General Relativity II – Tests

{**Abstract:** *In this segment of the "How Fast Is It" video book, we cover several testable effects of general relativity and how they differ from what Newton's gravity predicts. Our first is the orbit of Mercury. It precesses more than Newtonian gravity predicts. To understand the non-Euclidian space that Mercury orbits in, we introduce the Schwarzschild metric and compare it to the Minkowski metric for flat space-time. We illustrate the positive curvature around the Sun using concentric circles with shrinking circumferences. We then show how this slight difference in curvature produces additional movement in the precessing perihelion of Mercury's orbit that exactly fits the measured number. Our next effect is the bending of light. We cover Arthur Eddington's famous measurement during a total eclipse of the Sun and show how the amount of starlight bending matched Einstein's calculations better than Newton's. And we show how this effect tips over light cones and changes world-lines. Our third effect is gravitational redshift that leads directly to gravitational time dilation. We show how it works, how it was tested and cover how our GPS uses it. We also cover the Pound-Rebka experiment used the Mossbauer Effect to showed how this time dilation impacts gravitational redshift. We end with an illustration on how this effect resolves the Twin Paradox we introduced in the Special Relativity segment.*}

Introduction

With GR we now have a theory of gravity quite different than Newton's. But is this a difference without a difference? Or does GR predict different physical phenomena than Newton's theory?

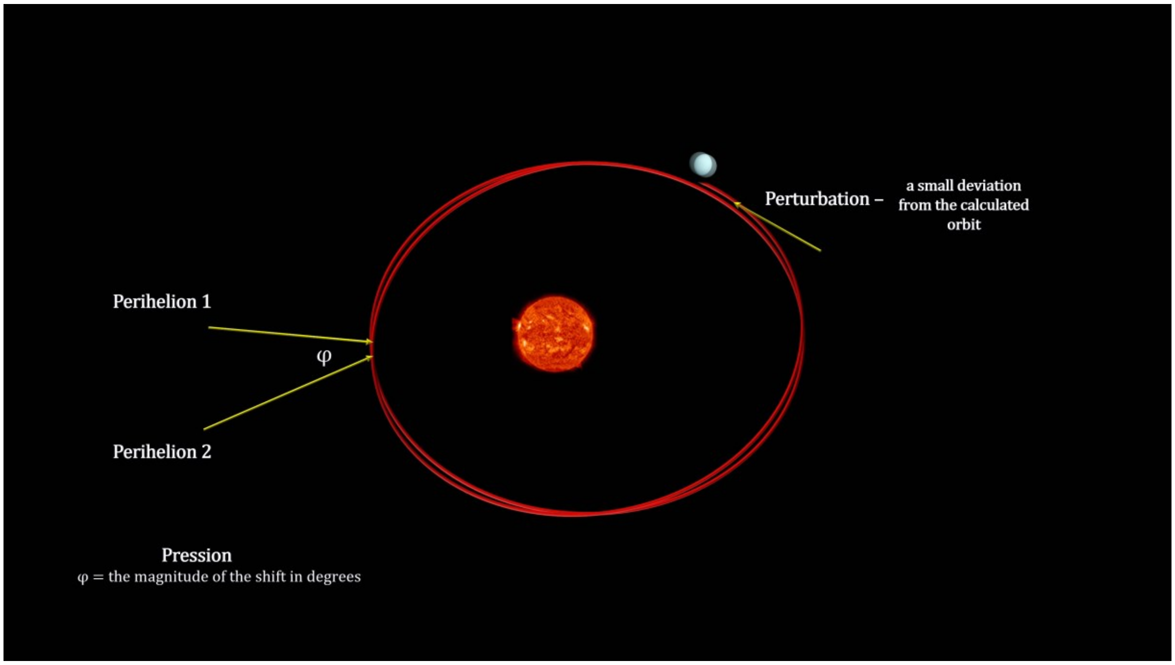
If you've seen the "How small is it" video book on quantum mechanics and the standard model, you may have noticed that much of the theory was developed to explain experimental evidence. In GR, we find that the theory was developed without much experimental evidence.

But the arrival of the theory, put experimental physics to work to prove or disprove it. Einstein himself showed that the field equations predict the orbit of Mercury better than Newton's. He also proposed two additional tests: one was the bending of light around the Sun, and the second was gravitational redshift. But first we'll start with Mercury.

