Gravitational Waves

{Abstract: *In this segment of the "How Fast Is It" video book, we cover Gravitational Waves. We examine just what a 'ripple in space-time' is. We'll cover the wave properties including its metric, speed, and the two transverse polarizations: h plus and h cross. We'll cover how it expands and contracts objects it encounters. We'll examine binary star systems and the waves they create. This includes the properties of the binary encoded in the created gravitational wave including frequency, frequency changes (the chirp), chirp mass, gravitational luminosity and luminosity distance. We'll use stars the size and mass of our Sun to calculate the expected amplitude and wavelength magnitudes. We'll move from normal stars to neutron stars to stellar mass black holes. Along that progression, we'll see their signature waveforms and build the gravitational wave sensitivity graph. We'll cover the Hulse-Taylor pulsar (PSR B1913+16) and how it provided indirect evidence for the existence of gravitational waves. We'll then cover direct detection: first with strain gauges and then with Michelson Interferometers. We'll cover the LIGO technology and sensitivity and then examine the GW150914 event. We'll show how it fit the waveform and magnitudes for merging stellar black holes. We'll also quantify the magnitude of the radiated energy. Then we'll cover how we located the sky location for the event. We'll end with a look at other observatories and the impact detecting gravitational waves will have on cosmology.*}

Introduction

At the end of the second segment on General Relativity, released at the end of 2015, I said that one of the last Einstein predictions was that accelerating masses can create gravitational waves. I pointed out that gravitational waves had yet to be directly detected. But in January 2016, it was announced that gravitational waves had in fact been detected. So I thought I should create this addition to the "How Fast Is It" video book to cover gravitational waves.

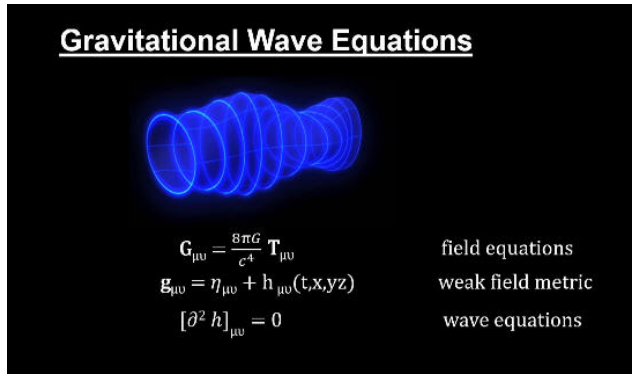
- We'll cover exactly what a gravitational wave (a ripple in space-time) is.
- We'll examine a top astronomical event that could create waves large enough to measure at great distances.
- We'll review how a Michelson Interferometer like the one we covered in "How Small Is It" can be used to detect gravitational waves.
- And we'll see how the Laser Interferometer Gravitational wave Observatory (LIGO) actually detected the gravitational wave from a merging pair of black holes.

We'll end with a look at the future of gravitational wave observatories.

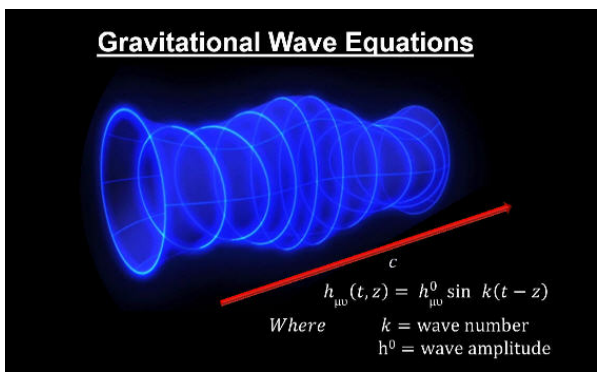
Gravitational Waves

[Music: *Puccini – Madama Butterfly***]**

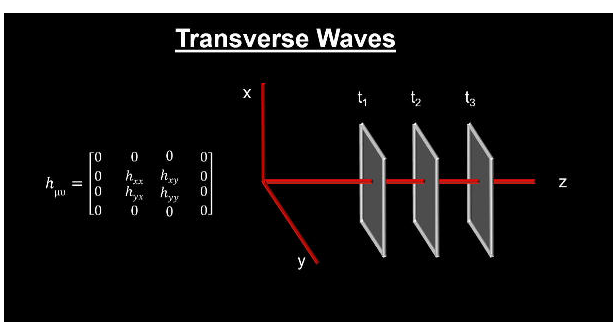
Our first step is to examine just what a ripple in space-time means. Here, far from an event that could create a GW, we have a relatively flat space with a Euclidian metric **g** that isn't changing with time. A ripple represents small deviations from this flat space-time metric. We use **h** to represent these deviations. **[**In other words, **h** is the metric tensor of the gravitational wave.**]**



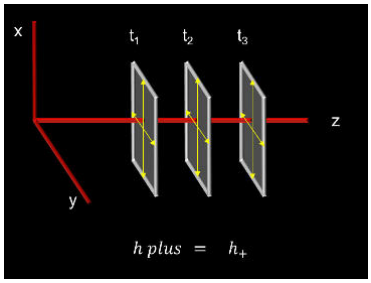
Solutions to Einstein's equations show that **h** oscillates sinusoidally just like light. And it travels at the same speed as light.

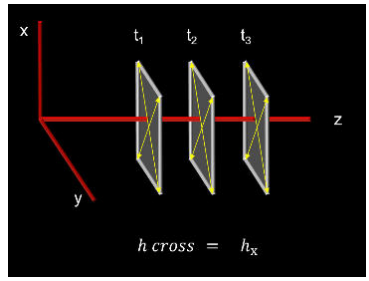


[It turns out that solving these equations in the weak limit also removes all components of **h** except for those that operate in the plane perpendicular to the direction of the wave.**]** As a wave moves down the z axis, planes at different times experience different values for the metric used to measure distance on the plane. This makes the wave a transvers wave just like light.

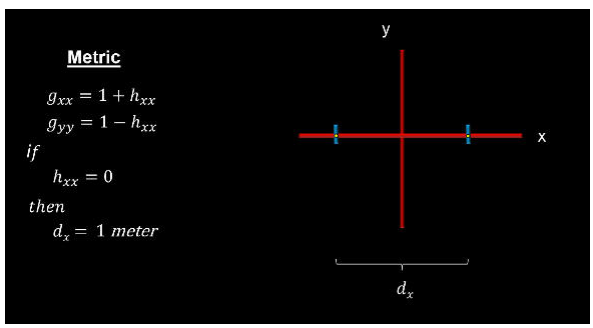


We see two possible polarizations for a gravitational wave. We call one **h plus (h_+)** for action along the x and y axis. We call the other **h cross (h_x)** for action along the diagonal. **[**This is again much like electromagnetic wave polarizations except that these two polarizations are 45 degrees apart and EM wave polarizations are 90 degrees apart.**]**

