. Second, it looks like you need a downward force on the object to get it to drop into the hole. That would be gravity – but that's what the lines were supposed to represent. So we'll avoid using this technique.



For over half a century before Einstein's time, it was known that there was something odd about the orbit of Mercury.

The elliptical path it carves around the sun shifts with each orbit, leaving its perihelion, or closest point to the sun, 56 arcseconds forward on each pass.



Newtonian equations accounted for all but around half an arc-second per year. And of course, they couldn't take into consideration the effects of curved space, because the idea that space wasn't flat hadn't been considered yet. With Schwarzschild's metric, Einstein came out with the exact number to cover the mysterious half an arc-sec.

$$\varphi = \frac{24\pi^3 a^2}{cT^2(1-e^2)} = 0.43\ arcsec$$

Where

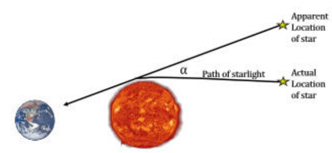
a = semi-major axis of Mercury's orbit
T = period of Mercury's orbit
e = eccentricity of Mercury's orbit
c = the speed of light

He had passed the first test of his new theory.

Bending of light

When light comes close to the Sun, the sun's gravity bends it inward. This makes the star look like it's further away from the sun in the sky than it really is. Both Einstein's and Newton's gravitation theories predicted this.

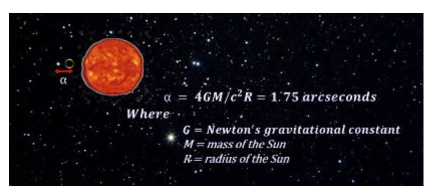
But the theories predict different values for the amount light would bend. Einstein's number was almost twice Newton's. Einstein suggested that a solar eclipse could be used to find the exact number.

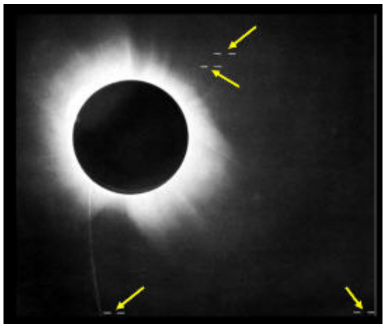


In 1919, a solar eclipse was slated to occur with the sun silhouetted against the Hyades star cluster - the nearest open cluster to our Solar System. Here's the Hyades star configuration with some of the brightest starts identified.

The British astrophysicist Arthur Eddington took up positions off the coast of Africa and in Brazil, and simultaneously measured the clusters light as it brushed past the sun.

The images were then superimposed on top of an image taken at night earlier in the year. When the eclipse and night images were compared, a gap was found. And when the gap was measured, it confirmed that Einstein's prediction was right.





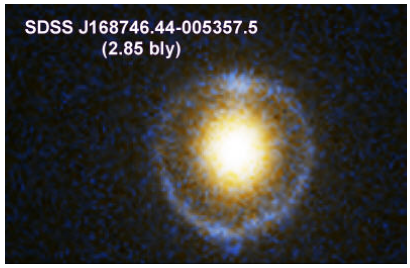
Here's one of Eddington's photographs. The lines highlight the Hyades stars used to calculate the shift.

Gravitational Lensing

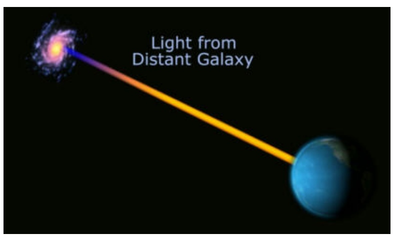
*[**Music**: David Arkenstone - Rob and Mary. From the movie Rob Roy.]*

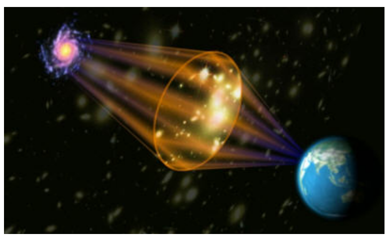
This same light bending leads to the warping of light from distant galaxies as the light encounters super-massive galaxies on their path to us. This is called gravitational lensing. [Einstein predicted that we would see 'rings' now called Einstein rings.

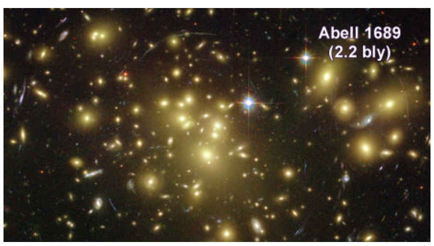
These rings are produced when two galaxies are almost perfectly aligned, one behind the other, giving an image like this with a reddish-white elliptical galaxy in the foreground and a thin ring of blue surrounding it — which is in fact the distorted light from another galaxy twice as far away. That would put it at 5.7 billion light years away.]