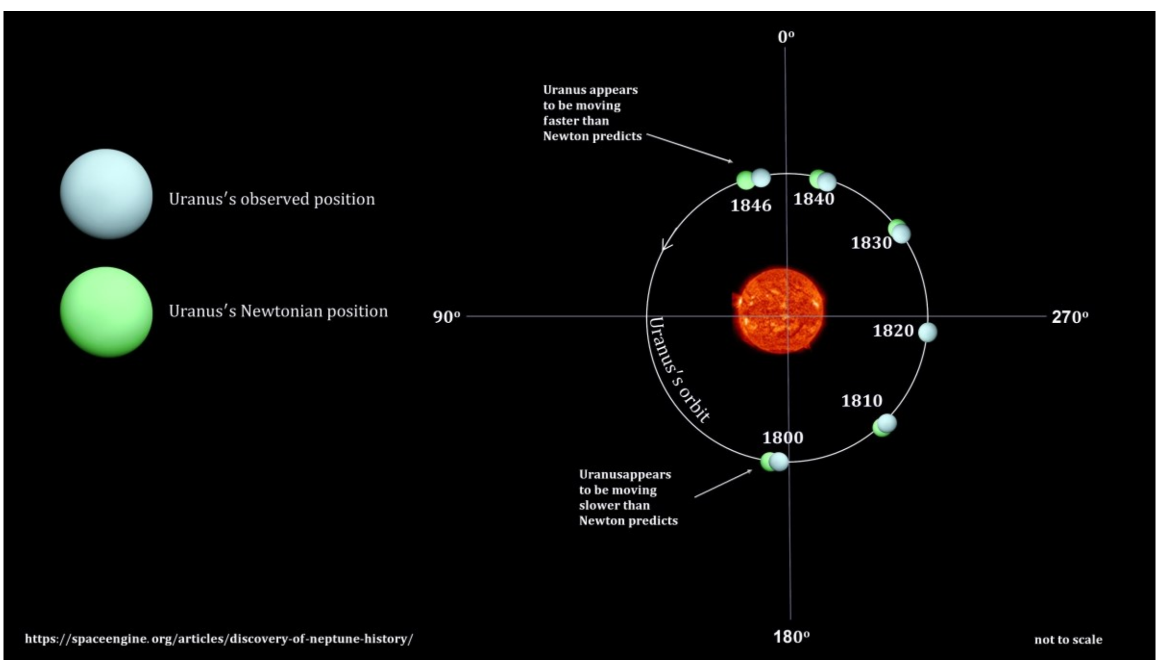
Orbit of Mercury

The planet Mercury's orbit test for Einstein's General Relativity Theory actually starts with the planet Uranus, discovered by William Herschel in 1781. By the early 1800s, it was understood that planetary orbits were elliptical with small deviations call perturbations. And that each orbit's closest approach to the sun, called the perihelion, shifted slightly over each orbit. This is called pression.

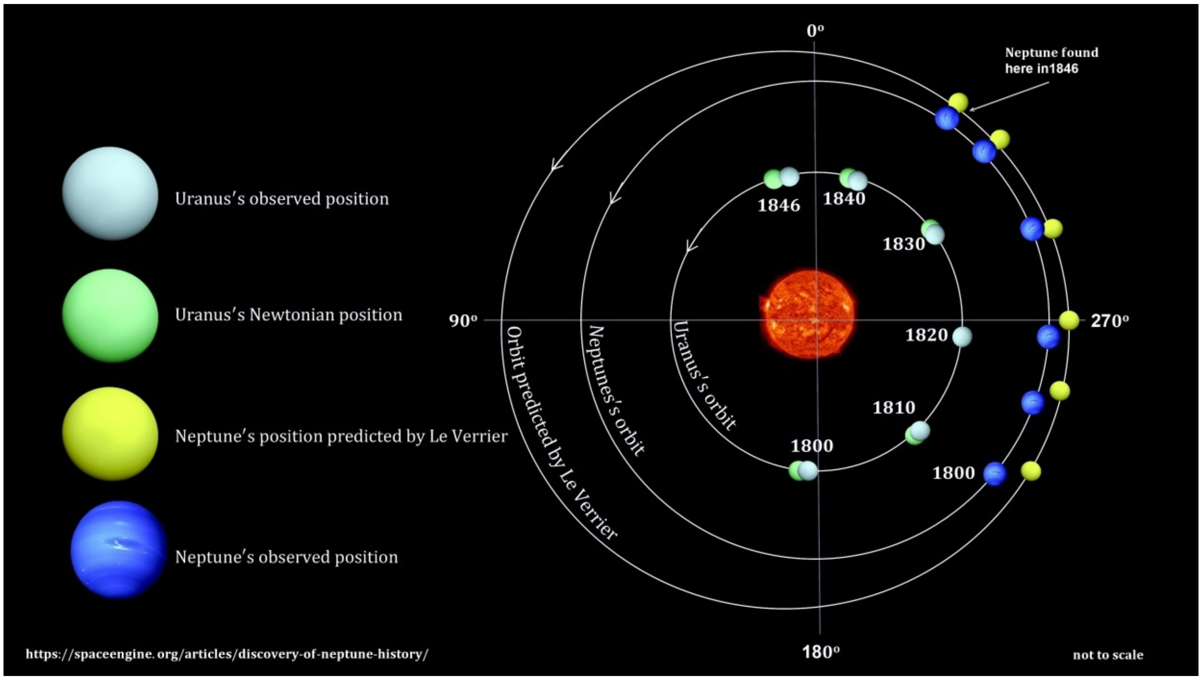
Using Newton's gravitational equations, all known perturbations and pressions were calculated and found to fit the observations for all the planets, except for one – Uranus.