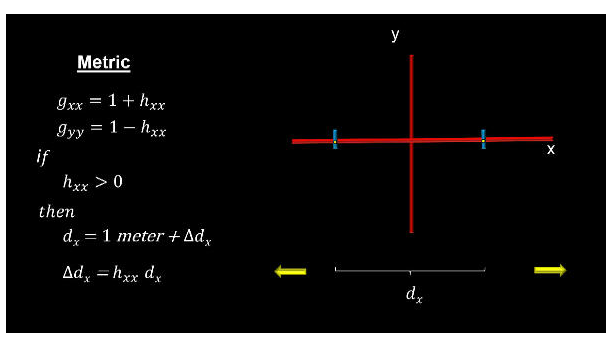




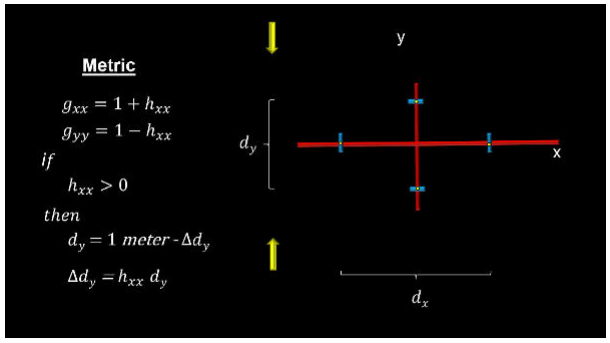
To see what an oscillating **h plus (h_+)** metric does, we'll measure the changes in the distance between points on the plane when a gravitational wave passes. Here we have an x-y plan with the wave passing into the page. We mark two points on the x axis 1 meter apart in Euclidian flat space where **h** is zero.



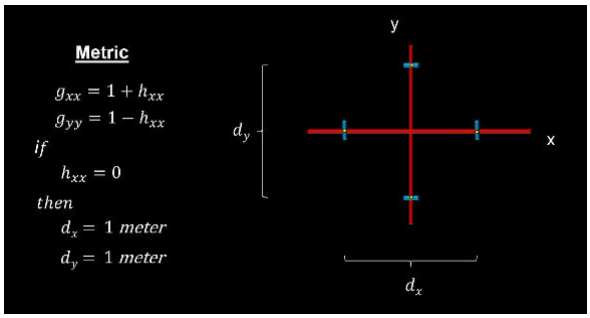
When **h** is greater than zero, the distance between the two points on the x axis becomes longer than 1 meter by an amount equal to **h times the original distance**.



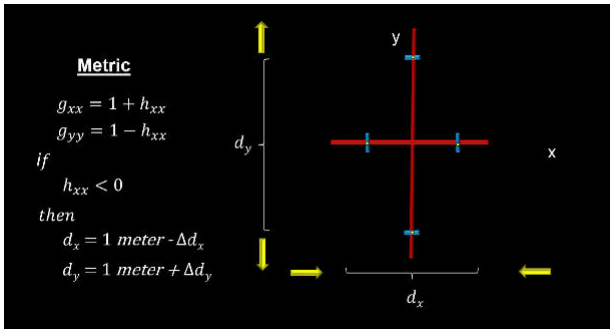
At the same time, a one meter distance on the y axis will shrink to less than 1 meter by the same amount.



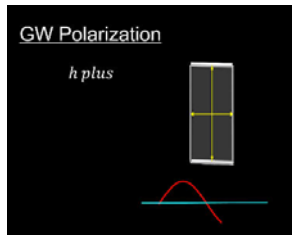
When **h** returns to zero, the distances between these points returns to 1 meter.



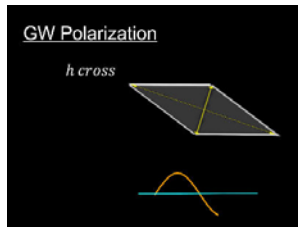
When **h** is less than zero, the distance between the two points on the x axis will become shorter than 1 meter, and the distance between the two points on the y axis will become longer than 1 meter.



Here's an exaggerated look at what an oscillating **h plus (h_+)** polarized gravitational wave does to a square plate it passes through. Again the wave is passing into the page.



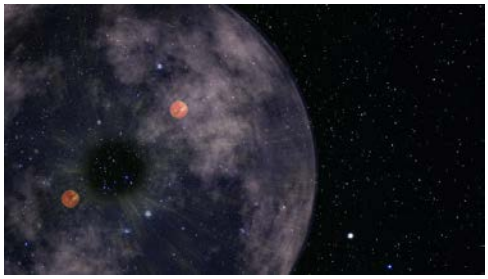
For an **h cross (h_x)** polarized wave, the effect would be similar but shifted 45 degrees.



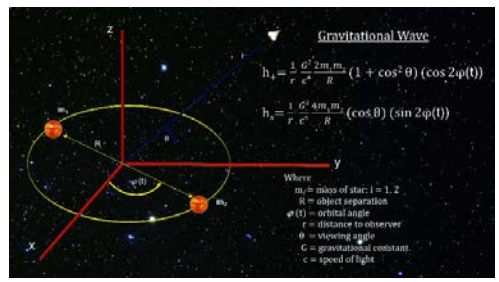
When describing a GW, we can now be more precise than "it's a ripple in space-time". A gravitational wave is an oscillating polarized metric that operates in the plane perpendicular to the direction of the wave as it moves through space at the speed of light. And we have seen what this means for the objects that encounter such a wave: they are stretched and squeezed in various directions.

<u>GW Sources</u> **[Music**: *Handel - Concerto Grosso - Larghetto***]**

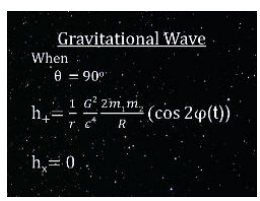
We'll now turn our attention to the kinds of massive accelerating objects that can create such a wave. In order to generate a gravitational wave, you need a non-spherically symmetric rotating system. For example, here's a binary star system with two masses revolving in a circular orbit around a common center of gravity. The stars' acceleration creates gravitational radiation that travels out from the system in all directions - just like the light they are generating.



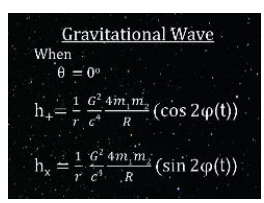
[The amount of matter involved; the distance between them; and their rotational angle determine the amplitude and frequency of the created waves. We have seen that the wave carries two polarities **h plus** and **h cross.** The amplitude of each polarity at any point in time depend on the orientation of the viewer to the plane of the orbiting objects.



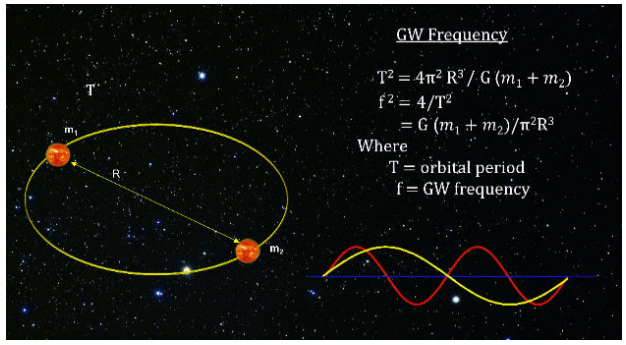
Viewing the binary system edge-on, the polarization is purely **h plus**.



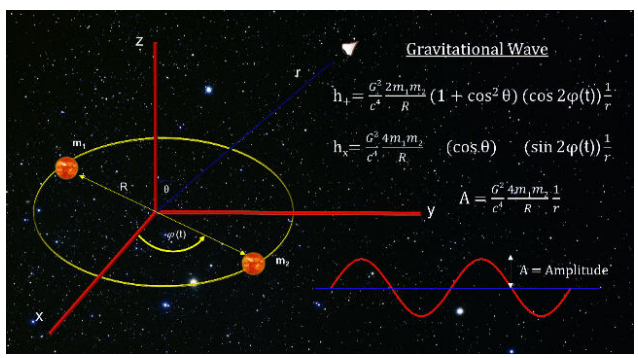
Viewing the binary system face-on, the polarization is circular. In between, the combinations of the two give us the viewing angle we have on the binary system.



