Special Relativity

{Abstract: *In this segment of the "How Fast Is It" video book we cover the Special Theory of Relativity. We start with the Lorentz Transformations developed after the Michelson-Morley experiment showed that the speed of light was the same for all inertial observers. We then use light clocks to illustrate some of the most striking implications of these new transformations - starting with time dilation and space contraction. As we work through the special relativity effects, we review the physical evidence such as GPS satellites for time dilation and cosmic ray muons for space contraction. We then cover how we add velocities in such a way as to always come up with a number less than or equal to the speed of light. We then use the Large Hadron Collider at CERN to illustrate mass-energy momentum increasing without bound as speeds approach the speed of light. The last special relativity effect that we cover is the moving of simultaneity to the realm of the relative.*

*With this done, we cover Albert Einstein's motivation for his two Special Theory of Relativity postulates. One was driven by Maxwell's equations and the other was driven by the inability to detect the Aether. We then cover the geometry of space-time called Minkowski Space. We close with a description of the famous Twin Paradox. For that we use a 50-year trip to Vega and back.***}**

Introduction

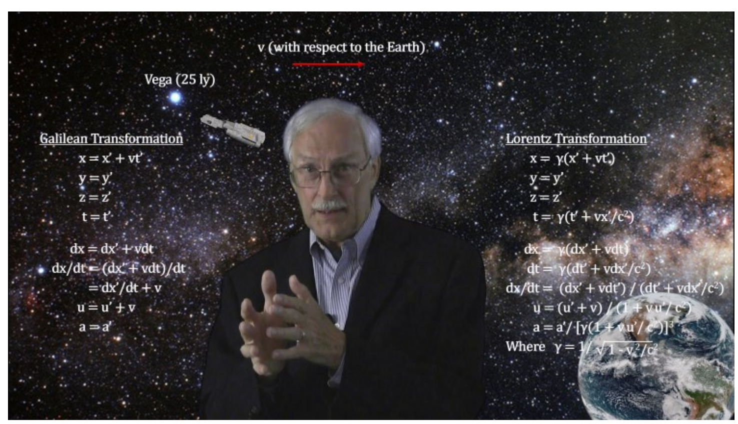
Hello, and welcome to our How Fast Is It segment on Special Relativity. In our last segment on the Speed of Light, we saw how the Michelson-Morley experiment showed that the speed of light was a constant for all inertial observers. This made it clear that the centuries old Galilean transformations between inertial reference frames were incorrect at speeds close to the speed of light.

New transformations were needed that had to satisfy two main requirements:

1) They had to add speeds in a way that always left the speed of light the same.

2) They had to produce the same answers that the Galilean transformations did when speeds were small compared to the speed of light.

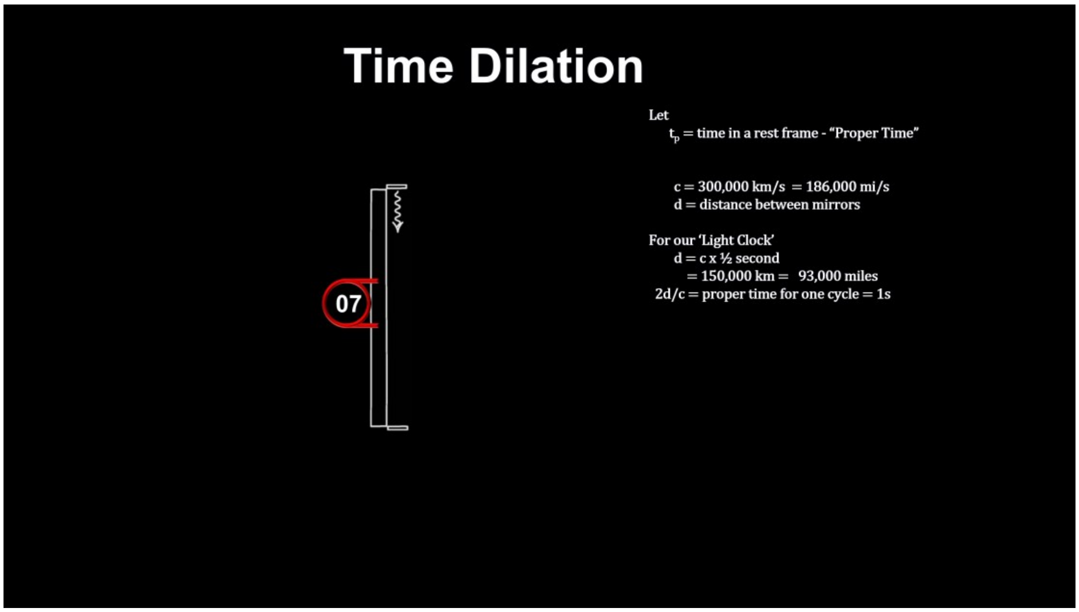
In the years following the experiment, physicists like George FitzGerald, Henri Poincaré and Hendrik Lorentz developed these new transformations - now called the Lorentz transformations. In this segment, we'll see how they needed to stretch time and shrink distance in order to meet these two requirements. And we'll cover how Albert Einstein put it all together with just two postulates.



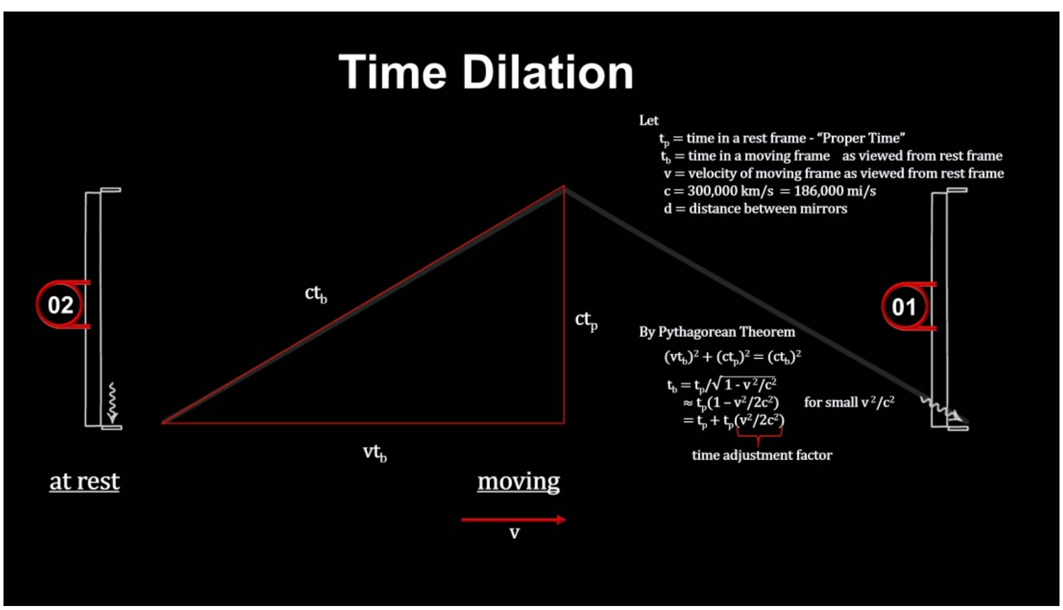
*[**Music**: Felix Mendelssohn - Concerto for Piano, Violin and String Orchestra: Mendelssohn composed this concerto in 1822 at the age of 13.]*

Time Dilation

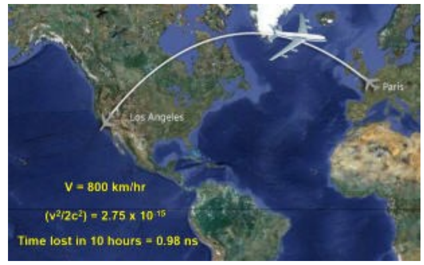
One of the most unusual consequences of the speed of light being a constant for all observers is that the flow of time itself was not immutable. It too must be relative to the inertial frame's motion. Here's a good way to see why this must now be the case. Here's a simple light clock. Light travels between mirrors at the end of a known length. For our purposes, we'll make it a very long length – say 150,000 km. That way, one round trip will mark 1 full second.



Now if we put the clock into motion with respect to a ground-based observer, we see that the light has to travel further, and because its speed is constant, it will take longer. The moving light clock is therefore running slower than a stationary duplicate. This is called time dilation. It's the geometry of the situation that gives us the conversion factor.



If I were on a 10-hour flight from LA to Paris traveling at 800 km/hr., I would lose just under a nanosecond – way too small to notice.



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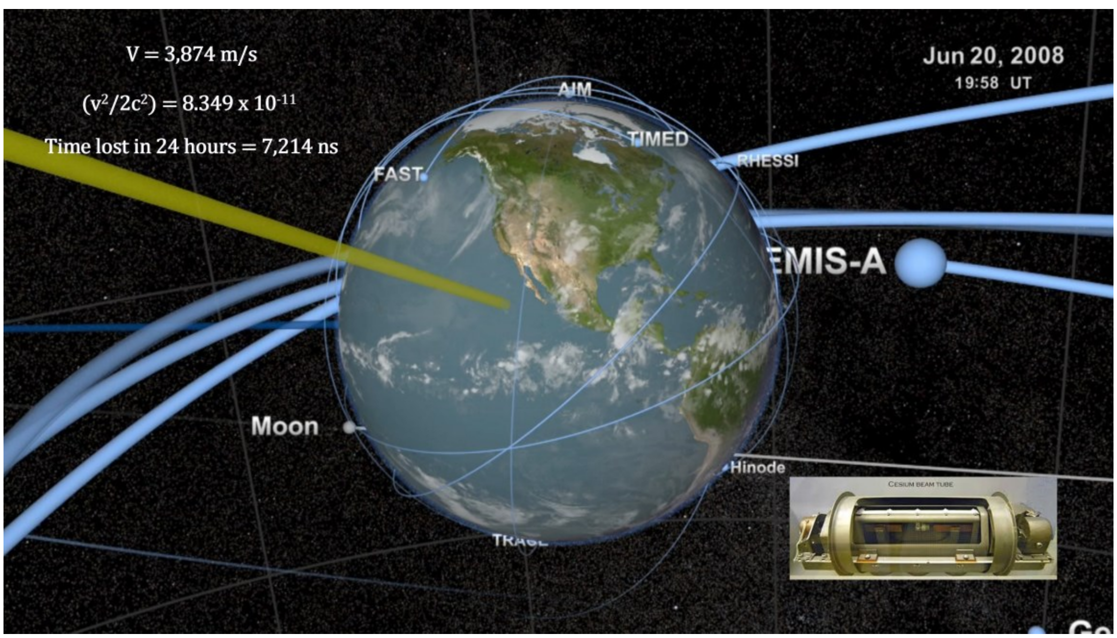
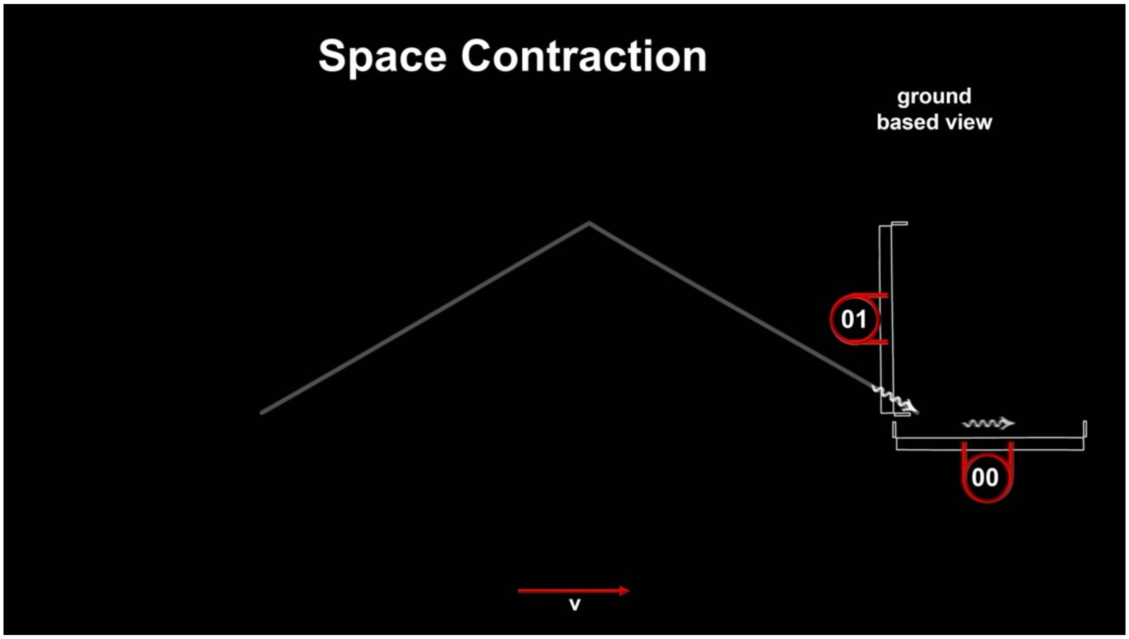
But for faster moving spacecraft it can be quite significant and critically important. For example, orbiting GPS satellites travel 3,874 m/s with respect to the Earth. They use cesium atomic clocks to keep time. These clocks use the exact frequency of the spectral line emitted by atoms of the element cesium. They measure time to within one second in 1,400,000 years. But these on-board clocks lose 7,214 ns a day due to time dilation. If the on-board clocks aren't corrected regularly based on Lorentz's time dilation formula, the position data they produce would be off by kilometers in less than a day.