Careful study over decades showed that Uranus' orbit did not fit Newton's equations. At times it was moving faster than predicted, and at other times, it was moving slower. There were two schools of thought at that time. One held that Newton's theory did not hold up that far from the Sun – indicating that a new theory was needed. The other proposed that there's another planet beyond Uranus that pulled on it. The observed deviations could be explained as perturbations. If correct, this would keep Newton's theory intact.

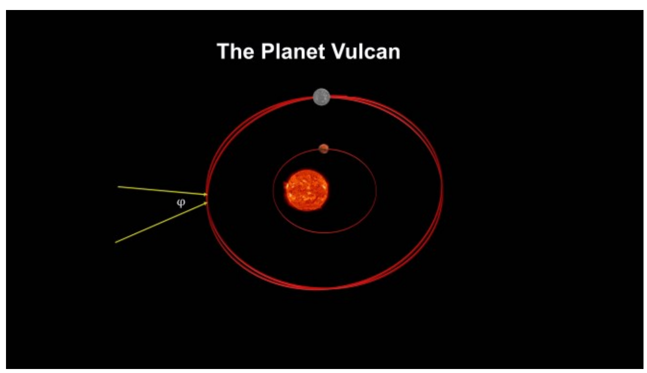


Astronomer Urbain Le Verrier went to work to try discover this new planet. Early in 1846, he published calculations that that came very close (to just over 1%) to where it actually was. On September 23 of that same year, Johann Galle, an astronomer at Berlin Observatory, and a student Heinrich Louis d'Arrest found the new planet looking where Le Verrier had placed it. This planet is now called Neptune.



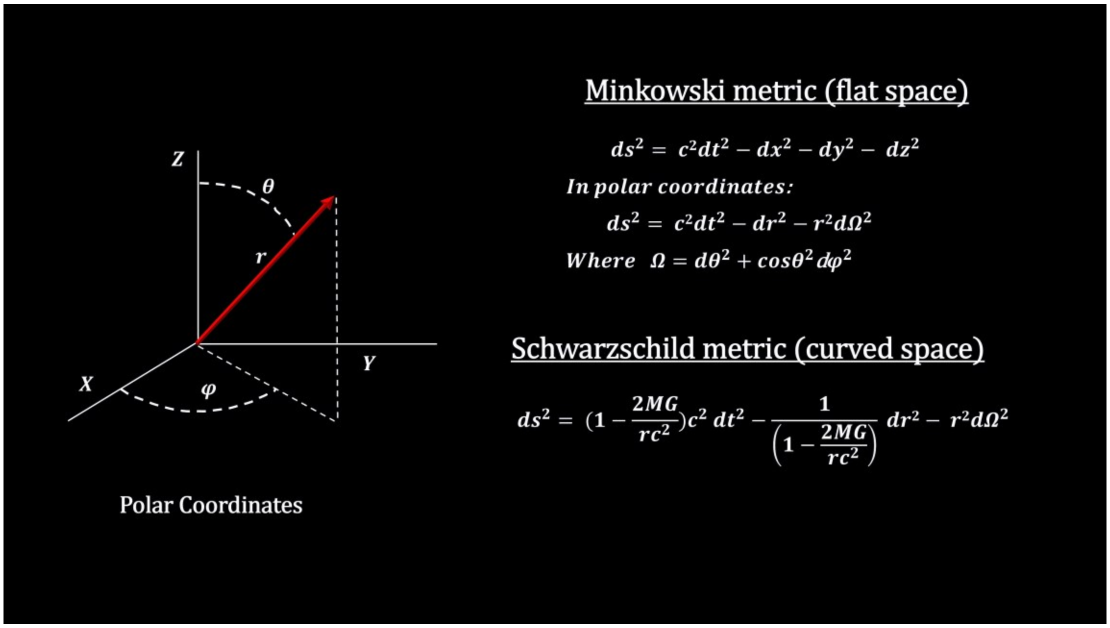
The reason this is relevant for Mercury is that the overall thinking at the time was similar. Newton's theory does not fully explain the observed precession of Mercury's perihelion.

In 1859, astronomers, including Le Verrier, theorized that another planet, inside the orbit of Mercury, could account for the observations – much like how Neptune explained Uranus' orbital irregularities. The proposed planet between the Sun and Mercury was even given a name – Vulcan. But no such planet was ever observed.