Here's a clip that shows how this lensing works on a grand scale. A distant galaxy would be seen here on Earth directly if there were no intervening massive cluster to bend its light. But with such a cluster, the light from the distant galaxy gets bent into rings and arches that continue on to Earth.





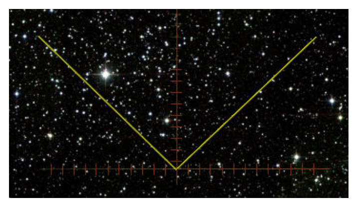


This is Abell 1689 [2.2 billion light years away]. It's one of the most massive galaxy clusters known. The gravity of its trillion stars, plus dark matter, acts like a 2-million-light-year-wide "lens" in space.

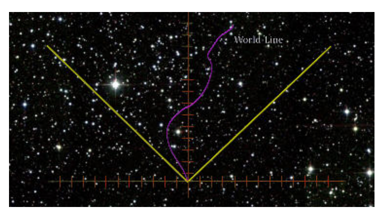
And here's MACS J0416.1–2403 5.47 billion light years away. It's the latest from Hubble on gravitational lensing released in late 2015. These foreground galaxy clusters are magnifying the light from the faint galaxies that lie far behind the clusters themselves. These faint lensed galaxies are around 12 billion lightyears away. It's the gravitational lensing that allows us to see that far back in time. Without the magnification, these galaxies would be invisible for us.

Light-cone tipping

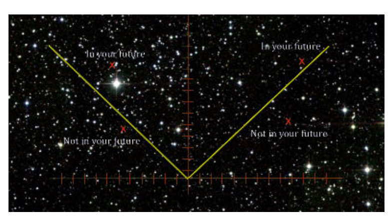
One of the key implications for the bending of light is its impact on what's physically possible in heavily curved space-time. Here's a two dimensional slice of the future light-cone that we developed in the previous segment on SR.



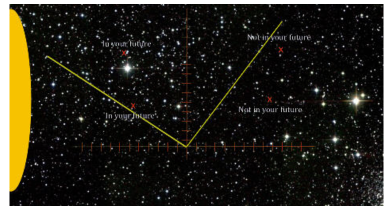
This purple line represents a path taken by anything with mass. It's called the world-line and can be anywhere inside the light-cone. In this representation, world-lines have to remain between the two arms of the light cone because nothing can travel faster than the speed of light.



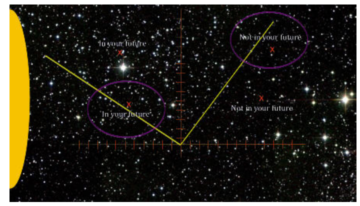
The speed of light lines are the divider between events that are in your future (if it's your light cone) and events that are not. By "in your future" I mean that you can be connected to them physically in some way.



Now suppose there is a great mass-energy density to the left of the cone. The light would be bent in its direction.



We see that points that were impossible to reach before, now fall inside the cone and are reachable. And we see that points that were reachable inside the cone now fall outside the cone and are unreachable.



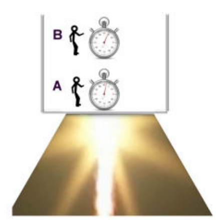
This is light-cone tipping. The closer we get to the source of the gravity, the greater the space-time curvature. And the larger the matter curving the space, the greater the curvature. We'll take another look at this when we get to black holes.

Gravitational Time Dilation

> *[**Music**: Grieg - Holberg Suite, Sarabande (Andante). Based on eighteenth century dance forms, this was written in 1884 to celebrate the 200th anniversary of the birth of Danish Norwegian **humanist playwright** Ludvig Holberg.]*

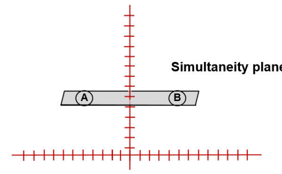
One of the most dramatic consequences of GR is how space-time curvature effects the flow of time.

We'll use the elevator thought experiment to illustrate how clocks run at different rates in the box according to their distance from the source of the gravity. We'll see that a clock closer to the source of the gravitational field runs slower than a clock further away.

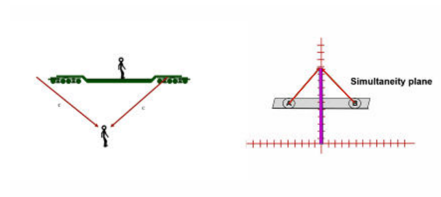


To help see how this works, we'll take another look at the lightning strike for the person on the train, and the person on the ground that we used in our segment on SR. Only this time, we'll map the events to our space-time graph. The world-line for the person standing on the ground is shown in purple.