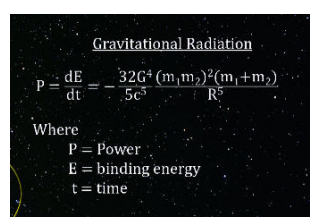
The GW solutions show that the frequency of the created GW is twice the rotation rate of the binary system.



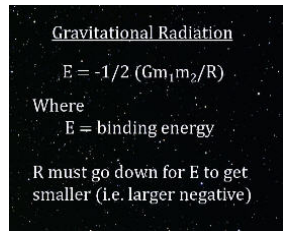
We also see that the maximum GW amplitude depends on the masses of the two objects; the distance between; their rotational velocity; the viewing angle; and how far away the system is from the observer.



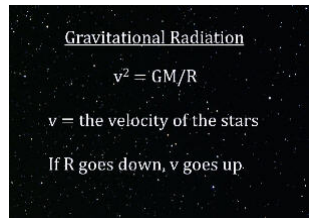
There is one more key factor to consider when it comes to binary systems: namely that GWs carry energy and momentum away from the system. We call this "**gravitational luminosity**".



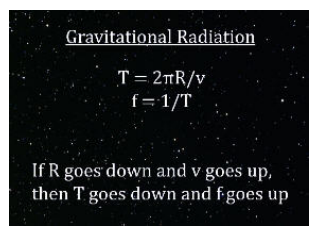
Newton and Kepler provided the mechanics for understanding what happens to the orbit when gravitational energy is lost. Because binding energy is negative, a loss of energy will make it a larger negative. This has the effect of reducing the distance between the two objects.



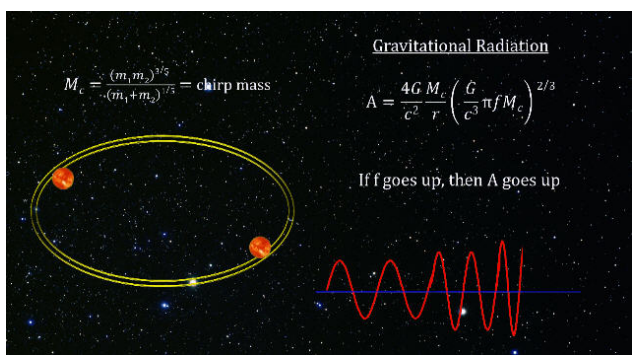
This in turn increases their velocity.



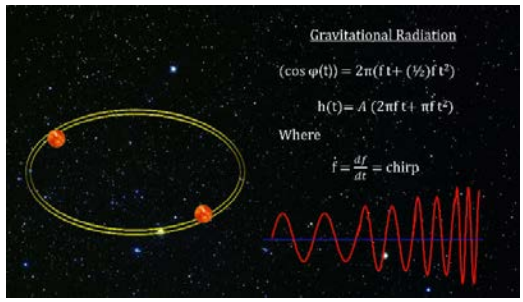
A shorter circumference and faster velocity reduces the time it takes for a full orbit and therefore increases the frequency of rotation and therefore the frequency of the GW.



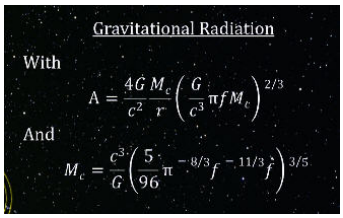
And the wave equations show that the amplitude of the GW will increase with the frequency.

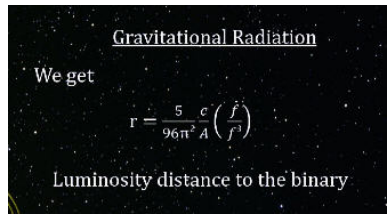


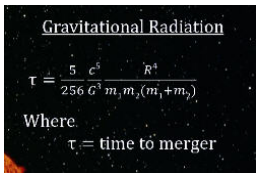
The rate that the frequency is changing is called the **chirp**. It gives us the ability to express the amplitude of the GW in terms of the frequency and the rate that the frequency is changing, instead of the masses and the distance between the masses. This is critical, because for most cases, we will have no way of knowing directly what the masses are or how far apart they are. But measuring the frequencies might be possible.



If we can also measure the amplitude, we can even calculate the distance to the binary system. Because this distance is based on GW luminosity, it is called the "Luminosity Distance". For most all GW sources, this will be the **only** way to figure out how far away they are.

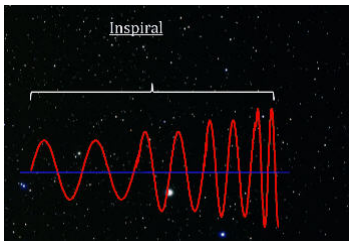


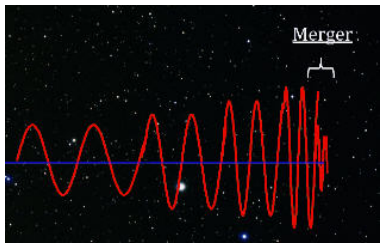


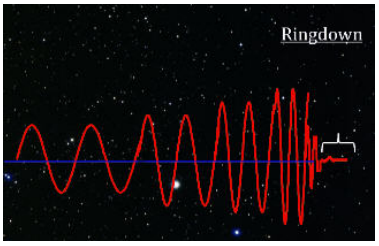


With a decaying orbit, the objects will eventually collide and coalesce. (We can even calculate how long that would take.)

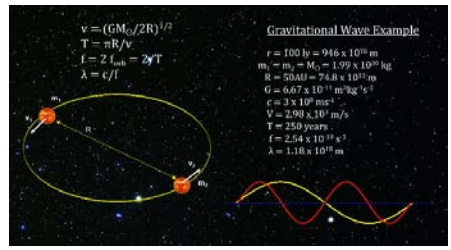
The resulting waveform (called a coalescing waveform) serves as a signature for this kind of GW source. It has three phases: the in-spiral; the merger; and the ring-down to an object that is no longer asymmetric and therefore no longer radiating gravitational waves.



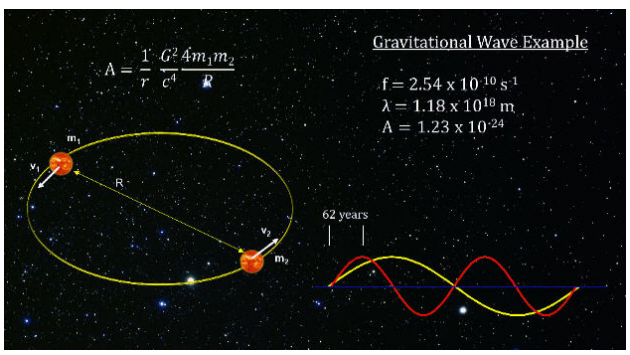




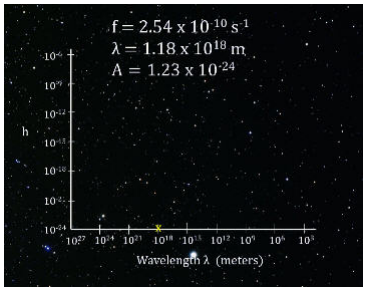
[To get an idea on the expected amplitudes and frequencies for GWs created by a system like this one, let's put in some numbers. Suppose this system is 100 light years away and each star is the mass and size of our sun; and the distance between them is 50 times the distance between the Earth and the Sun (50 Astronomical Units). From the masses and the distance between them, we can calculate the stars' velocity. And with that, we can calculate the orbital frequency; which gives us the frequency and wavelength of the resulting GW.