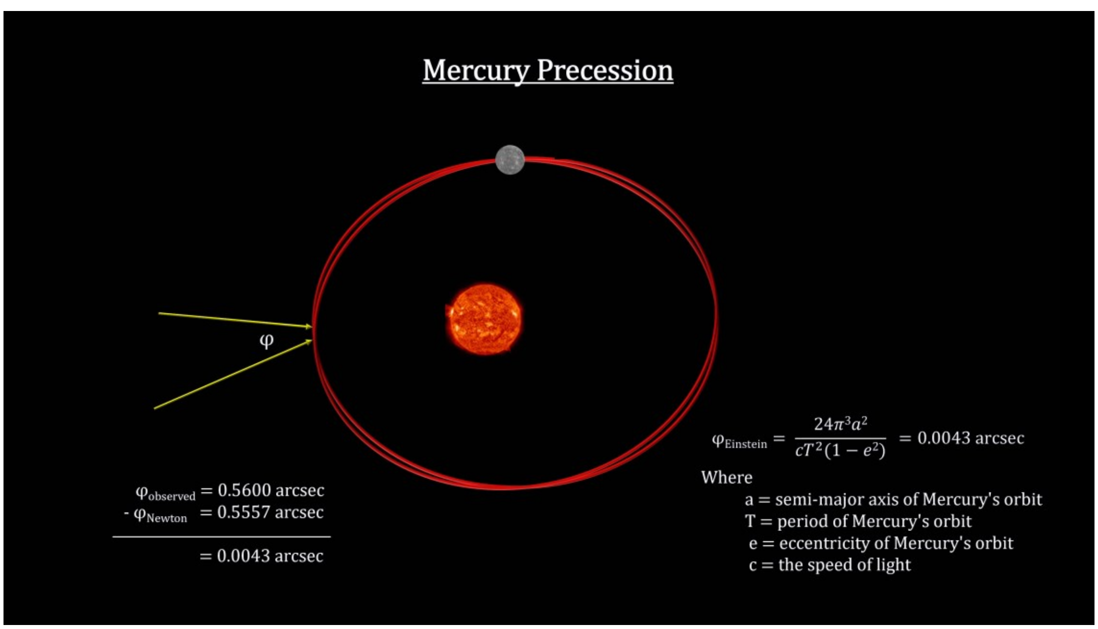


Another school of thought held that Newton's theory simply did not hold up that close to the Sun. Einstein was one of them, and his General Theory of Relativity describing the impact of the curved space near the Sun provides a full explanation for the observed precession without the need for an extra planet. Einstein himself thought that this result was the most critical test of his theory. Here's how it works. [When working with GR, it is easiest to use polar coordinates because we are always dealing with the structure of space-time some distance away from a large mass.]

In 1916, the same year that Einstein published his GR paper, Karl Schwarzschild published his exact solution for space around a large non-rotating mass. His metric is now called the Schwarzschild metric and it works quite well for slowly rotating masses like the Earth and Sun and planets in our solar system. We'll use this metric for the first 3 tests.

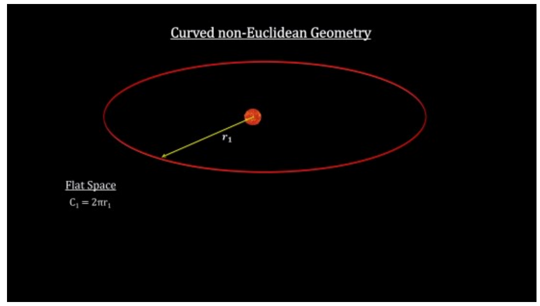


As seen from Earth, the precession of Mercury's orbit is measured to be 0.5600 arcsec per orbit (an arcsec is =1/3600 of a degree). Taking into account all the perturbation effects from the other planets, as well as a very slight deformation of the sun due to its rotation, and the fact that the Earth is not an inertial frame of reference, Newton's equations predict a precession of 0.5557 arcsec. That's .43 arcsec short. With Schwarzschild's metric, Einstein came out with the exact number to cover the difference. He had passed the first test of his new theory.

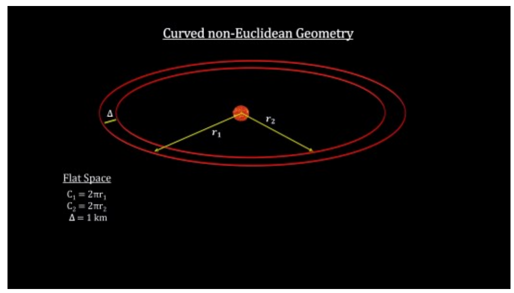


It's the curved space around the sun defined by the Schwarzschild metric that produces this small additional pression on each orbit. Here's what it looks like.

If we draw the circumference of the Earth's orbit, we get a length that is 2 π times our distance from the sun.