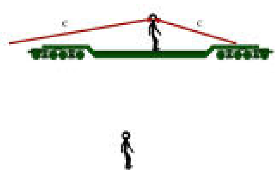
We label the lightning strikes A and B, and place the two events on the space-time graph with A to the left of the person on the ground and B to the right. The plan containing A and B contains all the points that are simultaneous for the person on the ground at the time of the two strikes. We call this the simultaneity plan.

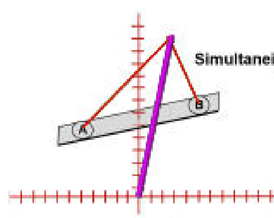


The light from both events travels at the speed of light so their world-line always moves at a 45 degree angle. They reach the person on the ground at the same time. This of course is what makes them simultaneous from the point of view of the person on the ground.



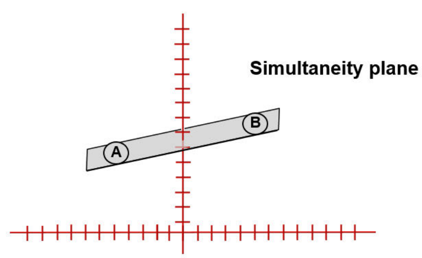
Now let's repeat the lightning strikes so that, from the point of view of the person on the moving train, they strike at the same time.



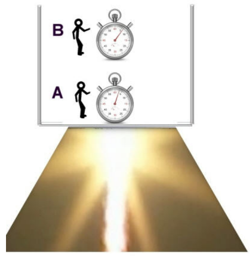


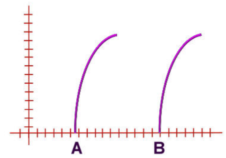
In order for the light to reach the person on the train at the same time, the strike behind him will need to hit first from the person on the ground's point of view, because it will have to travel further to get to the moving person than the light from the strike that hits in front of him.

So we see that the Simultaneity plan, for the moving person, is necessarily tilted up on the right.



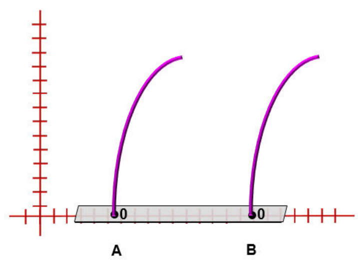
Now we can map the movements of A and B in the accelerating elevator to the space-time graph. The center is the source of the acceleration (aka gravity). A is to the right of it and B is a bit further to the right reflecting their distances from the source of the gravity. As the elevator accelerates, the world-lines on the space-time graph are not straight lines. They curve outwards because their velocity increases with every second.



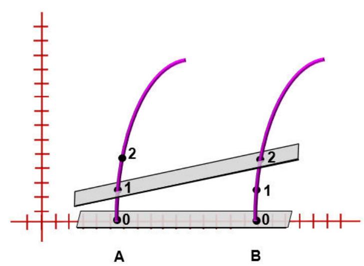


*[**Music**: Korsakov - Capriccio Espagnol. Written in 1887, this is an orchestral suite, based on Spanish folk melodies.]*

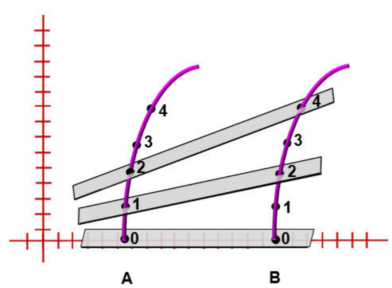
Here we have clocks that measure the proper time elapsed along each person's world-line. They mark the time in their own reference frame. At the start, they were both at rest, so their simultaneity plan is horizontal and they see each other's clocks reading zero.



In this example, we see that after 2 seconds, we have a slightly tilted simultaneity plan. B sees that 'at the same time' his clock ticks 2, A's clock ticks 1. 'A' also sees his own clock reading 1 when B's clock reads 2.



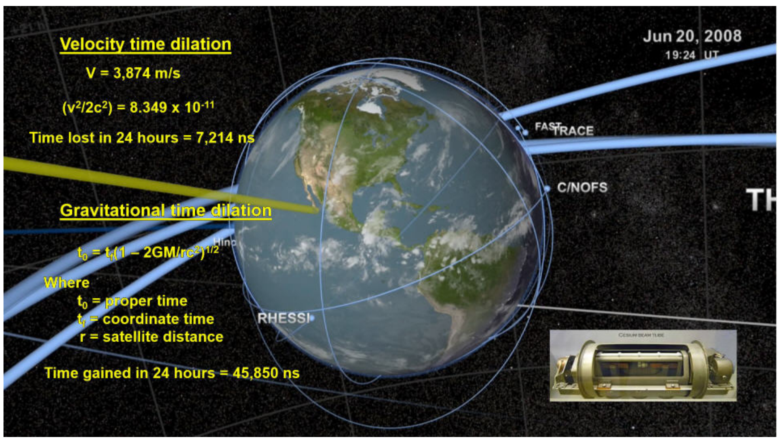
Continuing to a higher velocity, with the steeper slope for the simultaneity plan, B sees A's clock reading 2 when his own clock reads 4 seconds. 'A' also sees his own clock reading 2 when B's clock reads 4.