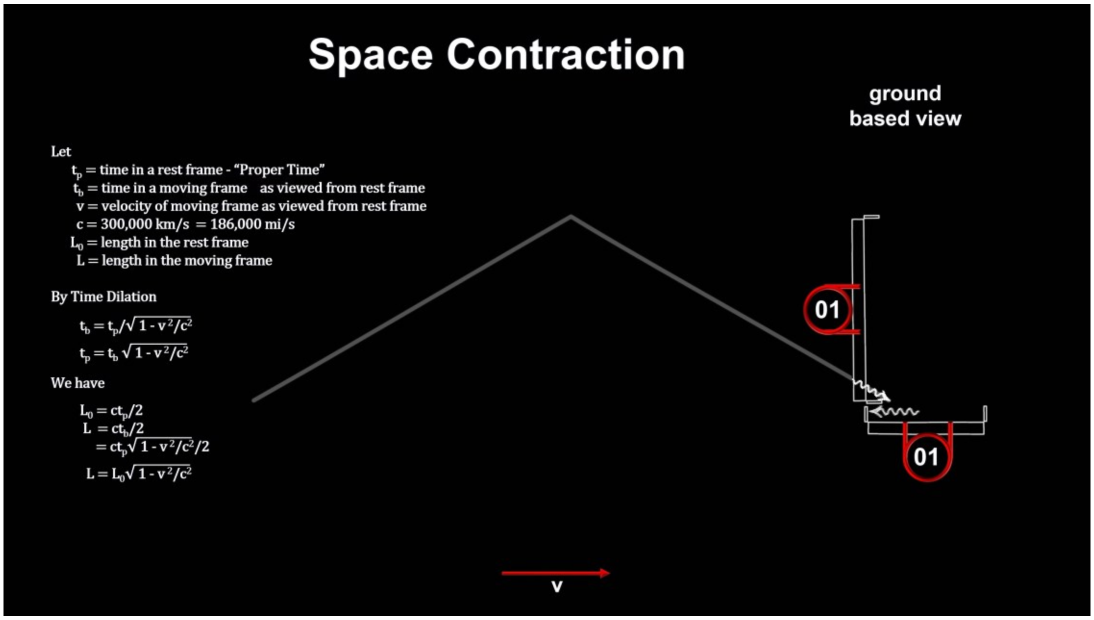


Space Contraction

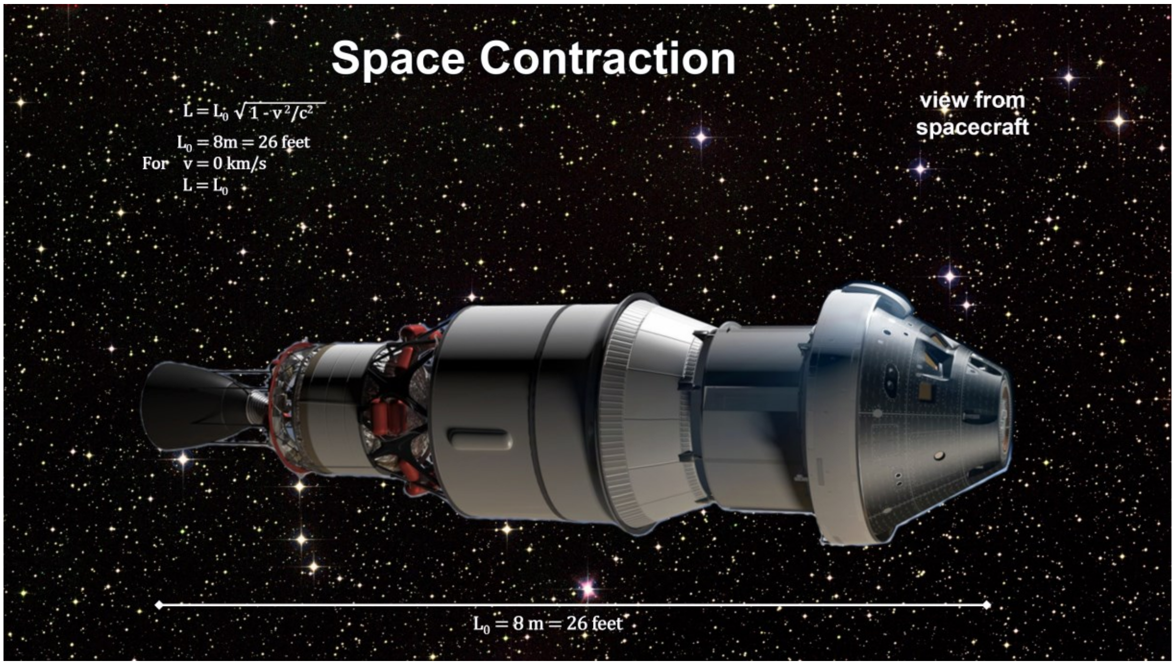
Another consequence of the speed of light being a constant for all observers is that the size of an object shrinks along the line of motion when viewed by an observer in a different inertial frame. To see how this must also be true, we put a second identical light clock on the moving frame, but this time we align it with the direction of motion instead of perpendicular to it. They tick at the same rate because they are identical clocks at rest together. Now if we put the whole system into motion in the direction of the horizontal clock, they will still tick at the same rate. They will also tick at the same rate for an observer on the ground, only they will tick slower as we have already established. But the motion of the horizontal clock's mirror makes the distance the light has to travel different from the distance the light travels in the perpendicular clock. Here we see the perpendicular clock ticking one before the light in the other clock has returned.

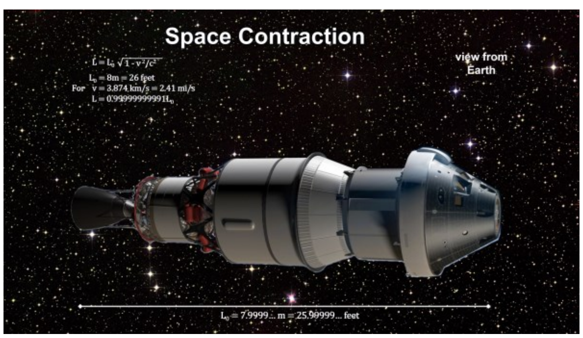


The only way to bring the clocks back into synch is for the length of the distance traveled horizontally to shrink. This is called space contraction.

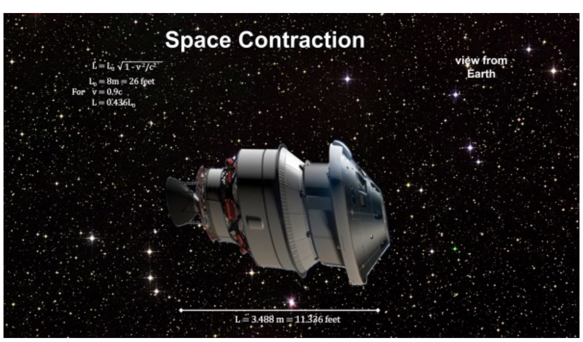


Here's a look at the new Orion spacecraft fully decked out for the planned long journeys to the moon, asteroids, and Mars.





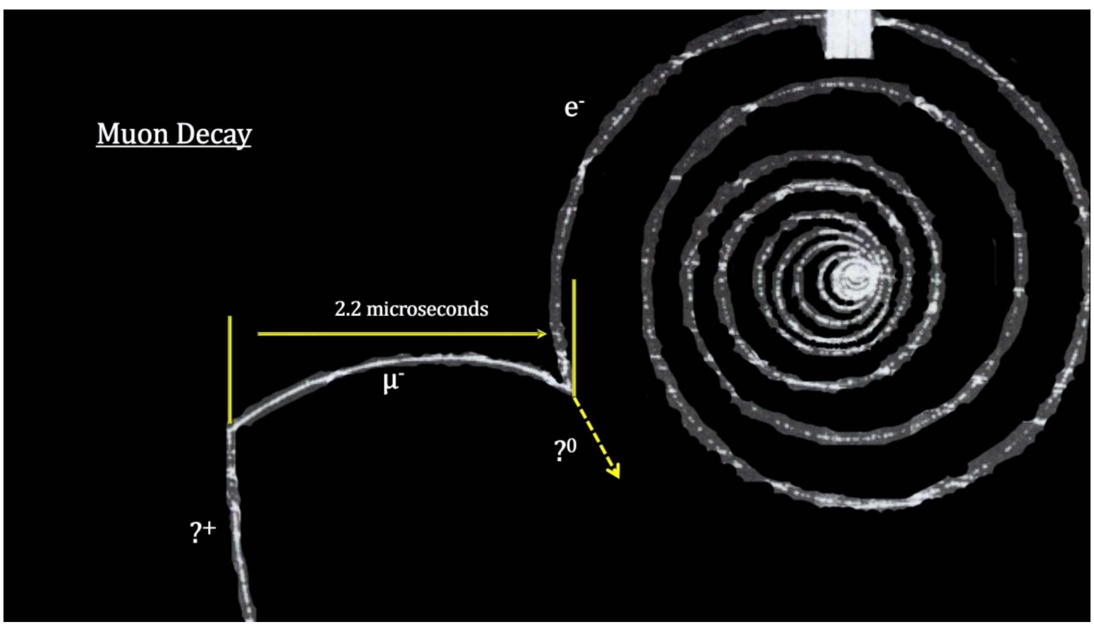
If it were traveling at the same speed as GPS satellites, it would look like this.