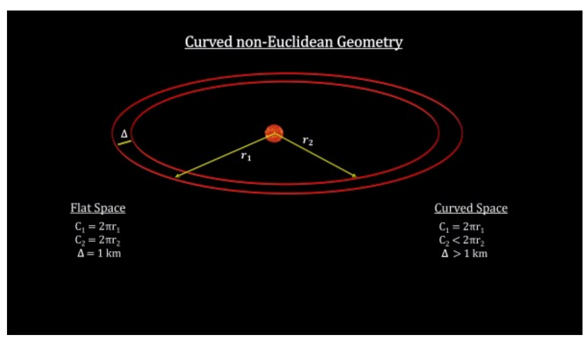


If we existed in flat Euclidean space, we would calculate the circumference of an orbit one km closer to the sun and see that the distance between the orbits is one km.

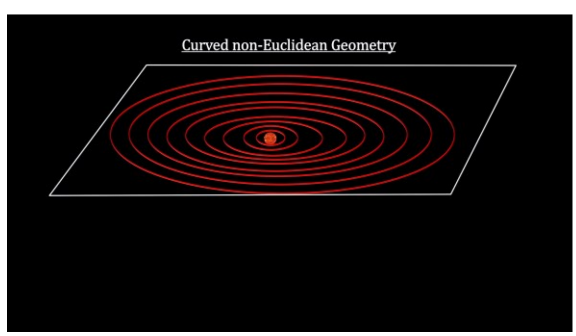


But because of our positive curvature, if we were to measure the circumference with a radius that is 1 km shorter than the first, we'd find that it is less than 2 π times the shorter radius.

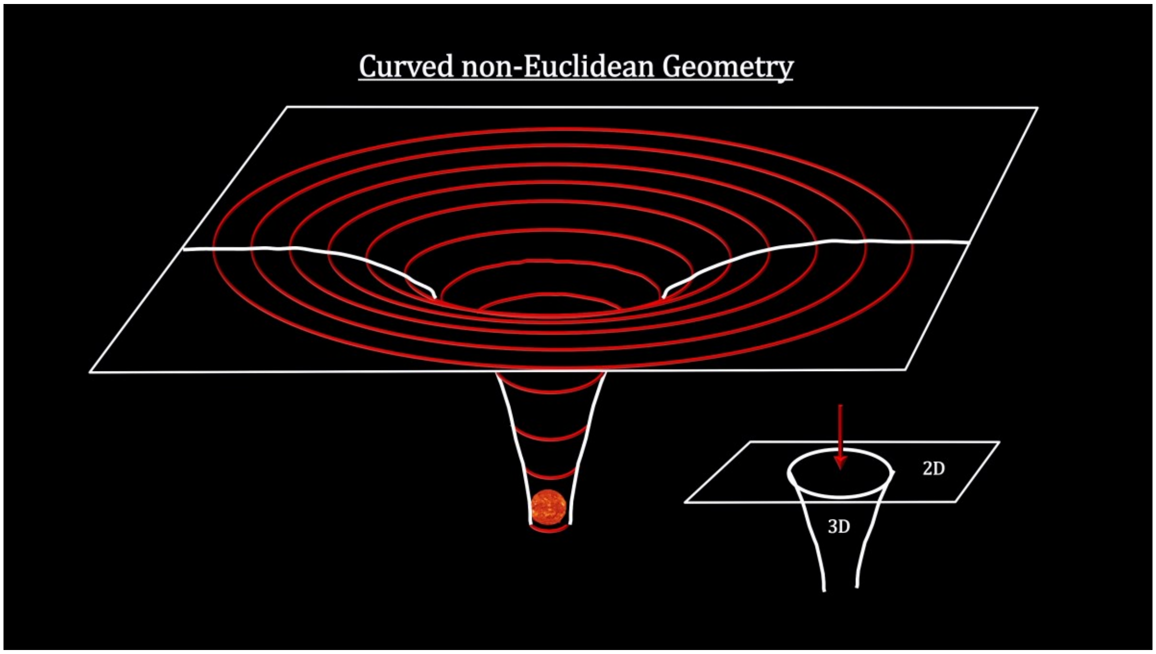
Which means that the distance between the circumferences would be greater than the 1 km difference in the radii! But only a little.



We can repeat this process all the way to the surface of the sun. With each successive radius, the difference between the orbits would increasingly diverge from the Euclidian numbers.