A and B **both** agree that A's clock is ticking slower than B's clock. [We are skipping the SR effects of time dilation and space contraction here. They play a big role as the velocities approach the speed of light.] The equivalence principle tells us that the same thing will happen near a massive body. Gravity slows down time. Newton's gravitation has no such implication.

We see this with our GPS systems. In our segment on SR, we saw that time dilation due to velocity differences have GPS satellites losing every day. Time that must be corrected for to get the right positions on the surface of the Earth.

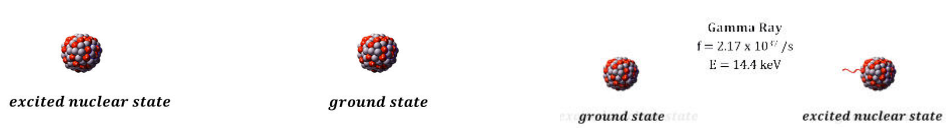
They must also take into account gravitational time dilation due to their being further away from the Earth than clocks on the ground. Based on the Schwarzschild metric, calculations show that the satellites' clocks will gain over 45,000 nanoseconds a day due to this general relativity effect. The accuracy of our GPS system is strong evidence for the correctness of the GTR. [So the total relativity effect is the difference between the two (45850 – 7214) of 38636 ns per day.]



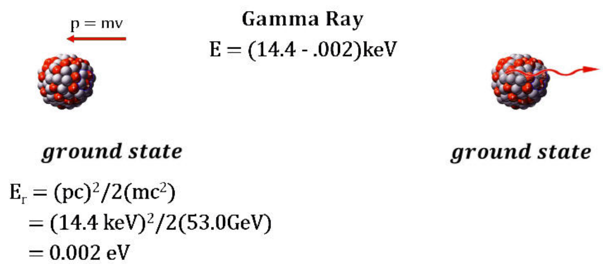
Gravitational Redshift Experiment

In 1959, physicists Robert Pound and Glen Rebka preformed an experiment in the Jefferson Physical Lab at Harvard to demonstrate gravitational redshift. It was based on physicist Rudolph Mossbauer's effect discovered two years earlier that involves the emission and absorption of gamma rays from the excited states of an iron nucleus.

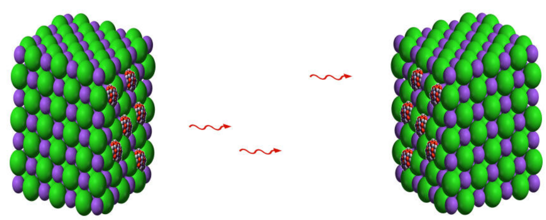
Here we have an iron atom's nucleus in an excited state. When it falls to a lower energy level, a gamma ray photon carrying the energy is emitted. Once this photon encounters a like atom, it will be absorbed – raising the energy level of the encountered atom's nucleus.



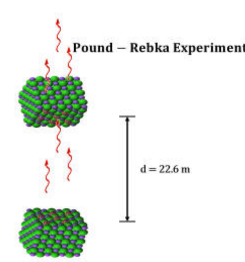
The problem is that when the gamma ray is ejected, the nucleus recoils. Because of energy momentum conservation, the recoil energy reduces the energy of the gamma ray. The gamma ray is no longer a match for the other nucleus and it moves right through. There is **no** absorption.



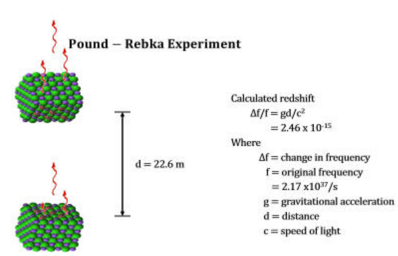
What Mossbauer discovered was that if he imbeds the atoms in a crystal, the recoil is reduced dramatically, and absorption can be established.



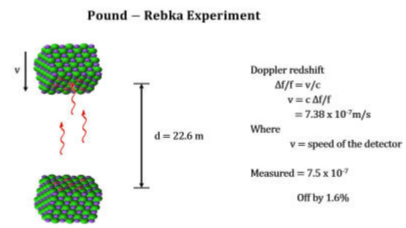
Pound and Rebka use this Mossbauer Effect. They placed an emitter at the bottom of a tower in the Laboratory and installed a detector 22.6 meters above. No absorption was detected because gravitational time dilation changed the frequency of the emitted gamma rays so no energy match existed in the detector.