And with the masses and frequency, we can calculate the amplitude of the resulting GW. Here we have a very small number. It would add around a hundredth of the diameter of an electron to a meter stick. What's more, it would take over 62 years to reach this miniscule stretched length!



Here is where this data point fits on a graph with wavelength decreasing along the x axis and amplitude increasing along the y axis.



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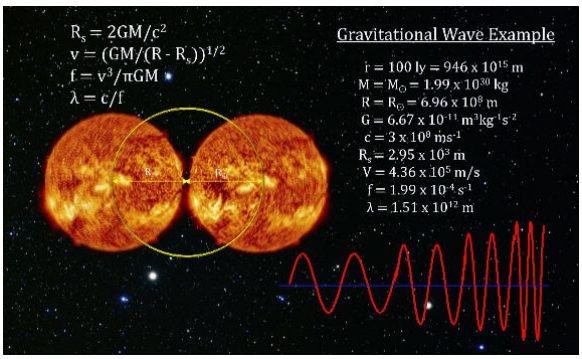
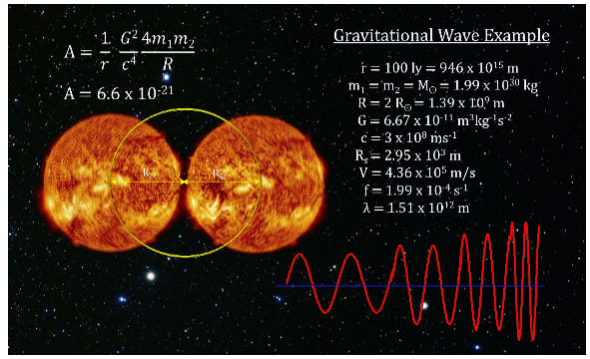
GW Sources Example **[Music**: *Debussy - Rêverie***]**

To get an idea on the expected amplitudes and frequencies for GWs created by a system like this one, let's put in some numbers. Suppose this system is 100 light years away and each star is the mass and size of our sun. At the point that they are about to touch, we would see the maximum frequency and amplitude. In this example, we get 10^{-21}.

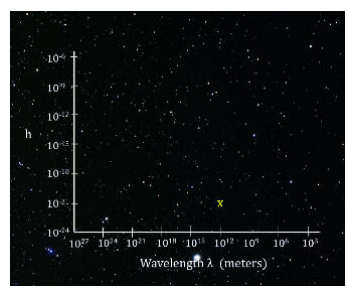




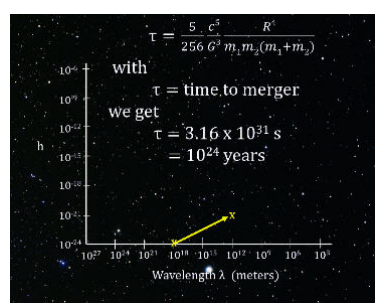
This is a very small number. It is approximately the ratio of the width of a human hair to the distance to Alpha Centauri 4 lightyears away!

$$\frac{human\ hair}{4\ ly} = \frac{100 \times 10^{-6}\,m}{4 \times 9.46 \times 10^{15}\,m} = 2.6 \times 10^{-21}$$

Here is where this data point fits on a graph with wavelength decreasing along the x axis and amplitude increasing along the y axis.



[We can even calculate how long it takes a binary system to decay from a given orbit to merger. In this example we get that it would take a trillion trillion years. This entire epoch of our Universe is only around 12 billion years. So in reality, this kind of system will never merge.



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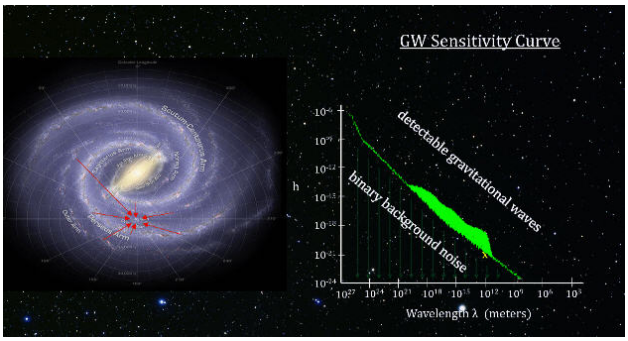
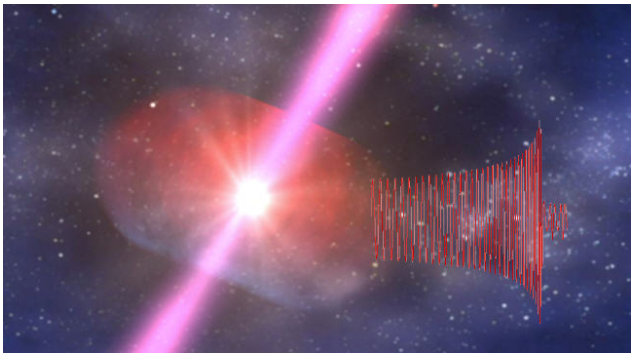
Binary systems like this one are plentiful and all around us. There are literally billions of them sending gravitational waves our way from every direction. But the gravitational waves they create are weak and totally indisguisable from one another. They just wind up contributing to a background noise level. In our GW sensitivity graph we see that in order to detect a gravitational wave, a binary system will have to create waves with greater amplitudes and higher frequencies to generate smaller wavelengths than the noise level marked in green. To stand out, a binary system is needed that can achieve much higher velocities. And as we have seen from our example, the large diameters of stars prevent them from ever getting close enough to reach the needed velocities.

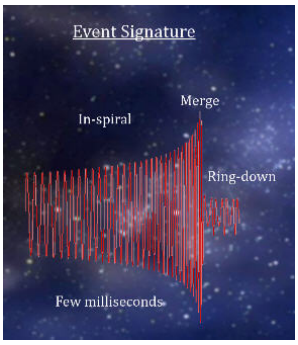


But rotating neutron stars might be small enough to achieve the needed speed. **[**We covered neutron stars in the "Globular clusters and Supernova" chapter of our "How far away is it" video book. Annotated with link to video segment.**]**

Here's a system with two equal-mass neutron stars that have reached the point where they are whirling around each other 10,000 times a second. The stars merge in a few milliseconds, sending out a burst of gravitational waves and a brief, intense gamma-ray burst.