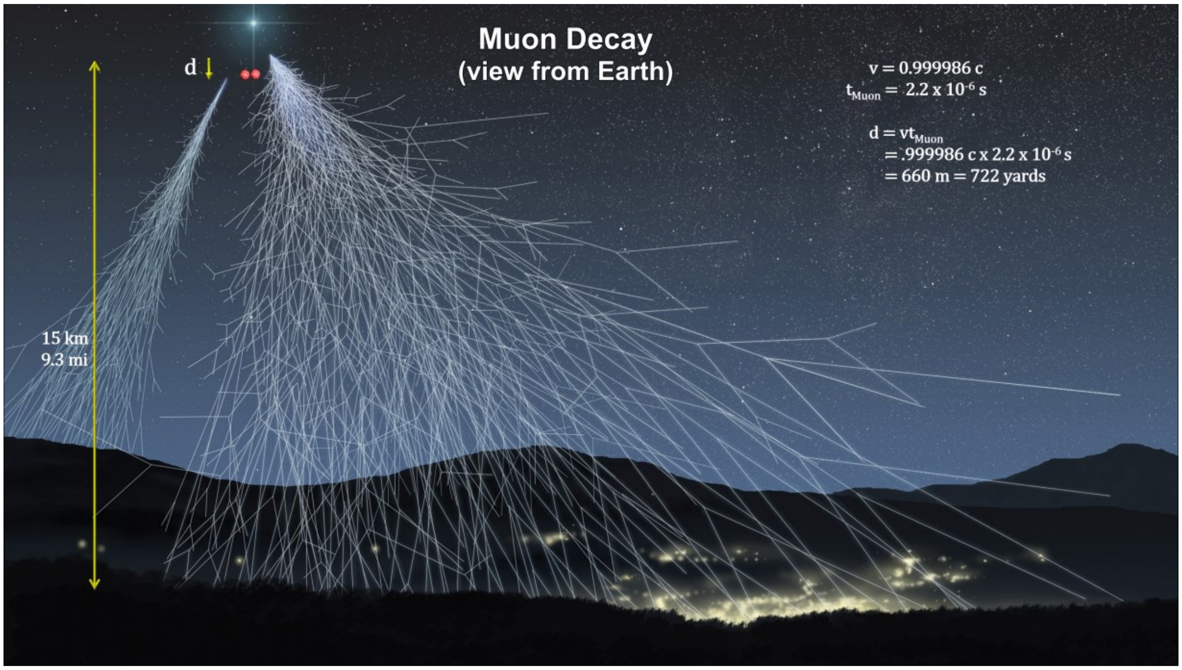
If it were traveling at .9 times the speed of light it would look like this.

And if it were traveling at .99 times the speed of light it would look like this.

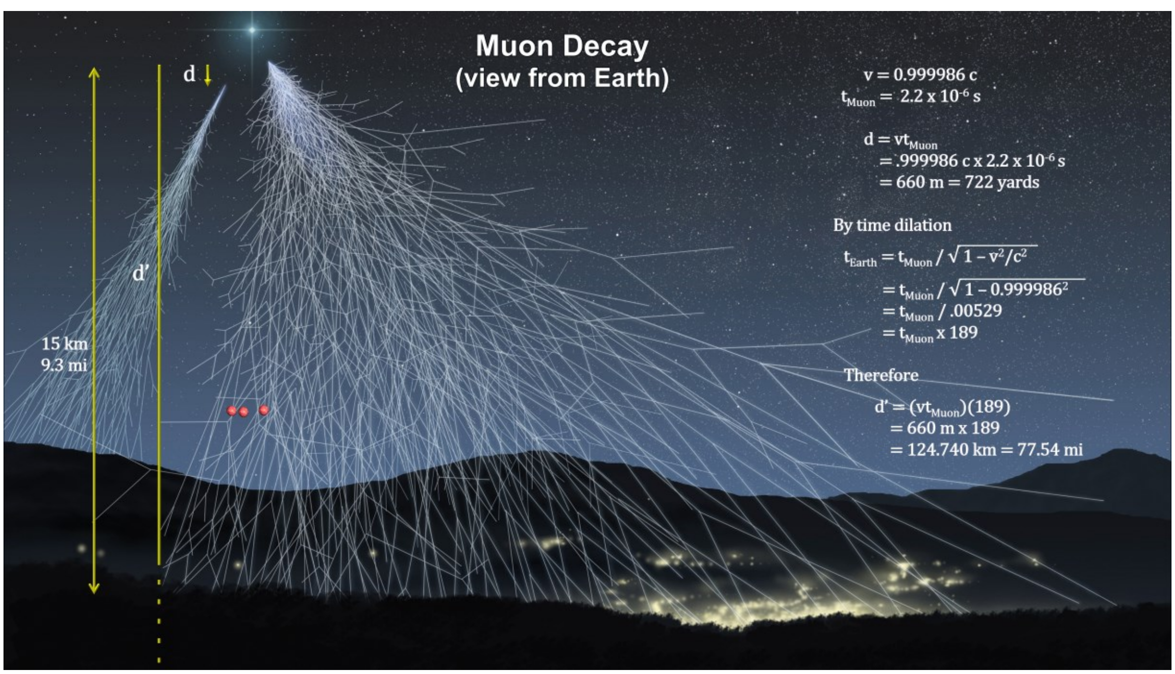
Unfortunately, we don't have the technology to take a measurable rod and accelerate it to almost the speed of light in order to measure how much is shrinks. But we do have the interesting example of muons created by cosmic rays in the upper atmosphere, 15 km above the ground. You'll recall from our segment on elementary particles in the 'How small is it video book', that we discovered muons with cloud chambers on mountain tops and in high flying balloons. We found that their half-life is only 2.2 microseconds.



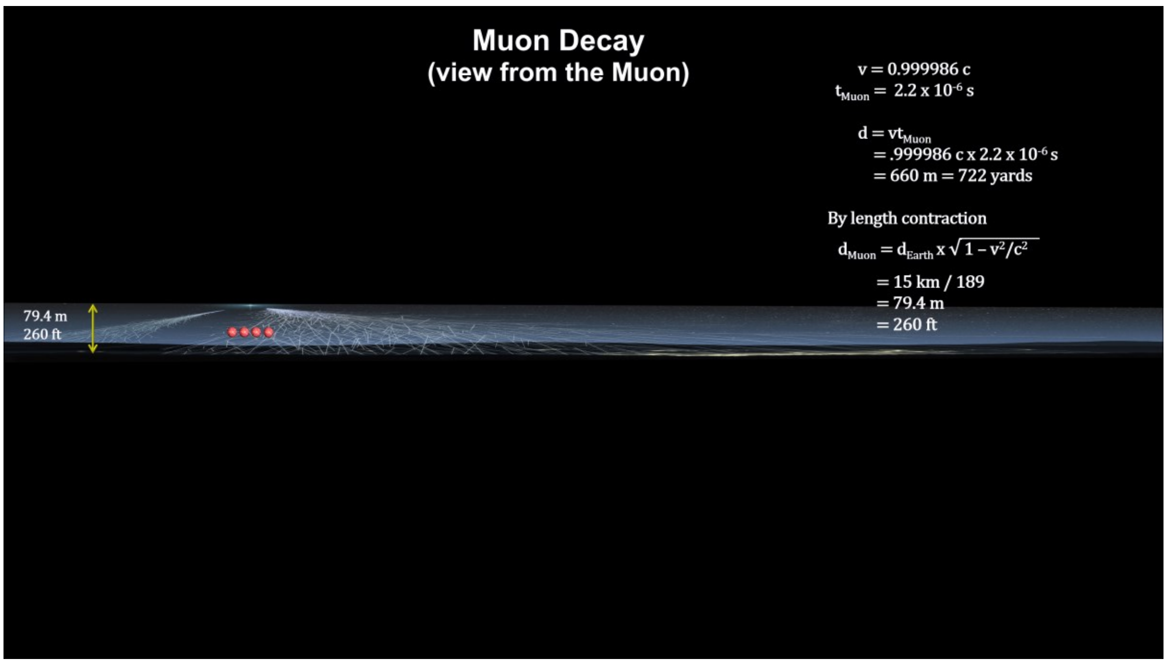
Traveling at near the speed of light, half of the muons would decay with each 660 meters traveled. That would leave few if any reaching the Earth's surface 15 km away. Yet we measure about 1 per square centimeter per minute. That's around 90% of the estimated muons produced in the upper atmosphere, which is way too high given the short half-life.



This high-count rate is explained using relativistic time dilation. The measured velocity of the created muons is .999986 c. So, by time dilation, the half-life would be 189 times longer. This gives the muons time to travel up to 125 km before half of them have decayed. And that's plenty of time for most the muons to reach the surface in line with the numbers observed.



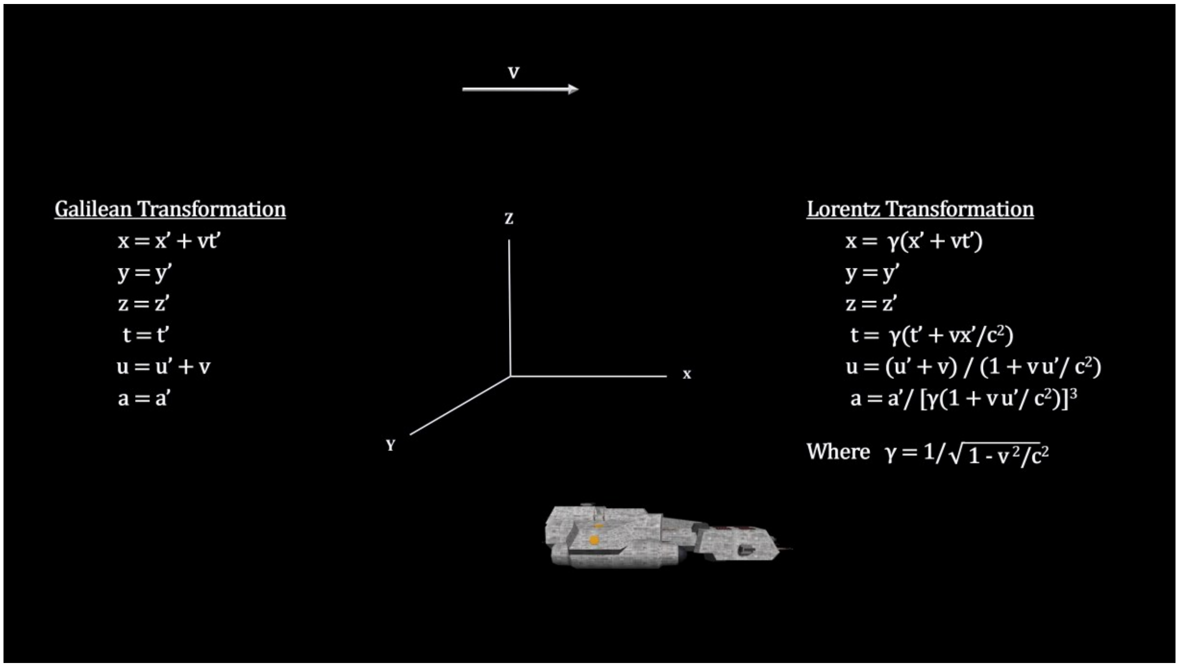
But what does this look like in the muon's own reference frame where it is standing still, and the Earth is approaching it at .999986 times the speed of light. It will only last 2.2 microseconds. How will it make it to the surface? This is where space contraction comes in. The 15 km of atmosphere, is shrunk by the same factor, from its frame of reference, that time is dilated, from our frame of reference. For the muon, the distance to the surface is only 79.4 meters. Most of them will reach it in 2.2 microseconds.