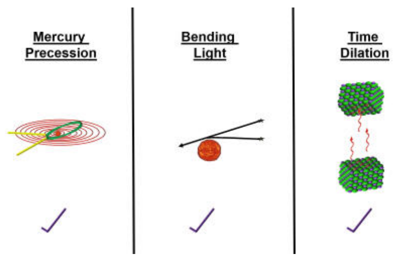


The calculated shift was extremely small, but the Mossbauer Effect is sensitive enough to measure it.



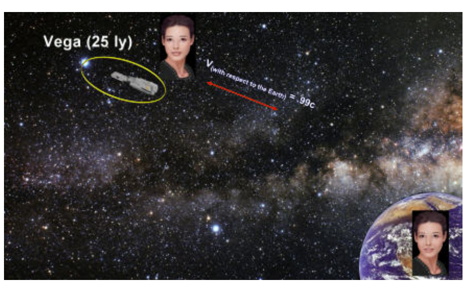
They then adjusted the detector's velocity down until absorption occurred. We get the amount the frequency changed using the well understood relativistic Doppler redshift equation just like the Doppler shift in starlight. Their results came to within 1.6% of the value predicted by Einstein's field equations using Schwarzschild's metric.



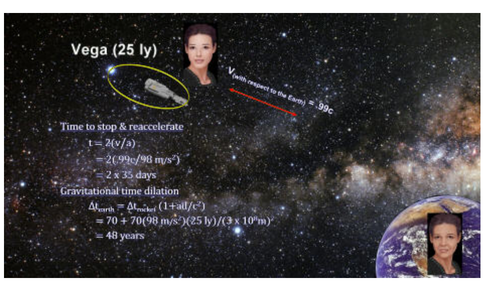


Although this experiment did not produce new results, it showed that gravitational time dilation, one of GR's most significant findings, was consistent with all physical conservation laws. This gave the GTR 3 successes out of 3 tests.

Twin Paradox Resolved



Gravitational time dilation is the answer to the Twin Paradox that we covered in the previous segment on SR. The key interval is at the half-way point. As the spaceship approaches Vega it decelerates to a stop and then re-accelerates back to Earth. The traveling twin finds that she is in a gravitational field.



Let's say her acceleration is 10 g's or 98 m/s^2. At this rate it will take her 35 days to decelerate to 0 and another 35 days to reaccelerate back to 99% of the speed of light. Gravitational time dilation shows that as her clock ticks 70 days, her twin's clock on Earth will have ticked 18,134 days. That's 48 years. The twin on Earth agrees.

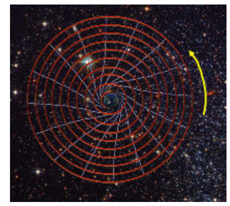
So instead of both twins thinking the other should be younger, they both agree that the twin on the rocket to Vega and back is younger. No contradiction is involved and the paradox is resolved.

Frame Dragging

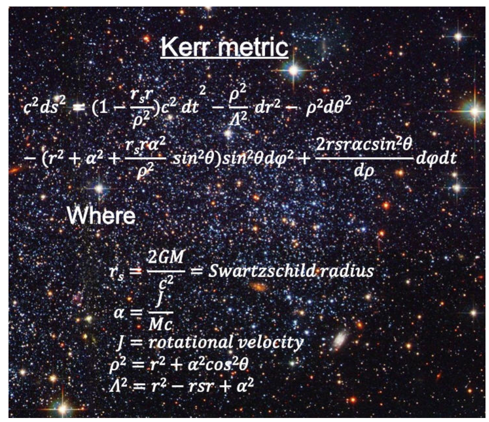
> *[**Music**: Heuberger - Midnight Bells: from the operetta Der Opernball written in 1898.]*

Our last test is the most recent. It was designed to measure the twisting of space around a rotating mass. This twisting is called frame-dragging, where space is literally dragged along with the rotating mass. The effect was derived in 1918 by physicists Josef Lense and Hans Thirring, and is also known as the Lense–Thirring effect.

They predicted that the rotation of a massive object would distort the space-time metric, making the orbit of a nearby test particle precess like a gyroscope. This does not happen with Newtonian gravity where the gravitational field of a body depends only on its mass, not on its rotation.

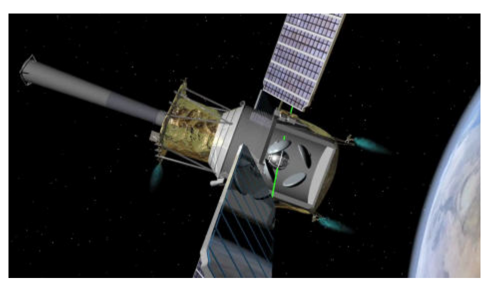


Up till now, we've been using the Schwarzschild metric which does not show this effect. It wasn't until 1963 that a mathematician named Roy Kerr discovered the significantly more complicated metric for rotating bodies that made it possible to calculate the precession one can expect from a given mass and rotation of an object like the Earth.