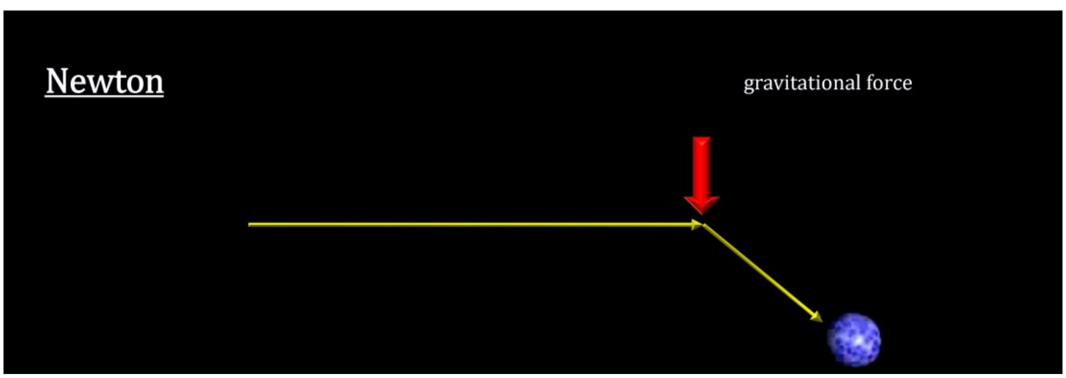


If we were to telescope this picture, you'd see the standard diagrams that are used to help explain GR. But diagrams like this are misleading in two ways. First, they represent an external curvature into another dimension, when in fact, we are talking about intrinsic curvature. There is no evidence for the existence of a fourth special dimension. Second, it looks like you need a downward force on the object to get it to drop into the hole. That would be gravity – but that's what the lines were supposed to represent. So, we'll avoid using this technique as we move on to the bending of light by the Sun.

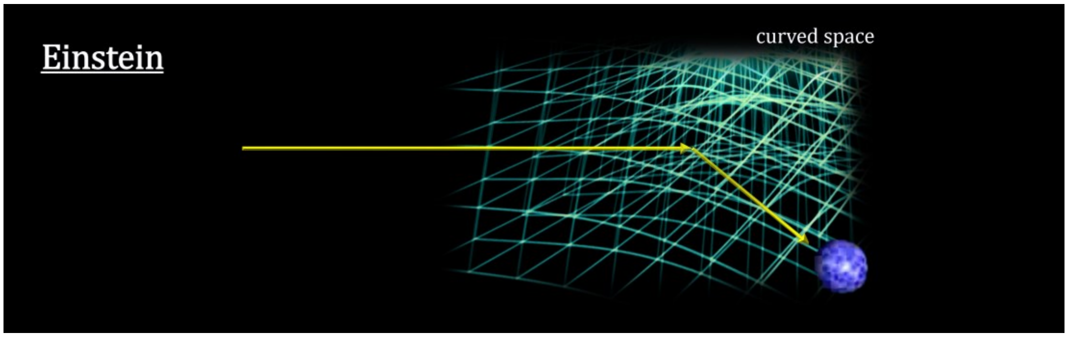


Sun Bending Light Test

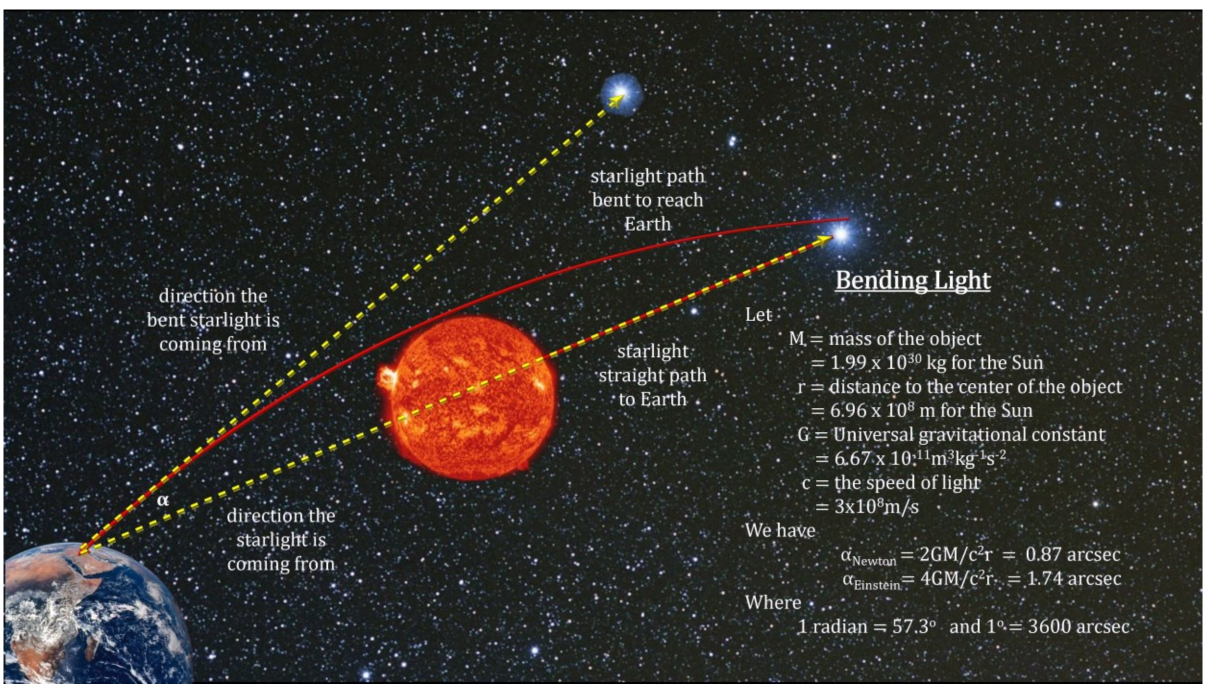
One of Newton's laws of motion states that "an object in motion remains in motion at a constant speed in a straight line unless acted on by a force." In this view, gravity is a force that can act on light and divert it from its usual straight-line motion.