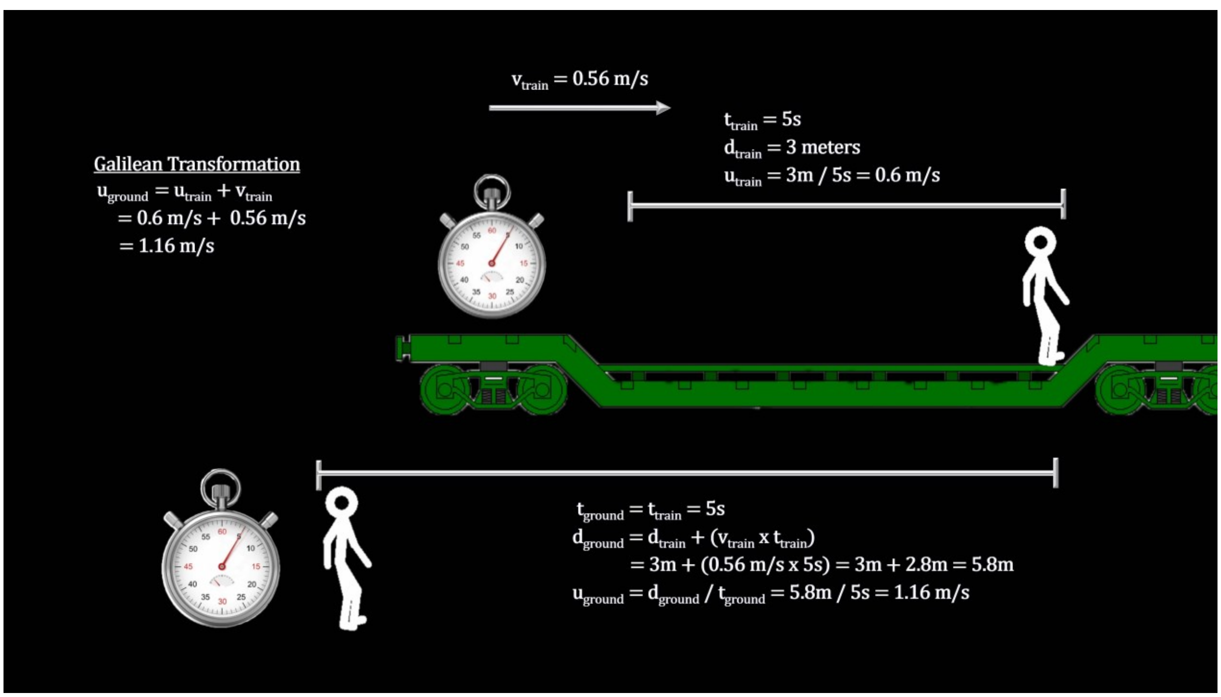
*[**Music**: Antonin Dvorák- Serenade for Strings, tempo di valse: Dvorák composed this serenade in just two weeks in May 1875. It remains one of the composer's more popular orchestral works to this day.]*

Velocity Addition

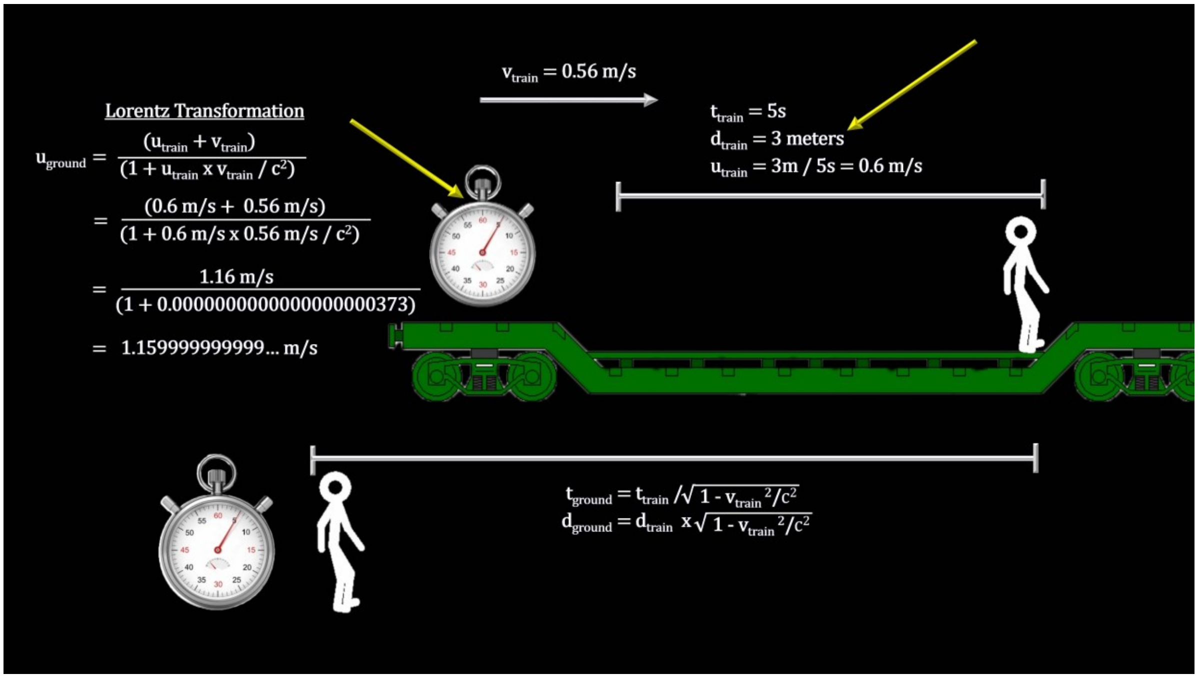
Now that we have a feel for the effects of special relativity, let's look at how fast things can go in this relativistic world. You'll recall from our previous segment on the speed of light that the classical transformations simply added the speed of movement in a reference frame to the speed of the reference frame itself. The Lorentz transformation takes this and divides by a reduction factor. This reduction factor reduces the old sum by just enough to make the speed of light a constant no matter how fast things are moving.



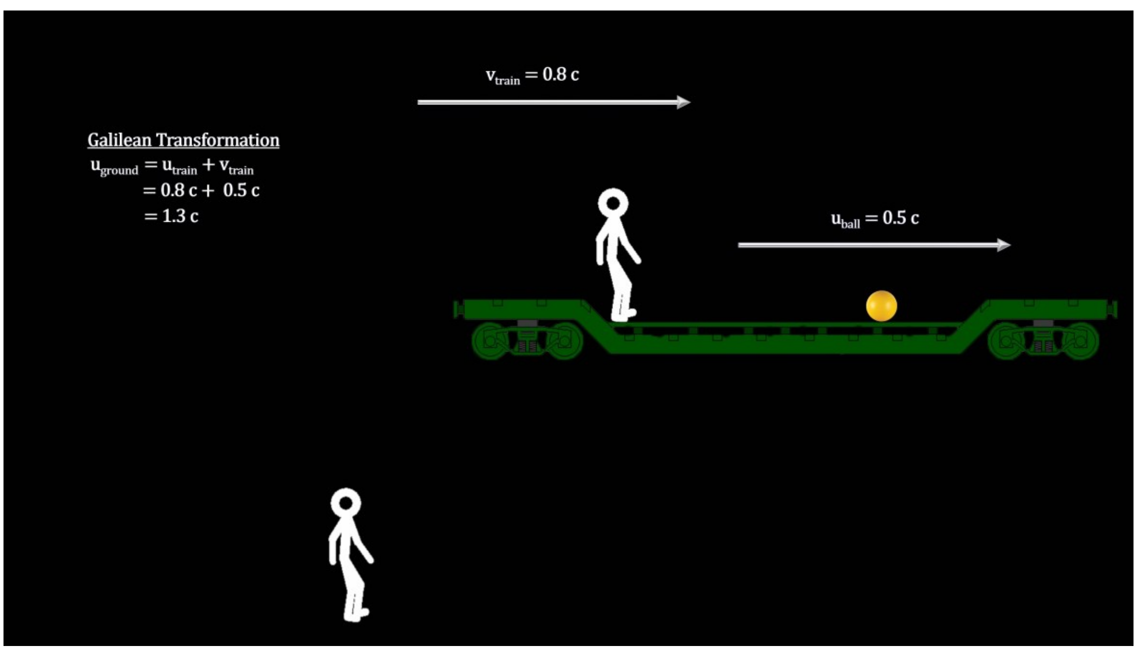
If we apply this to the example we used in the previous segment, we get speeds that are so close to the Galilean answer that we can hardly measure the difference. Here we see the person walking on the moving train. We simply added his walking speed to the train's speed to get the speed seen by the observer on the ground.



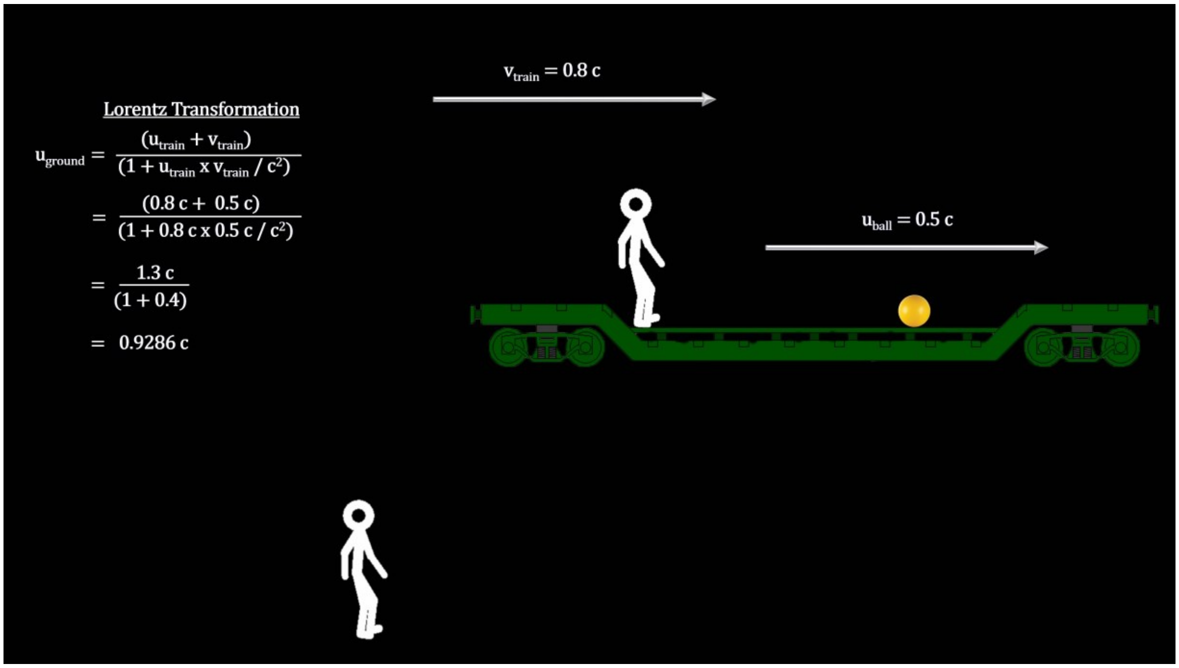
In Special Relativity, the clock on the train is running a bit slower and the distance the person walked is a little bit shorter as seen by the observer on the ground. So, his calculation of the total velocity is a bit less than the Galilean calculation. But at these speeds, it is too small a difference to notice.