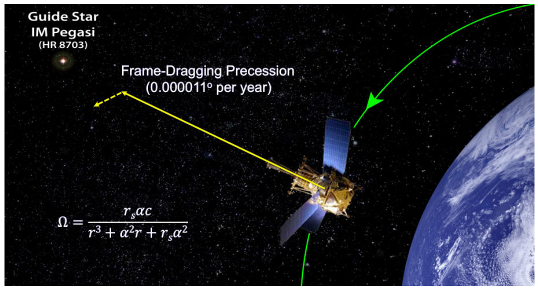


To test this effect, NASA developed a satellite called Gravity Probe B and put it into orbit 642 km above the Earth in 2004 where it operated for a year.

It used a set of super sensitive gyroscopes to measure precession due to frame-dragging. It also included a non-gravitational drag identification gyro and compensation micro thrusters to maintain a non-gravitational drag free environment. It compensated for solar radiation drag and atmospheric disturbances drag.



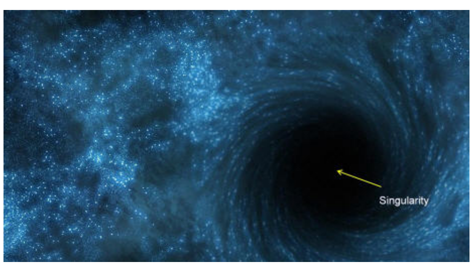
By 2011, data analysis had confirmed that frame-dragging did occur and measured it to within 15% of the amount predicted by the Kerr metric for Einstein's field equations.



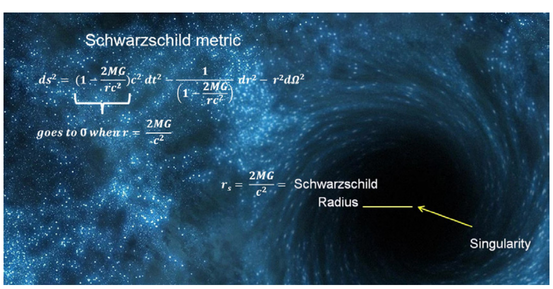
Anatomy of a Black Hole

[Music: Franz Liszt - Les Préludes - Symphonic Poem No3: published in 1856, it is the earliest example of an orchestral work entitled "symphonic poem".]

One of the most interesting consequences of GR is the structure and impact of a Black Hole.



In the Milky Way segment of the How Far Away Is It video book, we discussed how they are formed from collapsing massive stars, too big for neutron pressure to halt their collapse to a point, called a singularity.

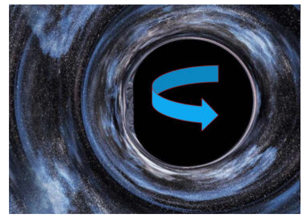


The Schwarzschild metric showed that if the mass of the body should contract to a small enough radius, it would capture light itself. It would go dark – hence the name Black Hole. This radius is known as the Schwarzschild radius and forms a sphere known as the event horizon.

One of the best illustrations of a black hole was created for the 2015 movie "Interstellar" with the help of theoretical physicist Kip Thorne. This black hole, called Gargantua, was given a mass of 100 million suns and a super high rotation rate of 99.8 percent of the speed of light. With this kind of rotation, we see that Gargantua is a Kerr black hole.

At 100 million solar masses, the Schwarzschild radius is around the distance from the Sun to the Earth. That's far enough away to makes the tidal forces at the horizon quite unnoticeable. We'll use Gargantua to illustrate the properties of GR that we have discussed in this segment.

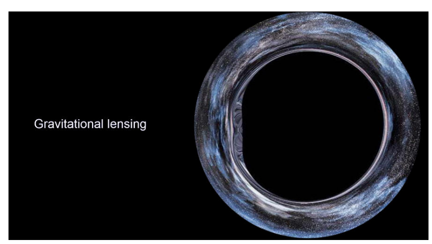
So let's build this black hole from the ground up. We are viewing it from the equatorial plane and the object is rotating in on the left an out on the right. Its center is dark out to the Schwarzschild radius.



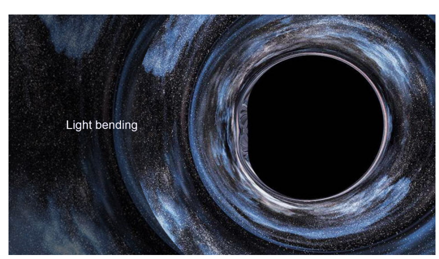
The Kerr metric shows that light can also be captured in stable **orbits** outside the event horizon. For a rapidly rotating black hole, the orbital volume around the black hole would be significant. This would produce a photon sphere shell incasing the black hole with light from all the stars in the universe accumulated over the entire age of the universe. It would be a sight to see. But given that the light is trapped in orbit, we can only see what leaks out.