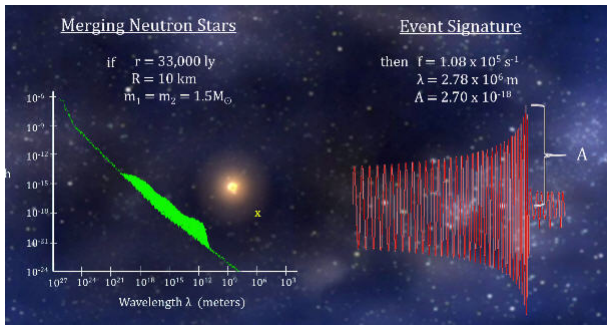
[Music: *Beethoven - The Creatures of Prometheus***]**





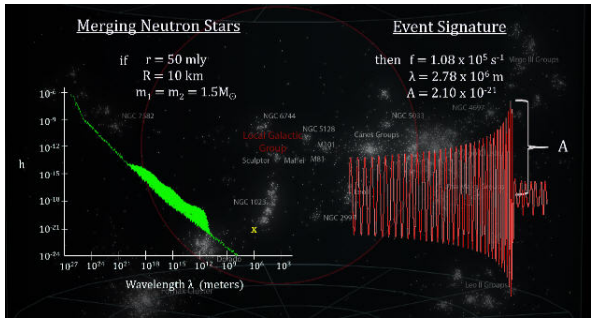
Here we have the signature curve for this coalescing binary neutron star scenario. You can see the three phases: the in-spiral; the coalesce or merger; and the ring-down to an object – most likely a Kerr Black Hole - that is no longer asymmetric and therefore no longer radiating gravitational waves. If we fed the waveform into an audio generator, it would sound like this {chirp}. We call it the chirp.

The mass of a typical neutron star is 1.5 times the mass of the sun with a radius of only 10 km. If the system is 33 thousand light years away (an average distance for a Milky Way object), it would give us a theoretically detectable wavelength and amplitude.

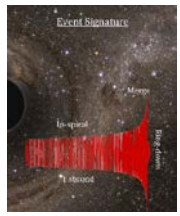


But coalescing neutron stars are not common events. **[**You'd have to have two massive stars both ending with supernovas that left them still in orbit around each other.**]**Astronomers estimate that there might be one of these neutron star mergers every 50 years inside the Milky Way. To get a higher rate, we have to move outside the galaxy into the Virgo Supercluster (our local supercluster that we covered in the 'How far away is it" video book (annotate link)).

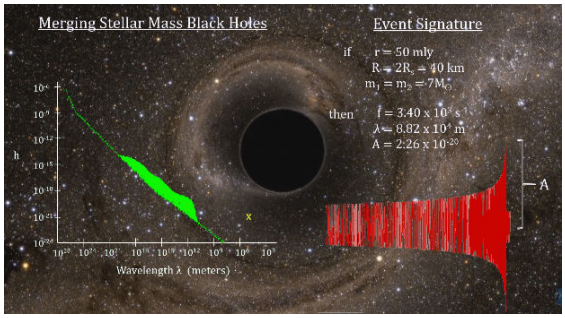
Within a 50 MLY radius we expect to have as many as 10 or more neutron star mergers per year because we're including thousands of galaxies. At this extended distance, the amplitude drops to the 10^{-21} range.



Stellar mass black holes can't get as close as neutron stars because their Schwarzschild radius is larger than the radius of a neutron star. But their mass alone can create larger GW amplitudes. Here we see a black hole merger simulation.

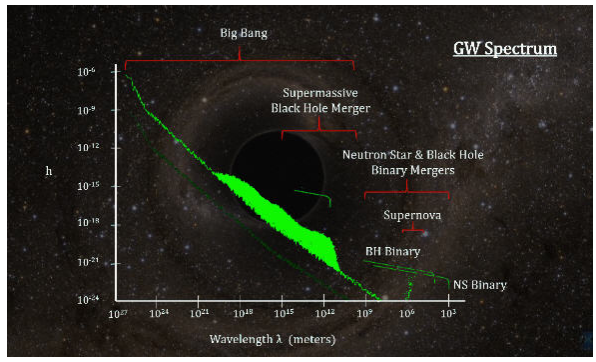


If each black hole has a mass of 7 Suns, the Schwarzschild radius is 20 km – twice the radius of the neutron stars. As the orbital radius shrinks to twice the Schwarzschild radius, and the black holes are approaching each other's photon sphere, their velocity approaches 70% of the speed of light. This produces a shorter GW wavelength with a larger amplitude – putting this kind of event well into the theoretically detectable area.



A number of other major cosmological events can also create gravitational waves. Here is a chart of some of the events and their expected wavelengths and amplitudes:

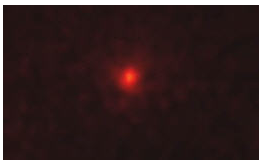
- Supernova
- Binary mergers like the ones we've analyzed
- Supermassive black hole mergers
- And remnants from the Big Bang



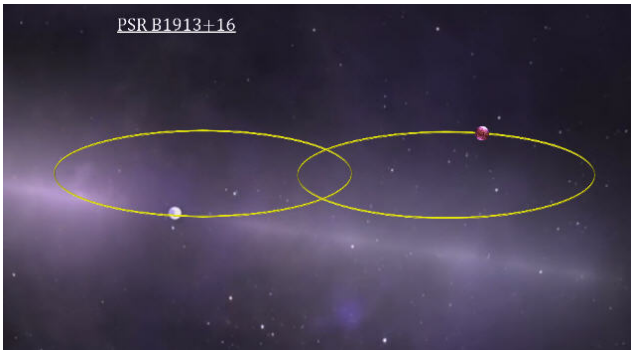
Gravitational Wave Evidence **[Music**: *Offenbach - Barcarolle - The Tales of Hoffman***]**

In 1974, 58 years after Einstein predicted the existence of gravitational waves, two radio astronomers (Joe Taylor and Russell Hulse) were looking for new pulsars using the 305 meter Arecibo Radio Telescope in Puerto Rico. They found one. It's named PSR B1913+16 and it lead to the first indirect verification of Einstein's prediction.

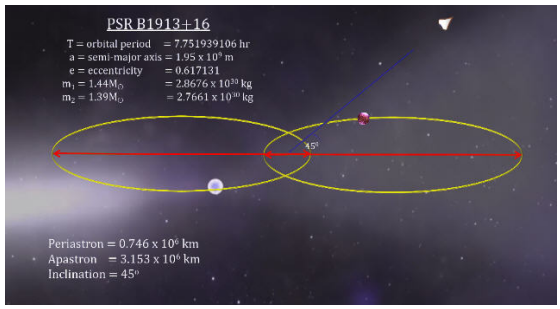
You'll recall from the "Globular Clusters and Supernova" chapter in the "How far away is it" video book (annotate link), that a pulsar is a rapidly rotating neutron star with a powerful magnetic field that accelerates charged particles as it rotates. The accelerating charged particles produce electromagnetic radiation primarily at radio wavelengths. The result is a sort of magnetic lighthouse, which if aligned correctly, flashes in our direction twice each cycle. These signals are highly regular. In fact, pulsars are some of the best clocks in nature, and this allows extremely precise measurements of their motion.



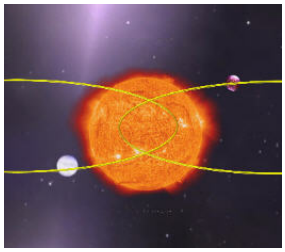
This one was pulsing every 59 milliseconds indicating that the pulsar rotates 17 times per second. But Hulse and Taylor noticed that the pulses varied regularly every 7.75 hours with pulses arriving 3 seconds earlier at some times relative to others. This meat that the pulsar was in an elliptical orbit with another neutron star! This was the first binary neutron star system ever discovered.