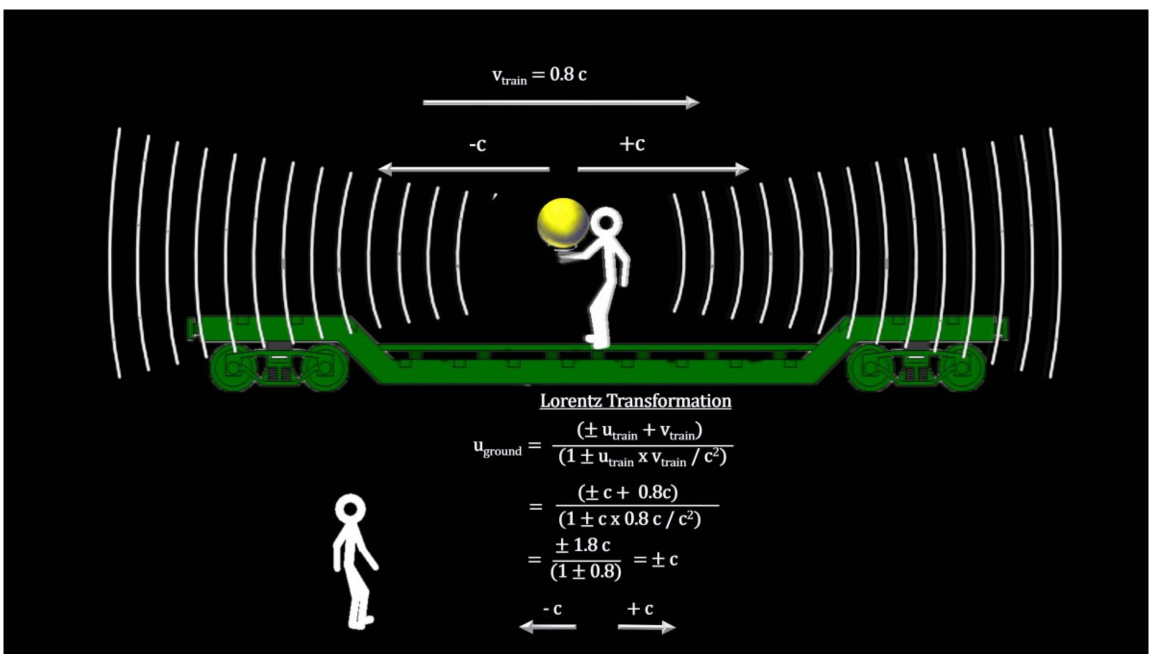
Now let's shift this super train into high gear and say its velocity is 8 tenths of the speed of light. [That would be 240,000,000 m/s.] And the person on the train kicks the ball at ½ of the speed of light. [That would be 150,000,000 m/s.] If the Galilean transformations were correct, the person on the ground would add these together and see the ball traveling faster than the speed of light.

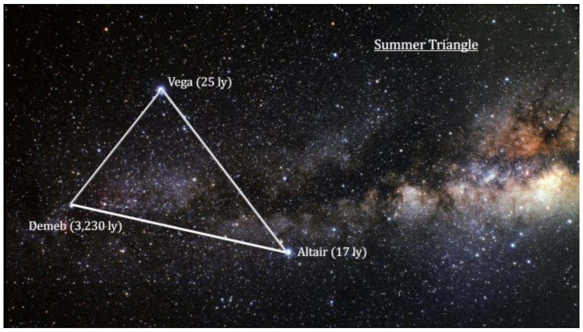


But to keep the speed of light a constant, the new relativistic equations tell us the person on the ground would see the ball traveling a lot slower than that at 9 tenths the speed of light. [That's 278,571,429 m/s.]

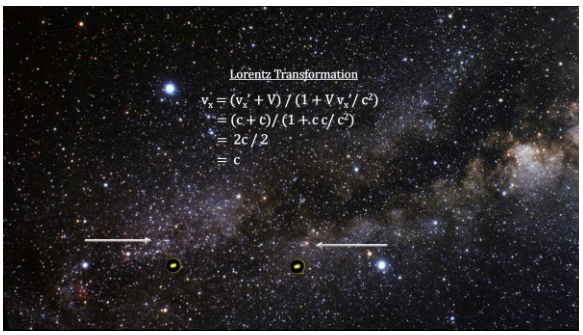


You can even see that, with the relativistic equations, if the train was traveling at .8 c, and a light was turned on, the person on the ground will see the light traveling at the speed of light just like the person on the train does.





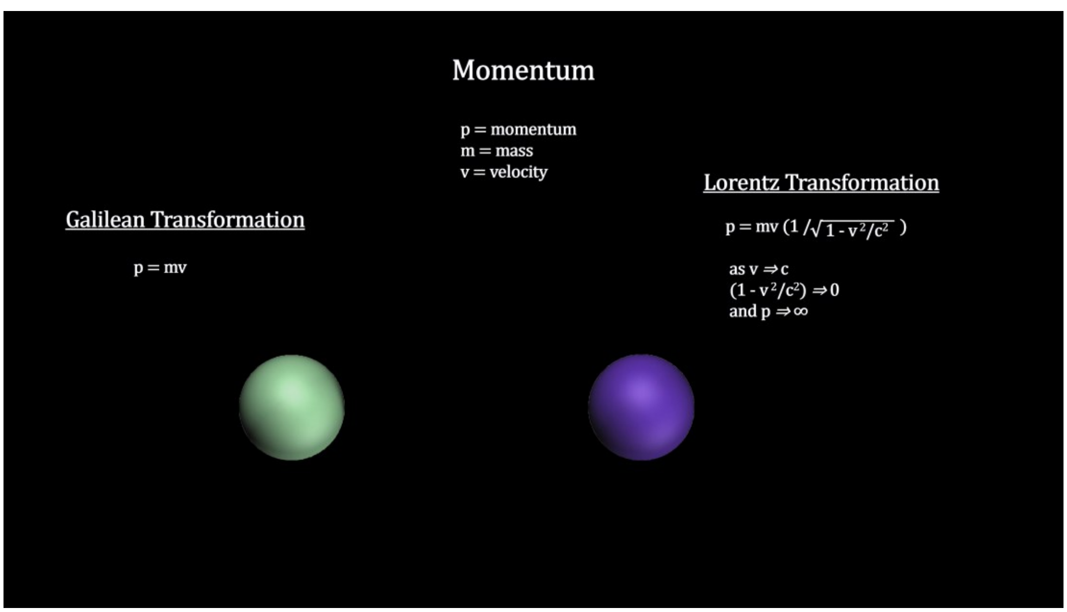
In a most extreme case, suppose we had two photons traveling towards each other. Here's the northern hemisphere's Summer Triangle with Vega, Altair, and Deneb.



Let's say one photon is traveling from Deneb to Altair and the other one is traveling from Altair to Deneb. Of course, each of them is traveling at the speed of light, so we would see their closing speed as 2 times the speed of light. But if we look at it from the point of view of one of the photons, we see that it sees then closing at the speed of light, not twice the speed of light.

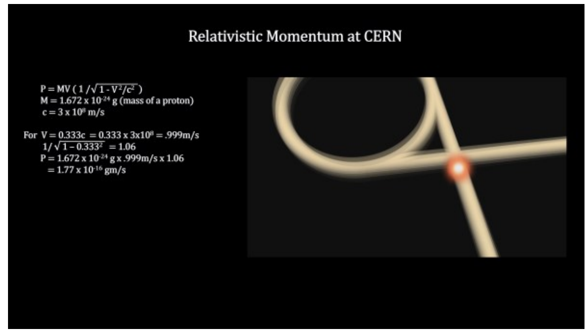
Momentum Increases

You'll recall from our segment on 'The Microscopic' in the 'How Small Is It' video book that momentum is mass times velocity. Another interesting consequence of the speed of light being a constant for all observers is that the momentum of an object increases without bound as its velocity approaches the speed of light. This increases the amount of energy it takes to increase its speed even more.

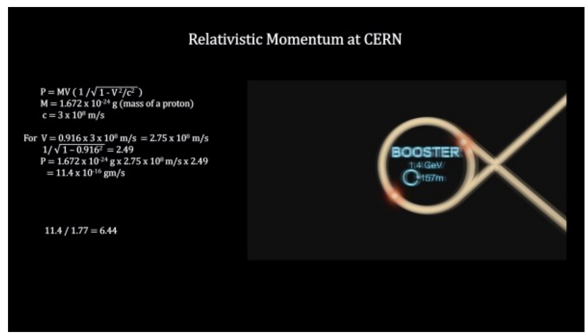


Our best example of this is the LHC at CERN where the Higgs Boson was found.

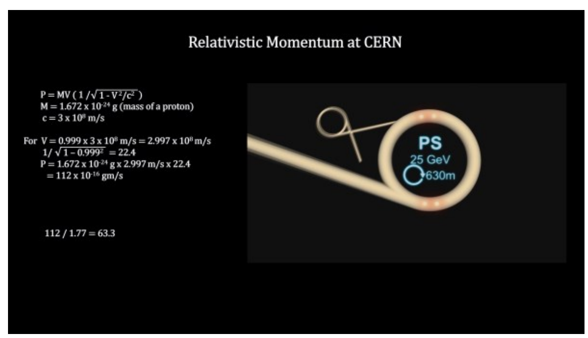
To begin with, protons are accelerated down a linear accelerator. By the time they reach the first cyclotron, they're traveling at 1/3 the speed of light. We can use this as the starting momentum [1.77 x 10^{-16} gm/s]. If we used Galilean transformations, we would be off by over 5% and the linear accelerator would not work.



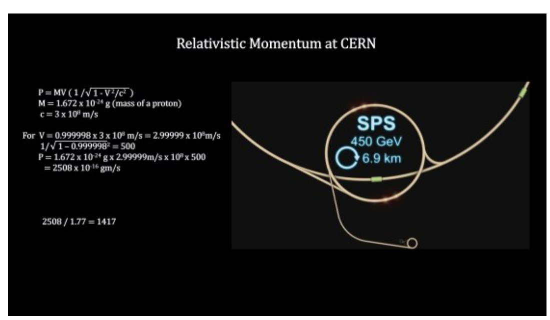
The first booster accelerates the protons to 91.6% of the speed of light. This speed more than doubles their momentum [to 11.4 x 10^{-16} gm/s].



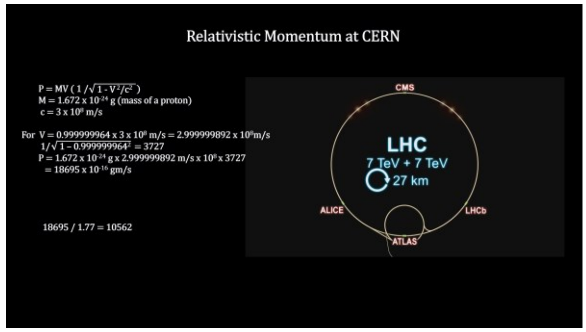
The protons are then flung into the proton synchrotron. They circulate here for 1.2 seconds reaching 99.9 % of the speed of light. At this speed their momentum has increased by more than 60 times [to 112 x 10^{-16} gm/s].



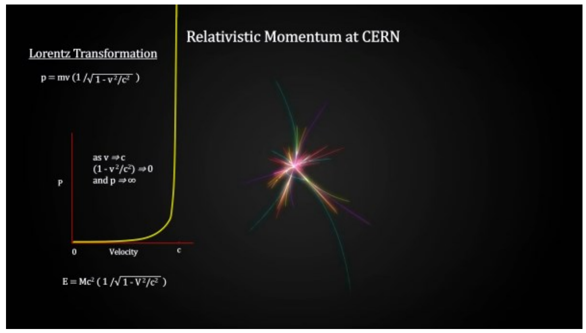
The protons are then channeled into the Super Proton Synchrotron. This is a huge ring, almost 7 kilometers in circumference. The best that it can do is to increase the velocity of the protons by around 8% to 99.9998% of the speed of light. The protons' momentum has now jumped by over 1,400 times its momentum at the end of the first acceleration. [2,508 x 10^{-16} gm/s].



Finally, the LHC accelerates these incoming protons to 99.9999964% of the speed of light. The proton momentum tops out at 10,562 times its original momentum [18,695 x 10^{-16} gm/s]. At this level, Galilean transformations would be off by over 99.9%.



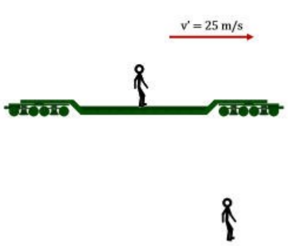
In fact, as the velocity gets closer to the speed of light, the momentum increases without bound, and the energy required to get it even closer to the speed of light grows to be more energy than exists in the universe. This is why we say that nothing with mass can ever reach the speed of light.