This thin ring around the black hole represents the cross section of this shell we'd see because of light that leaks out in our direction. It is flattened on the left because light rotating with the Black Hole's rotation can get closer to the horizon than light rotating against the black holes' rotation.

Next, we see a dense sprinkling of stars with a pattern of concentric shells. This is the pattern produced by the gravitational lensing.



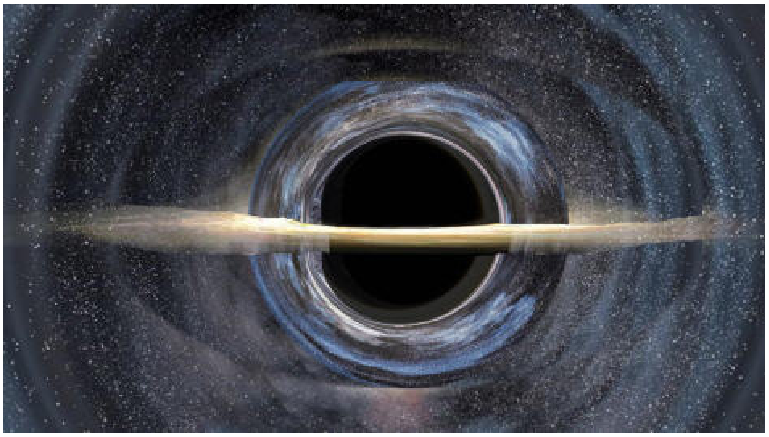
Further out we see the dislocation of star positions due to the bending of light by the gravity of the black hole.



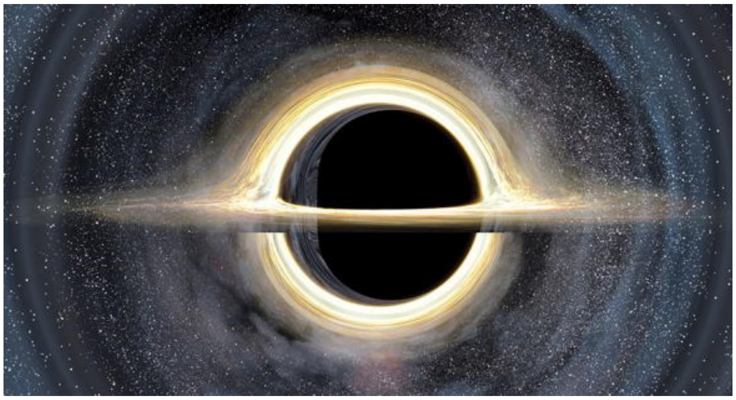
Disks and jets

This black hole has the remnants of an accretion disk that is no longer feeding the black hole. [Instead of being a hundred million degrees like a typical black hole's disk, this disk is only a few thousand degrees, like the Sun's surface.]

If the disk were not gravitational lensed, the black hole would have looked like this.

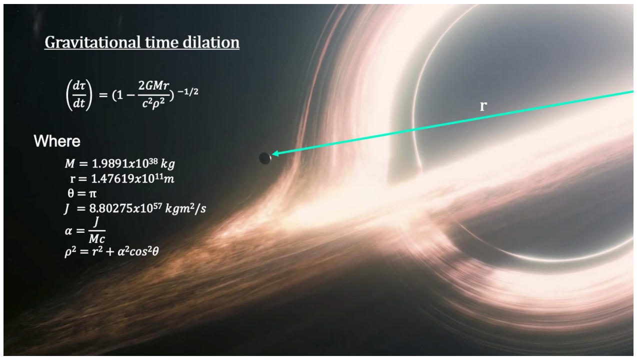


But, because of gravitationally lensing, the massive amount of light rays emitted from the disk's top face travel up and over the black hole, and light rays emitted from the disk's bottom face travel down and under the black hole. This combination gives us the full image of how the black hole would actually look.

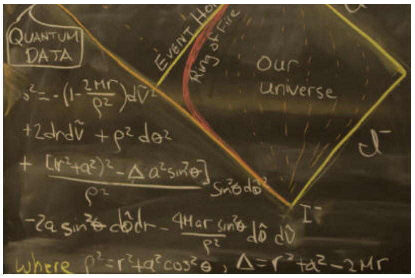


Black Hole Time Dilation

In the movie, one hour on Miller's plant equaled 7 years on Earth. Some of this came from time dilation due to the planet's speed. It's traveling at 55% of the speed of light in order to maintain its orbit. But the bulk of the time comes from gravitational time dilation. And the fact that Gargantua's rotational energy is so large, intensifies time dilation considerably.

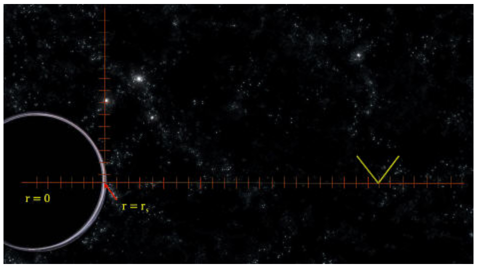


If you watch the movie again, you might note that it is the Kerr metric on the professor's blackboard.