Detailed analysis identified the shape and size of the highly elliptical binary orbits. Using the orbital motion, they calculated the star masses; their closest approach (called the pariastron (**pare**-e-astrin)); and their furthest distance apart (called the Apastron) as well as the system's inclination.

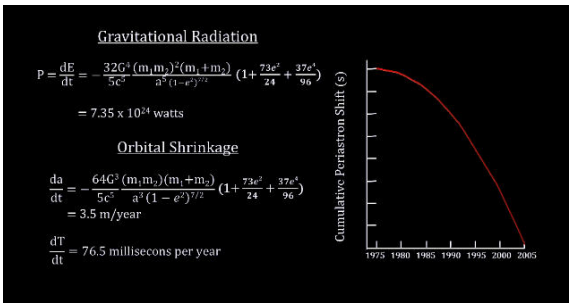


[These might look like large system, but in fact the pariastron distance is only 7/10 the diameter of our Sun.

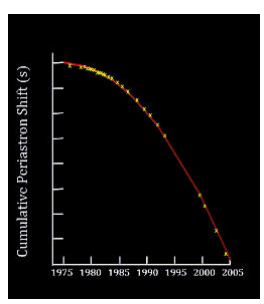


]

With this information and the GW equations, they were able to calculate the amount of gravitational radiation; the expected decay of the orbit due to the lost gravitational energy; and the corresponding reduction in the time it takes per orbit. This graph maps the accumulated reduction in orbital periods against time assuming that Einstein's GW equations are correct.



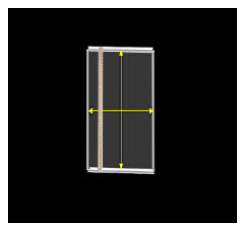
Hulse, Taylor and others have studied this binary system for 40 years now. This graph records their measurements. We see that the measurements fit the theory perfectly. **[**Joe Taylor and Russell Hulse received the Nobel Prize for this discovery in 1993.**]** This gave scientists confidence that Einstein's gravitational waves do indeed exist.



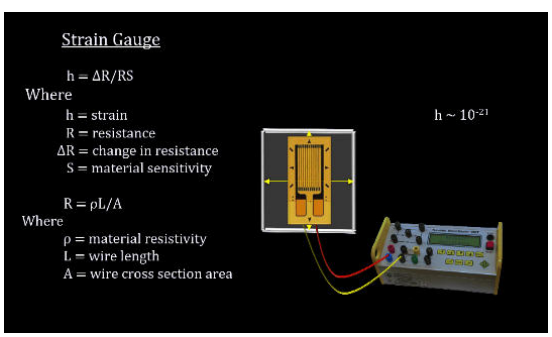
[The system is expected to merge in around 300 million years. Because neutron stars are the end product of supernova explosions, it was thought any binary partner would be ejected when the other explodes - making binary neutron stars unlikely. So this discovery showed that they do exist and they will merge in a timeframe that should make the event quite common – say 3 or 4 per year somewhere in the Universe.**]**

GW Direct Detection

But direct detection remained tricky for two main reasons. One is that the amplitudes of the waves are so small and the other is that the measuring sticks you might use to measure a change in length are changed themselves. In other words, the changed length will still read out as one meter.



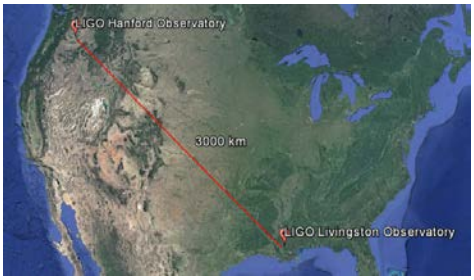
But the stretching and squeezing does put a strain on the plate that can be measured with an attached wire that acts as a resistor. It's called a strain gauge. If we attach wires along the plate instead of meter sticks, we can measure changes in the resistance of the wire as it is stretched and squeezed. A longer thinner wire will provide more resistance to an electric current, and a shorter fatter wire will provide less resistance to an electric current thus giving us a measure of the strain. Unfortunately, this technique is literally millions of times too insensitive to measure the tiny GW amplitudes **h**. But this technique is why we call **h** a measure of strain.



LIGO **[Music**: *Mozart - Symphony No 40 First Movement*]

Michelson Interferometers look like the best chance to detect these waves. You'll recall that we covered interferometers in the first chapter of this video book (annotate with link). The arms on that one were 11 meters long and its sensitivity was nowhere near what is needed for GWs.

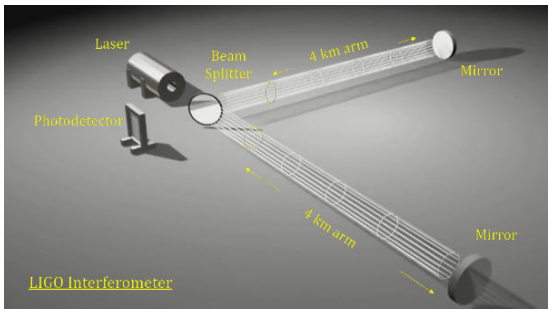
Today we have LIGO (the Laser Interferometer Gravitational-wave Observatory) that has built two identical interferometers 3000 km apart: with one near Hartford, Washington and the other near Livingston, Louisiana.



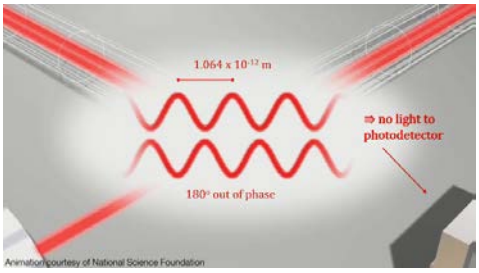
Here are the L-shaped LIGO instrument components:



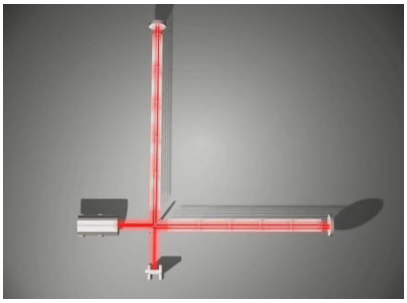

- It has a powerful near-infrared laser with an output, after amplification that reaches 200 watts of 1064 nanometer light.
- The beam splitter and mirrors (that act as test masses) are 40 kg objects suspended via fused silica glass fibers to minimize noise due to vibrations. Additional internal and external active vibration minimization technologies eliminate the effects of everything from nearby traffic to lunar tidal forces.
- The 4 km arms are 10,000 cubic cm of ultra-high vacuum equal to 1 trillionth of an atmosphere. In addition, each arm contains reflection mirrors that route the light back and forth inside the arms 280 times before it hits the exits for recombination.
- The photodetector is a state of the art indium-gallium-arsenide photodiode array with a high quantum efficiency designed to detect extremely small amounts of light at a wavelength of 1064 nm.



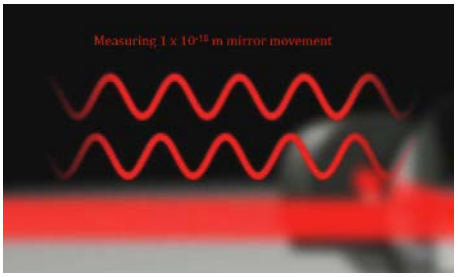
The laser light is split and sent to the two mirrors. On return, they are recombined and sent to the photodetector. The beams returning from the two arms are kept out of phase so that when the arms are both in synch (as when there is no gravitational wave passing through), their light waves subtract, and no light arrives at the photodetector.