The operations of the LHC at CERN confirm this relativistic momentum increase just as the operations of GPS satellites confirm relativistic time dilation. And cosmic ray created muon's confirm relativistic length contraction.

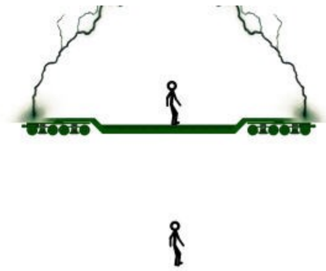
> ***[Music**: Edward Elgar - Cello Concerto: This piece was composed during the summer of 1919 at Elgar's secluded cottage, where he had heard the sound of the artillery of World War I rumbling across the English Channel at night from France in 1918.**]***

Simultaneity Lost

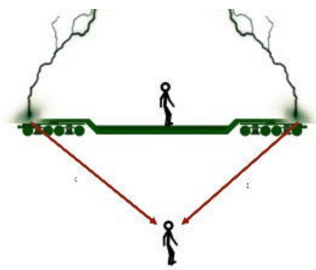


The last non-intuitive consequence of the speed of light being the same for all observers that we'll cover is that things happening 'at the same time' no longer has meaning across inertial frames of reference. In other words, there is no such thing as 'at the same time' for different inertial frames. To see this, we'll go back to our train example

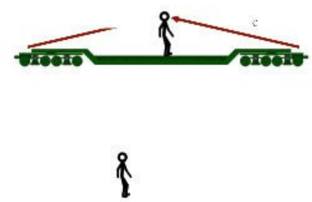
Here the observer on the train is in the middle of the car, and as he moves past the observer on the ground, lightning strikes the front and back of the train car.



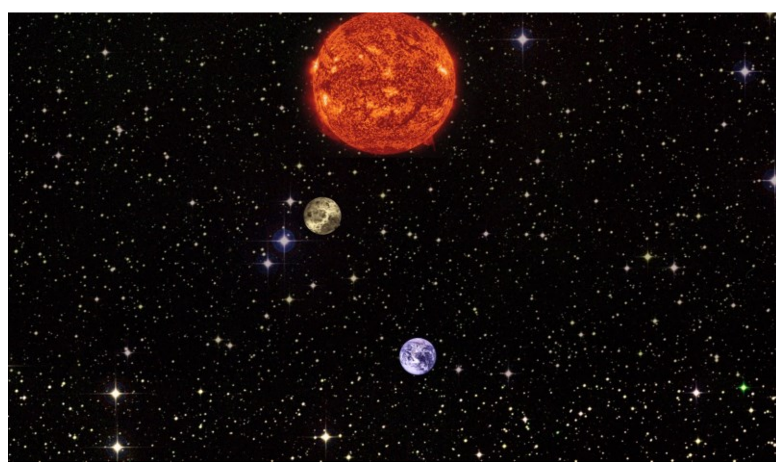
The light from both strikes travel the same distance at the same speed and reach the observer on the ground “at the same time”. He correctly concludes that lightning hit the front and the back of the train car at the same time.



But how does this look to the person on the train? In his reference frame, he is not moving forward. The person on the ground is moving backward. He knows that he is also the same distance from the two lightning strikes, and he knows that the speed of light is the same for both strikes. So, for the front strike to reach him first it must have happened first. If they had happened at the same time, they would have reached him at the same time.



Who is right? They are both right - in their own reference frame. They are simply measuring what they see. What is simultaneous at two locations in one reference frame may not be simultaneous in another reference frame. What happens in two places at the same time on earth, will not be happening at the same time if we’re viewing the events from Venus. Simultaneity is relative along with time, distance, speed, and momentum.

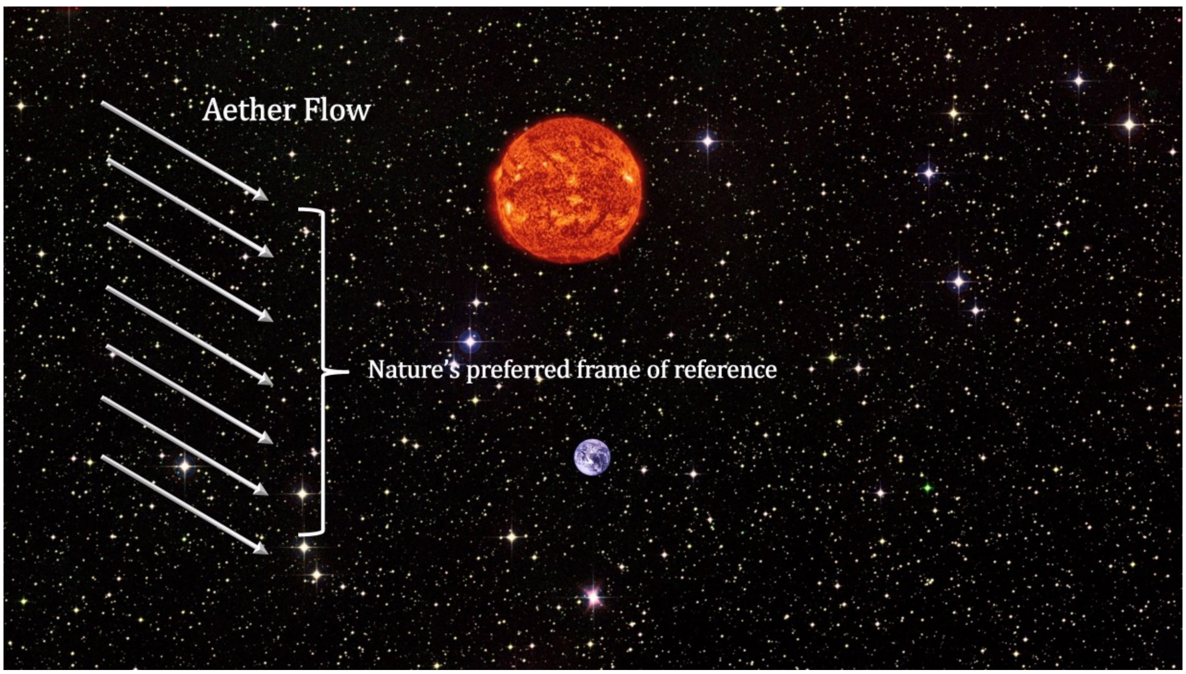


The end of the Aether

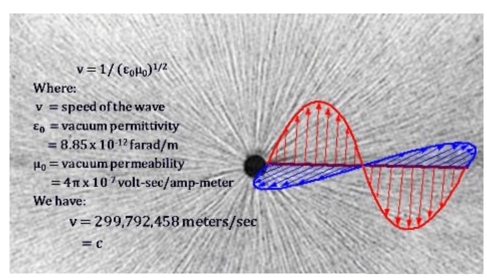
All these effects were well known before Albert Einstein's theory of relativity. And he was familiar with them. The thinking by 1905 was that there is still an aether that represented nature's absolute reference frame, but that time dilation and space contraction made it impossible to detect. The aether was held to exist for two main reasons:

- It gave light a medium to propagate through
- It provided for an absolute frame of reference like we use to have with the Earth.

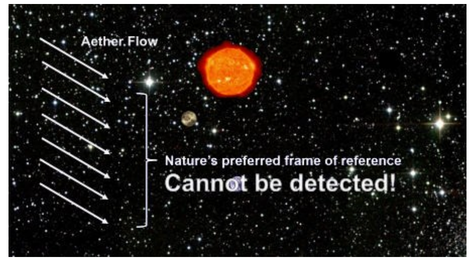
With it we could claim that there is such a thing as simultaneous events in two places in at least one (preferred by nature) reference frame.



Einstein parted with this theory for two reasons. One was that he studied James Maxwell's equations that showed the speed of light was the result of empty space's resistance to the formation of an electric field (permittivity) and a magnetic field (permeability) as we covered in The Microscopic segment of our "How small is it" video book. So, there was no need for a propagating medium like the aether. Empty space will do just fine. And relative motion would not change these basic properties of empty space thus making the speed of light the same for all observers.

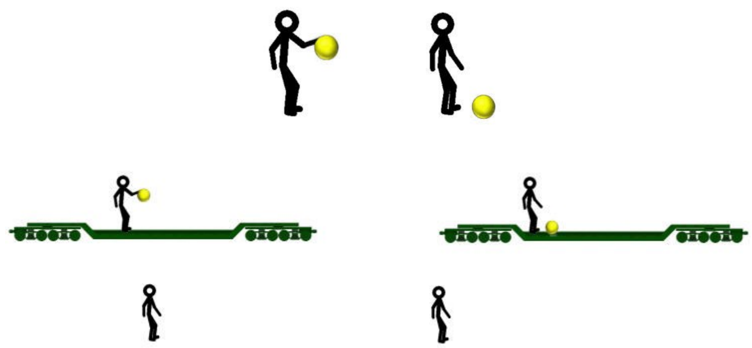


The other reason was more philosophical. What does it mean to postulate the existence of a thing and then explain that it cannot be detected? In the end, he accepted the idea that there is no universal frame of reference and all time, space, and simultaneity, were relative for all inertial frames.



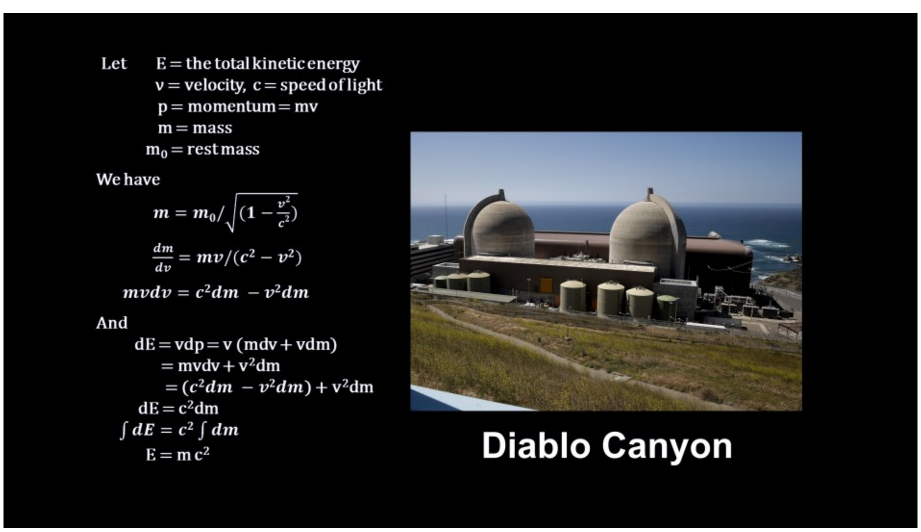
He articulated two postulates:

1) The laws of physics are the same in all inertial reference frames. This means that formulas like F = ma will hold in all inertial reference frames, and all experiments run the same in all inertial frames of reference. For example, if you drop a ball while standing up at home, it falls straight down to your feet. The same thing happens if you are standing on a moving train. The ball falls straight down to your feet.



2) The speed of light in a vacuum has the same value in all inertial reference frames.