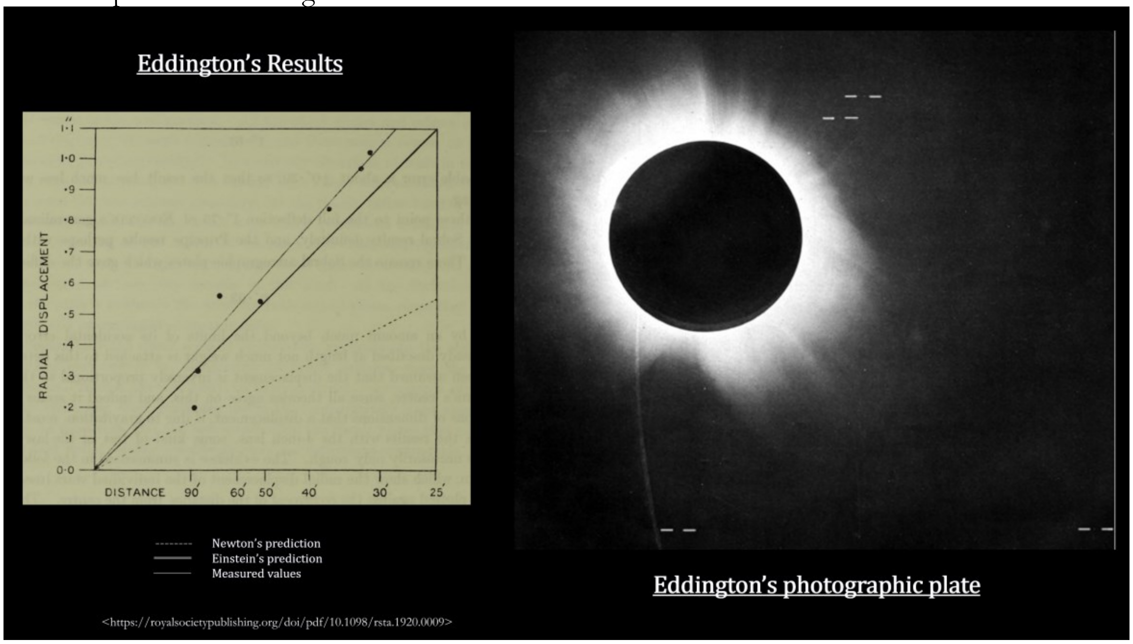
Einstein, on the other hand, has massive objects curving the space around them. An object in motion traveling through this curved space follows Geodesics, the shortest path between two points, unless acted on by a force. It's important to remember that in General Relativity, gravity is not a force. But light will bend.