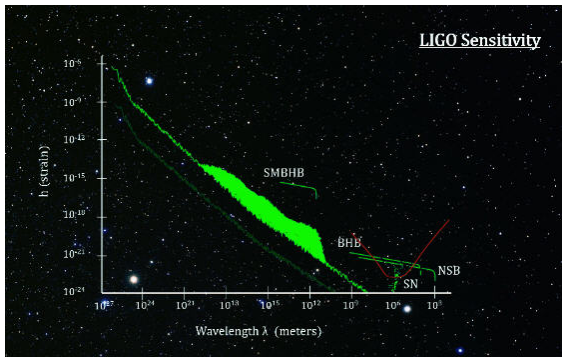
When a gravitational wave passes through the interferometer, the distances along the arms of the interferometer are shortened and lengthened, causing the beams to become slightly out of synch. Hence, some light arrives at the photodetector, indicating a signal.



Given LIGO's extra 280 passes through the tube, a GW strain amplitude of 10^{-21} would displace the mirrors 10^{-18} m (that's one thousandth the diameter of a proton).

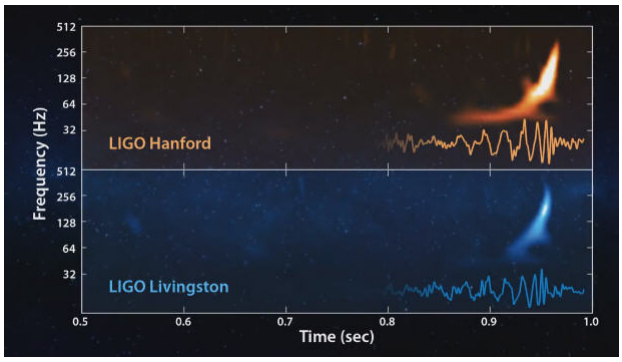


On our sensitivity graph, we see where LIGO's characteristics fit. This is a range where powerful binary system mergers within the Virgo Supercluster, our local supercluster [see How Far Away Is It – Virgo Supercluster with annotation] should be detectable.



The event - GW150914

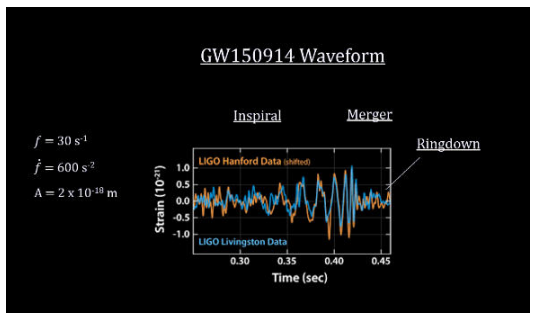
At 09:50:45 Coordinated Universal Time on the 14th of September 2015 a signal was detected by the LIGO detectors in Livingston and 6.9 milliseconds later in Hanford. It was a chirp signal that lasted just over 0.2 of a second. When we route the wave into a sound generator, here's what it sounds like.



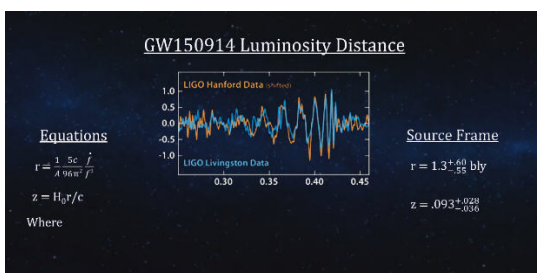
This plot combines the data from both sites. The waveform is consistent with coalescing masses with:

- a 10 cycle, 200 ms inspiral that gives us the frequency, the rate that the frequency changes, and a peak wave amplitude,
- a merger that takes around 2 ms
- and a ringdown as the coalesced objects cease to radiate gravitational energy.

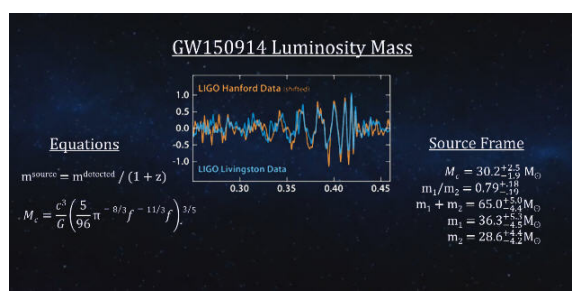
Detector noise introduces errors into all the calculations based on these figures. That's why we'll provide a range for each item.



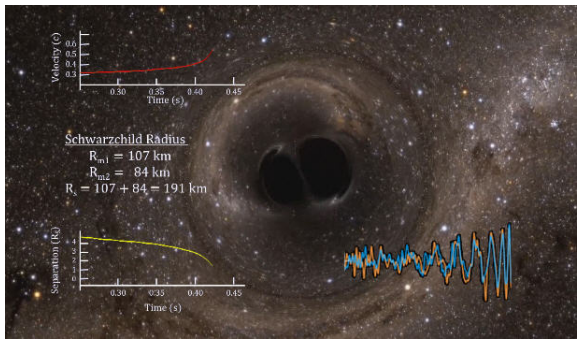
The amplitude and frequency data points give us the luminosity distance. It is important to note that gravitational waves experience redshifting as they travel across the cosmos just like light does. Having traveled around a billion light years, this wave would have experiences a redshift near .1. So the frequency we see here is a bit smaller than the frequency at the start of the wave's journey there.



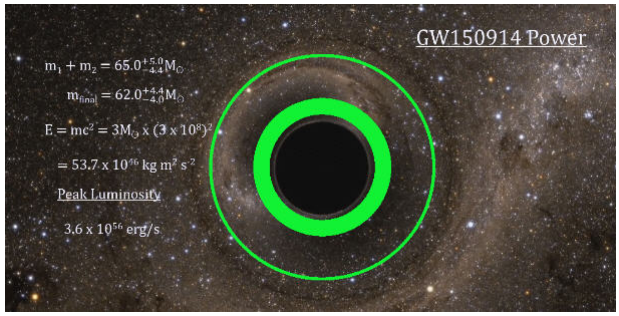
The frequency data also gives us the chirp mass. Taking the redshift and information gleamed from the merge and ringdown portions of the waveform, we get the binary system masses. These masses are too large for neutron stars that are only a few times the mass of the Sun. So we must be witnessing the merger of two large stellar black holes!

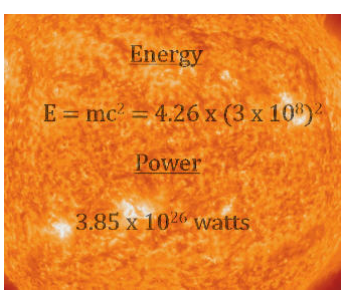


During the last 200 ms of their inspiral, the orbiting velocity of the black holes increased from 30% speed of light to 60% of the speed of light. Over the same period, the distance between the two black holes went from around a thousand km to just under 200 km when their event horizons made contact.