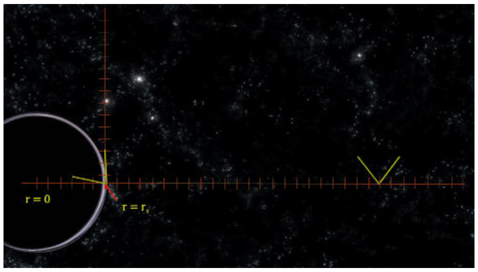
Entering a Black hole

We can use tipping light cones to show how all objects, unfortunate enough to cross the event horizon, are captured forever. Here's a light cone far from the black hole. The horizontal axis represents distance from the singularity.

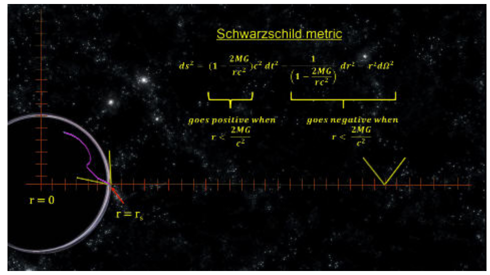


As we saw earlier, when space-time is curved by the presence of mass-energy, the light cone structure gets distorted. When the mass is a black hole, the tilting reaches 45^0 at the event horizon.

This means that all events beyond the horizon are no longer in the future light cone of any object that has gone past it. No possible world line gets you out. All remaining world lines lead to the singularity.

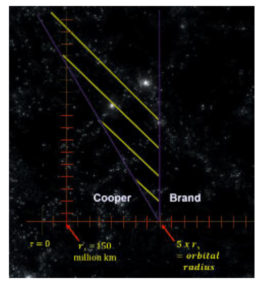


[It is interesting to note that as you cross the horizon, the time term goes from negative (which is time-like) to positive (which is space-like). Simultaneously, the space terms all go from positive (space-like) to negative (time-like).] Distance from the singularity decreases inside the black hole's horizon as surely as time increases outside the horizon.

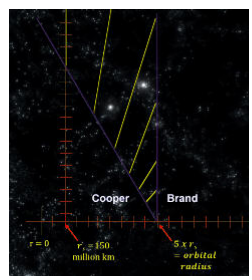


In "Interstellar", Cooper flies his ship into the black hole while Brand watches from a higher orbit. We can use our space-time diagram along with light cone bending again to illustrate what each of them would have seen.

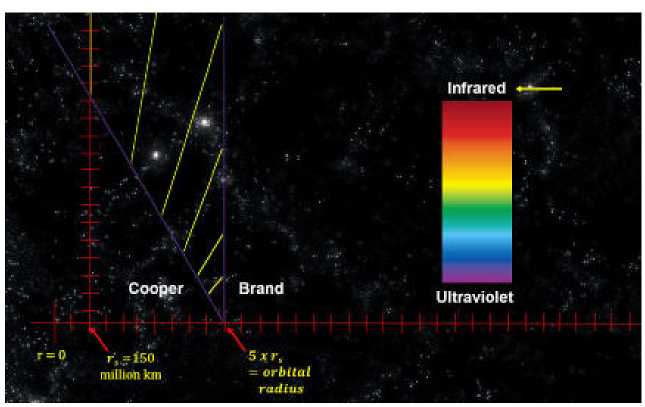
First we'll take a look at it from Cooper's point of view. As he heads directly into Gargantua he sees periodic signals from Brand. She is far enough away from the horizon for her light signals to all travel in a parallel manner at 45 degrees along her light cone boundary. Cooper crosses the event horizon without even noticing it, as signals continue to arrive at regular intervals. Eventually he will feel the tidal forces of the singularity.



Now things are quite different from Brand's point of view. As Cooper approaches the event horizon, his light cone tips towards the singularity. This means that his light signals back to Brand are taking longer with each km traveled. The effect is hyperbolic and the light signal he sends from the horizon itself will never get to Brand. She sees his clock slowing down to the point that it stops. She never 'sees' him enter the black hole!



What's more, because of gravitational redshift, the image of Cooper and his ship shift to the red. At the horizon, it has shifted into the infrared and can no longer be seen by Brand. For her, Cooper grinds to a halt and goes invisible. Quite different from what Cooper sees.



Conclusion

The GTR is now 100 years old. In spite of the fact that there have been a number of tests, questions remain. One of the theories most interesting predictions is gravitational waves. But as yet, no gravitational waves have been found. If they are ever found, would there be an associated elementary boson particle (the graviton) like photons for the electromagnetic force? A great deal of active research is under way to find out. The fate of GR remains in the hands of experimental physicists.