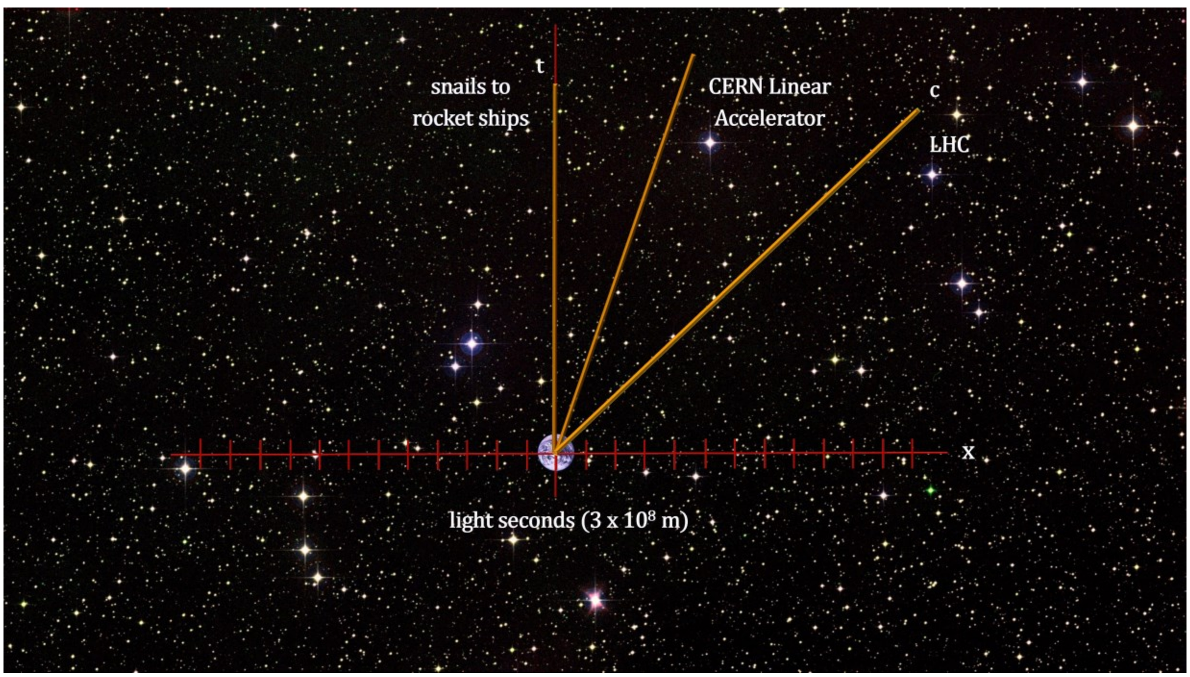
With these two postulates, Einstein regenerated all the relativity equations of Lorentz and others. It was his study of relativistic mass, momentum and energy that lead Einstein to the famous equation $E = mc^2$. No one else had made this relationship between mass and energy. Today's nuclear power industry is a testament to the accuracy of this special relativity formula.



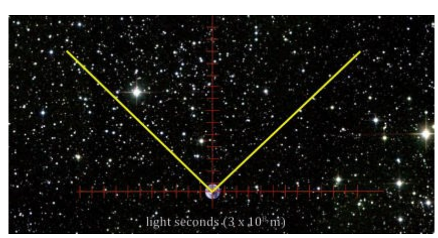
*[**Music**: Mozart - Eine Kleine Nachtmusik Romanze: The German title means "a little serenade. The serenade was completed in Vienna on 10 August 1787.]*

Minkowski Space-Time

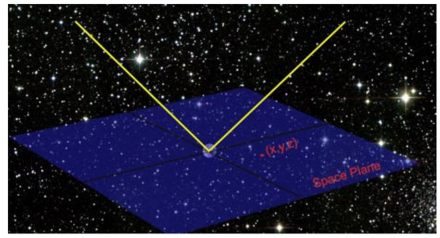
We can now graph light using our space-time coordinates. For this we use the distance light travels in one second for the units on the x axis. This gives us the line running diagonally up the right side for light moving in the positive direction. At this scale, we cannot distinguish the speeds achieved by snails, animals, cars, planes and rockets from the line that represents standing still. This is why we don't notice any of these relativistic effects in our daily lives. Here's the line for protons at the end of the LHC's linear accelerator traveling at 1/3 the speed of light. And the line for the protons in the LHC itself is indistinguishable from the speed of light.



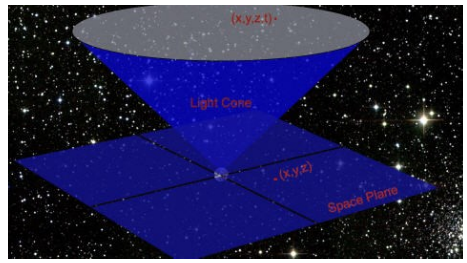
Now we can run the light line up the left side for light moving in the negative direction.



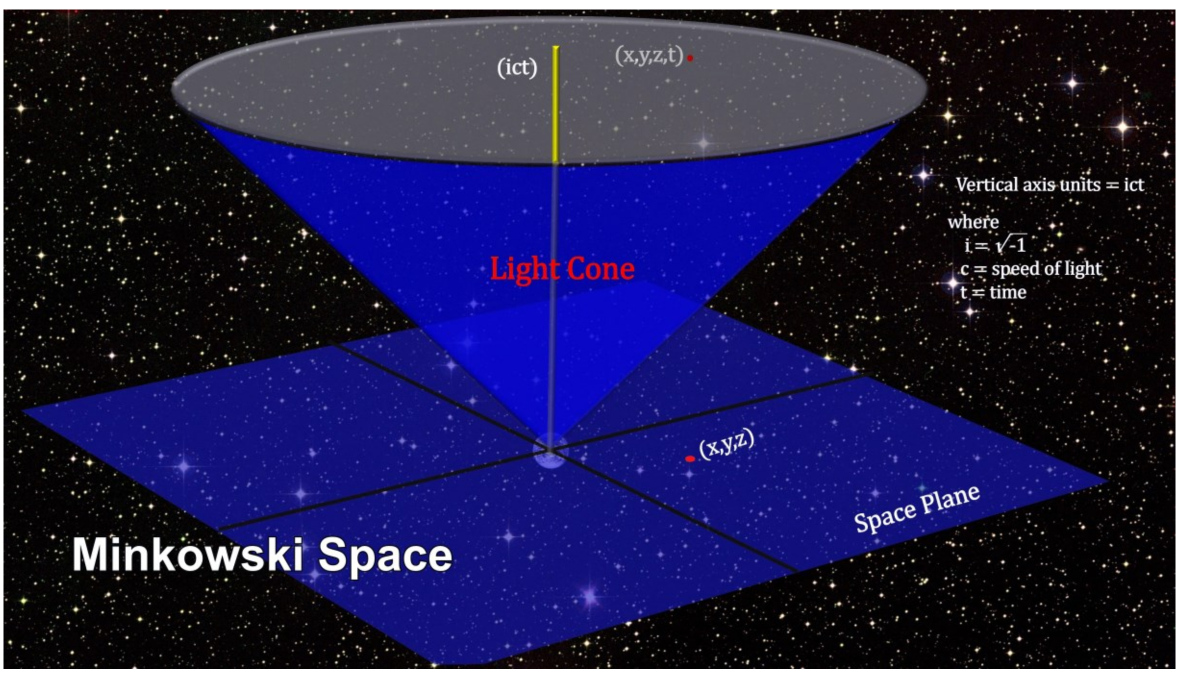
If we then extend the x axis line to be a plane that we'll use to represent 3-dimensional space.



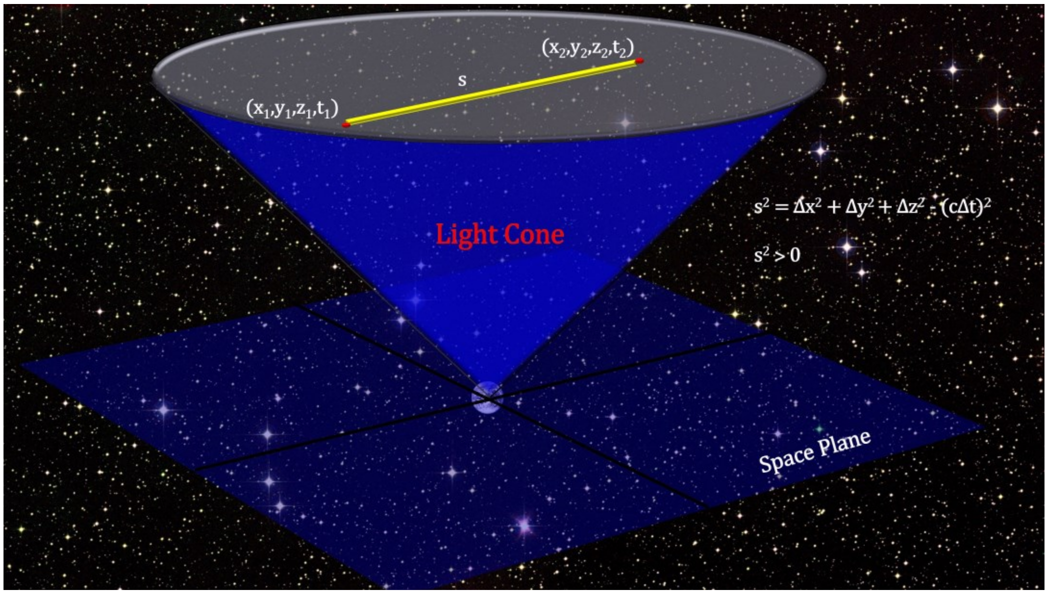
We can then rotate the light line 360 degrees to create a cone – the light cone. This is the 4-dimensional space-time graph.



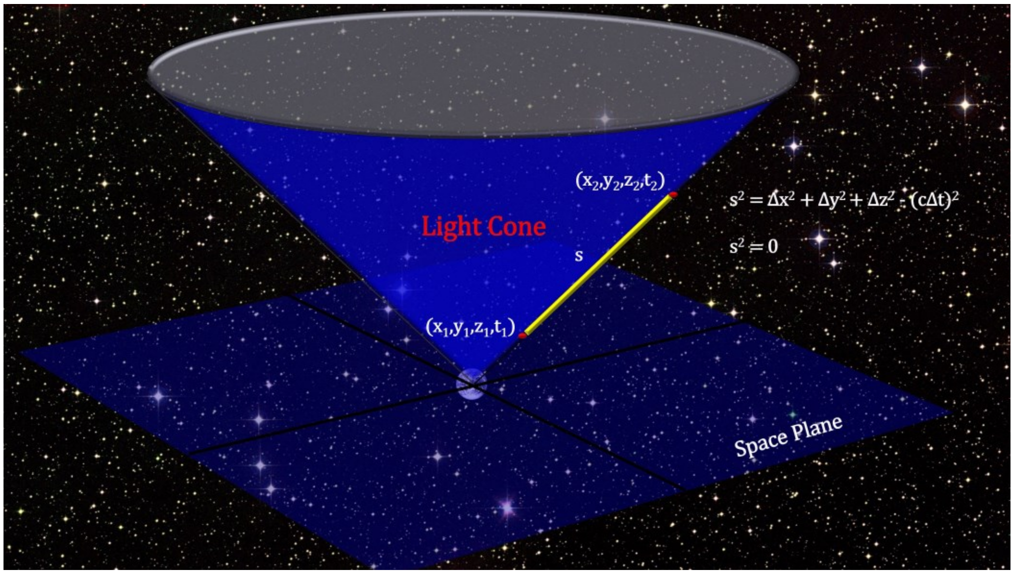
A point on this graph represents an event with 4 coordinates x, y, and z for space and t for time. A mathematician named Hermann Minkowski developed the geometry for Einstein's space-time [by using the speed of light times time times i (the square root of -1) (ict) units for the time axis.] So, you may sometimes hear space-time referred to as Minkowski space.