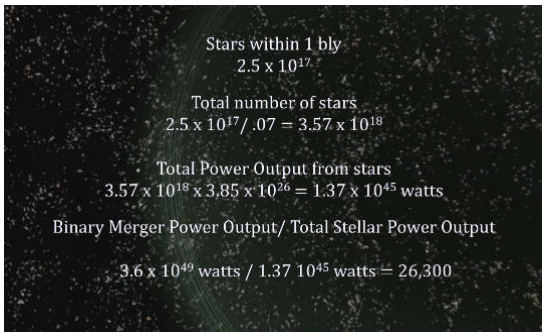
Modeling the final ringdown shows that the mass of the resulting Kerr Black Hole is around 62 solar masses. That's 3 solar masses less than the sum of the masses of the two inspiraling black holes. This mass was converted to the radiated gravitational energy. In other words, during the final 20 milliseconds of the merger, the power of the radiated gravitational waves peaked at about 3.6×10^{49} watts.





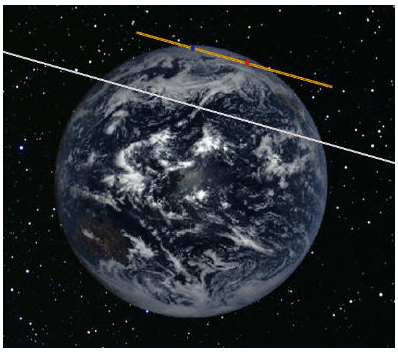
Let's take a second to get a feel for how large this number is. In our 'How far away is it" segment on Nearby Stars, we found that it converts 4.26 metric tons of matter into energy every second. The resulting power output is equal to 4 billion hydrogen bombs exploding every second! **[**3.85×10^{26} watts**]**. The Sun is an average star. So we can use this as an average stellar power output.

From our segment on Local Superclusters, we saw that there are 250 thousand trillion stars within one billion light years. This represented around 7% of the total number of stars in the Universe. We get the total power emitted by all stars in the visible Universe by multiply the average watts per star times the number of stars. The power generated by this merger of two stellar mass black holes is 26 thousand times greater than the combined power of all the light radiated by all the stars in the Universe. That's the signal we saw in September, 2015 a billion years after it happened (sound chirp).

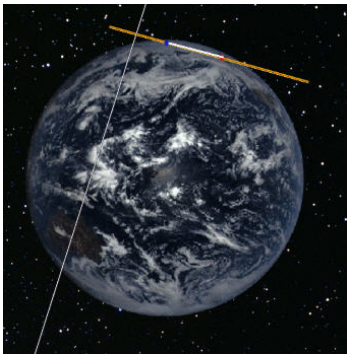


Binary Merger Sky Location [Music: *Puccini - Madame Butterfly - Un Bel Di Vedremo***]**

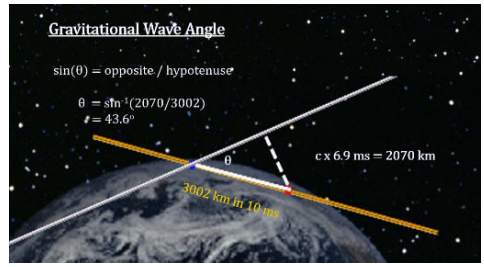
We have used the wave information to find the distance to this event. But the wave information does not tell us in which direction it came from because each interferometer is a whole-sky monitor with very little directional information. But having two detectors does give us some directional data. For example, if the wave came in parallel to the line between the two sites, the signals would have registered at the exact same time.



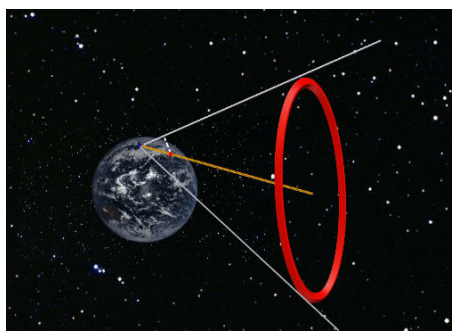
If the wave was perpendicular to the line, we would have seen a time delay of 10 ms, because the wave travels the through the Earth 3002 km at the speed of light.



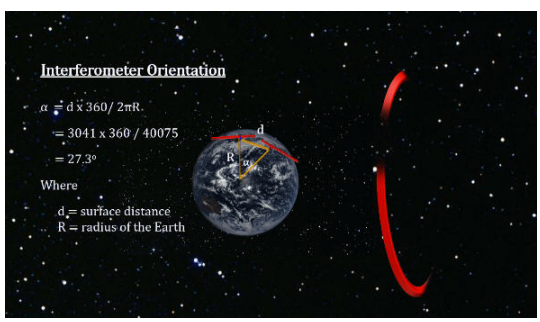
What we detected was a wave that came in at an angle that caused a delay of 6.9 ms. The dotted line represents the distance the wave had to travel for a piece if it to reach the Hanford interferometer. A little trigonometry gives us the angle.



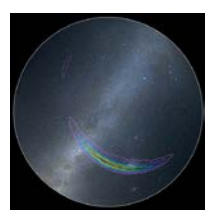
Of course this angle gives us a circle of possible directions.



Interferometers are most sensitive to waves that come in perpendicular to their two arms. Sensitivity drops off as the incident direction departs from the perpendicular. The curvature of the Earth gives the two LIGO interferometers an angle difference around 27°. This creates slight amplitude and phase inconsistencies across the two detectors that enable a narrowing of the probabilities to a smaller portion of the sky.



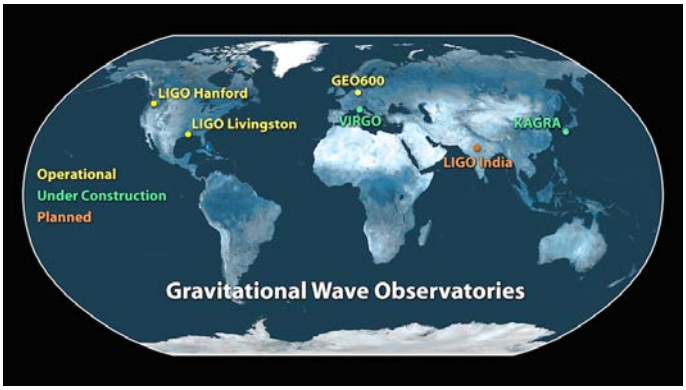
Here are the most probable directions as seen from Earth.



The Future

The best way to decrease the search area is to use a third gravitational wave detector to triangulate the source. Several are under construction or being upgraded to just that. Here is a map of current and future gravitational wave observatories on the Earth's surface.

[The Advanced LIGO is predicted to detect five more black hole mergers like GW150914 in its next observing campaign, and then 40 binary star mergers each year. Planned upgrades are expected to double the signal-to-noise ratio, expanding the volume of space in which events like GW150914 can be detected by a factor of ten. Additionally, Advanced Virgo, KAGRA, and a possible third LIGO detector in India will extend the network and significantly improve the position reconstruction and parameter estimation of sources.**]**



Impact on future cosmological observation

One of the greatest opportunities we have, now that we can detect gravitational waves, will be the ability to observe events that happened before light was traveling across the cosmos. The first 38,000 years after the big bang are known as "the dark ages". As you can imagine, there is a lot of guess work that goes into figuring out that period in our Universe's history was like. Gravitational waves created within the 'dark ages', may help us untangle that mystery. More time will tell.