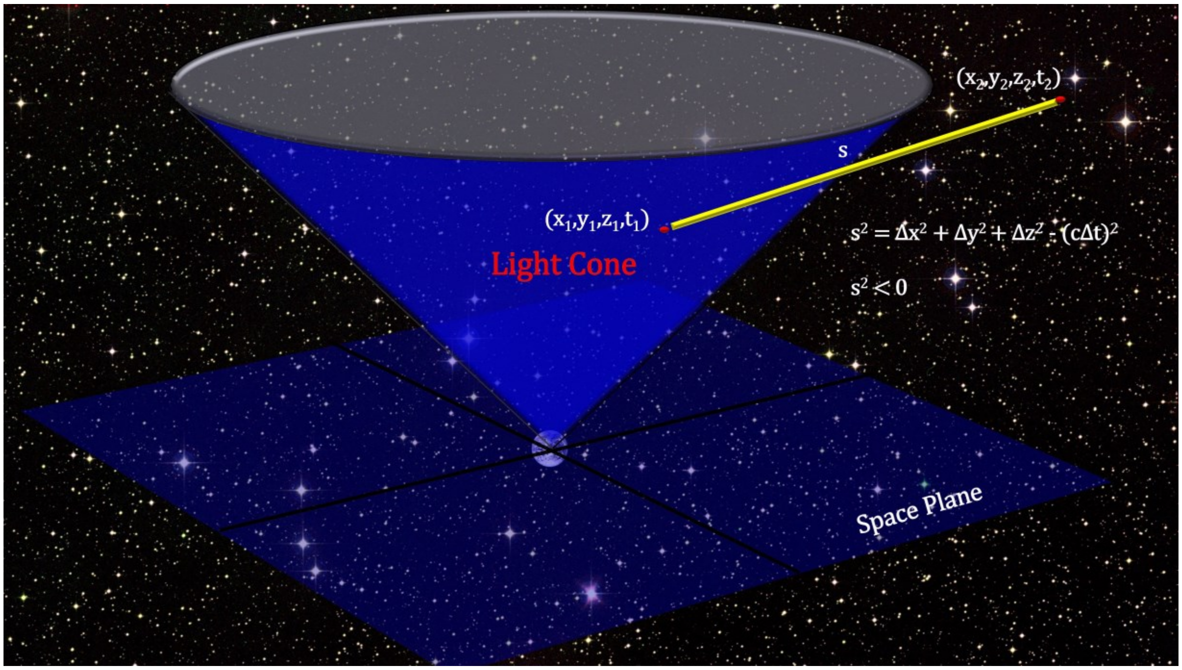
The 4-dimensional length of a line in Minkowski space is referred to as Proper Time or Proper distance. Space-time inside the cone is called 'time-like' because the length of connected points have positive Proper Time. This makes all points inside the light cone reachable. In other words, it is possible for an event at an earlier time within the cone to be the cause of an event at a later time within the cone.



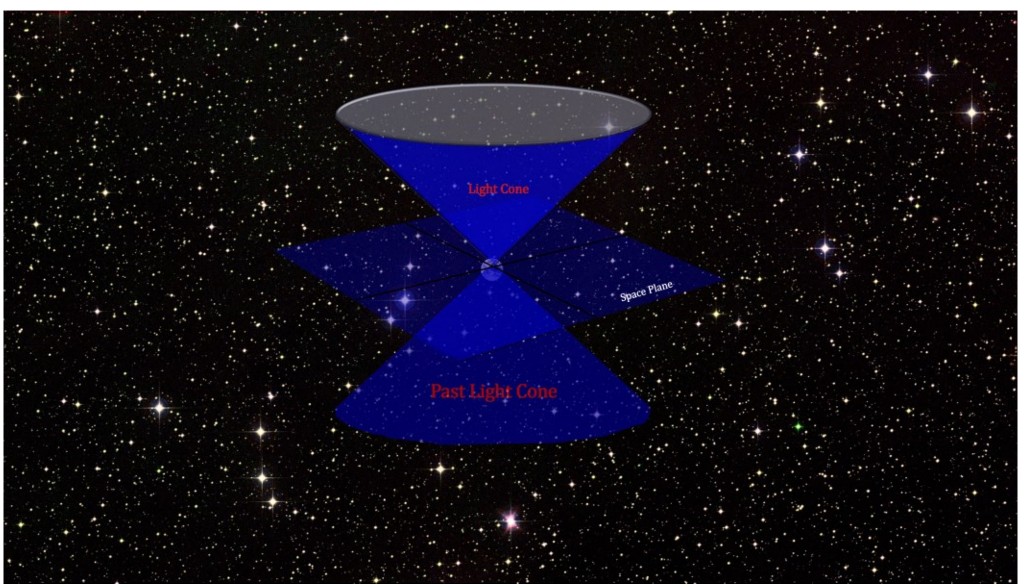
At the speed of light, the lines connecting the events runs up the edge of the cone, and the 4-dimensional length of the line connecting these events (Proper Time) is 0. Cause and effect can still hold.



The space outside the cone is called 'space-like'. The Proper Time of the line connecting a point inside the cone with a point outside the cone will have a negative length. And, like the boat example in our Speed of Light segment, where negative time simply meant that you could not get there from here, so the negative Proper Time simply means that you cannot get form one of these points inside the cone to the other point outside the cone. In other words, no event from within the cone can ever be the cause of an event outside the cone.



If we go back in time as well as forward in time, we get the full space-time diagram. Here we see that no past event outside the cone can be the cause for any event we see today.

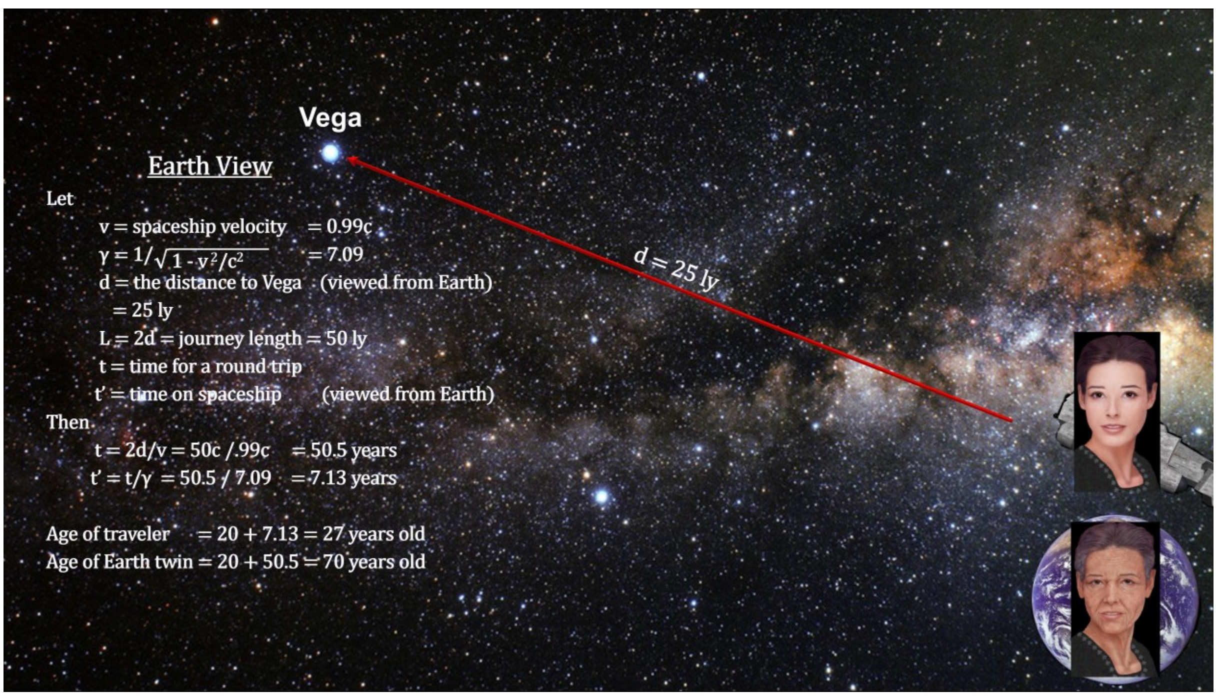


[Music**: Beethoven - Fur Elise: The score was not published until 1867, 40 years after the composer's death in 1827.**]

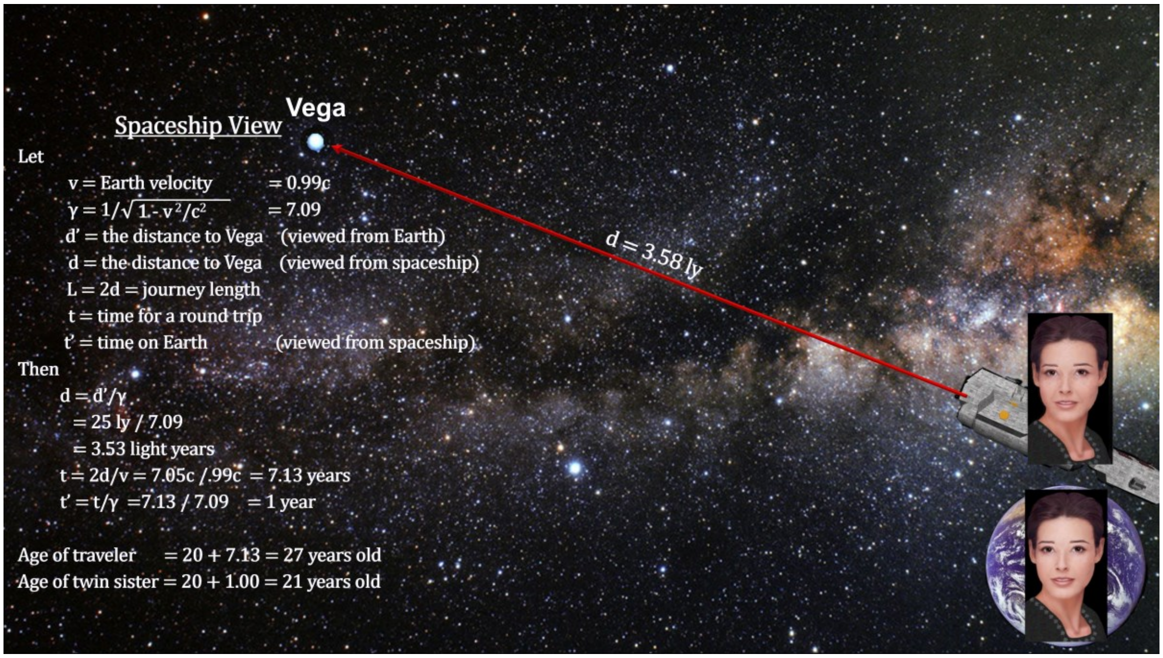
The Twin Paradox

We'll end this segment with a look at the famous Twin paradox. It goes like this.

Suppose two 20-year-old twins start out together on the Earth. One of them gets into a spaceship for a trip to Vega traveling at 99% of the speed of light. The person on the Earth sees the trip there taking just over 25 years, and the trip back taking the same amount of time. She is over 70 years old when the ship carrying her twin sister arrives back on Earth. But she also observes that her twin's clock ran a good deal slower than hers during the trip. Her twin is aging more slowly than she is. At 99% of the speed of light, time dilation would have the twin at just over 27 years old on her return - young enough to be her daughter rather than her twin.



From the point of view of the twin on the spaceship, she is motionless and the twin on the Earth is moving away and back. Like our cosmic ray muons, she sees the distance to Vega at only 3.5 light years due to space contraction. She also sees the twin on the ground aging slower than her over the 7-year journey. By her observations, her sister will be only 1 year older on her return due to time dilation. That's 6 years younger than she is – not 27 years older! How can it be that they are both older than the other? This is the paradox.



The first thing to notice is that the twin in the spaceship spends several periods of time being accelerated, meaning it is not an inertial frame. This paradox is fully resolved by Einstein's General Theory of Relativity. We'll cover that in the next segment.