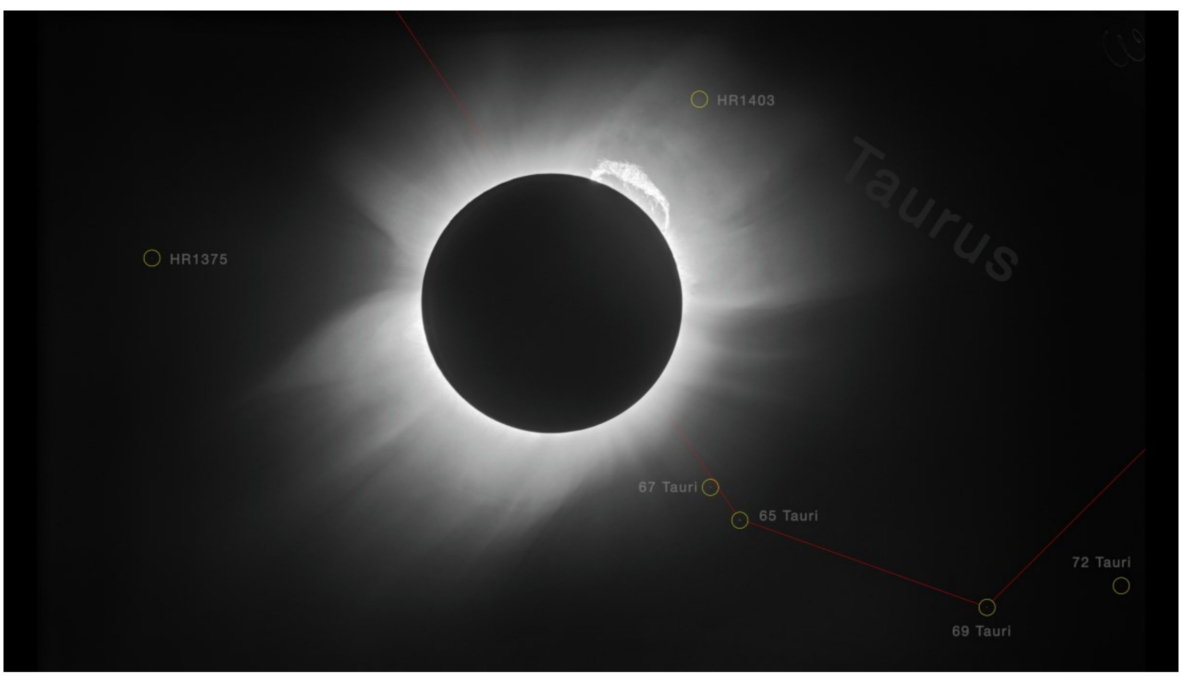
Both theories have light bending when traveling near a massive object. The larger the mass of the object, the larger the bending, and the closer to the center of the object, the larger the bending. But the two theories predict different amounts of bending for the same mass and distance measurements. For light passing near the surface of the Sun, Newton's theory predicts a deflection angle of 0.87 arcseconds. [This solution is a close approximation, since the full solution is an infinite sum of algebraic terms.] Einstein's theory predicts a deflection angle of 1.74 arcsec, twice Newton's prediction. Einstein pointed out that the best way to test his theory was to study apparent star locations during a total eclipse of the Sun.



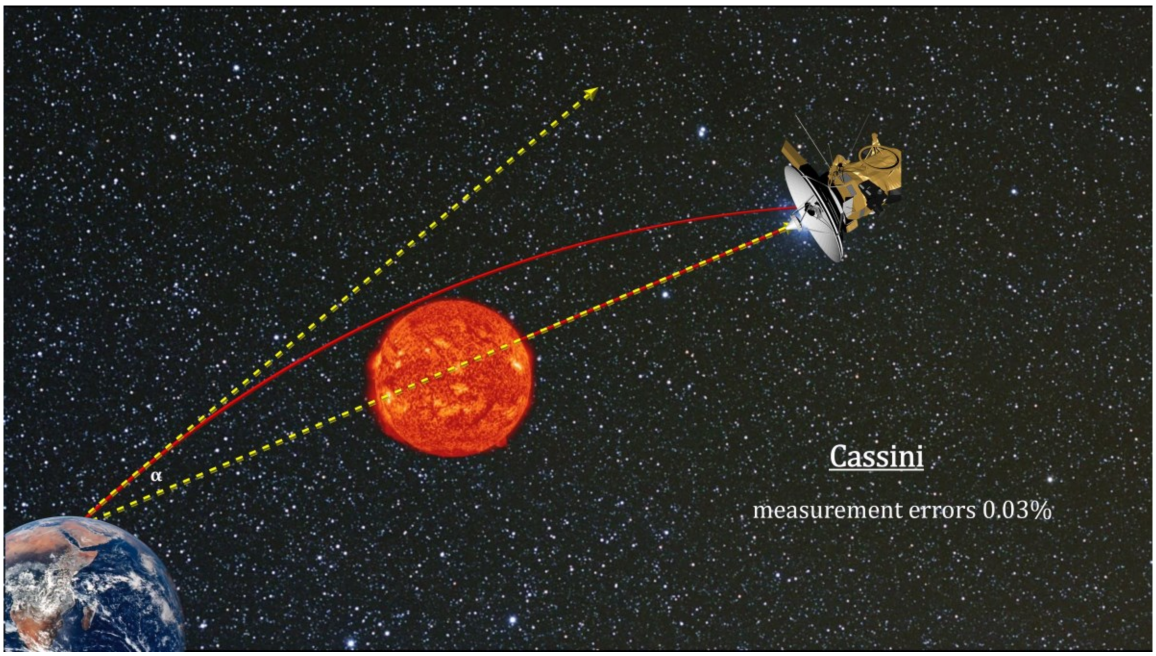
In 1919, a solar eclipse was slated to occur with the sun silhouetted against the Hyades star cluster - the nearest open cluster to our Solar System. The British astrophysicist Arthur Eddington took up positions off the coast of Africa and in Brazil, and simultaneously measured the clusters light as it brushed past the sun. The images were then superimposed on top of an image taken at night earlier in the year. When the eclipse and night images were compared, a gap was found. And when the gap was measured, it confirmed that Einstein's prediction was right.



Here's an enhanced picture produced a hundred years later by the Heidelberg Digitized Astronomical Plates project and released by the European Southern Observatory. They scanned one of Eddington's photographic glass plates and applied modern image processing techniques like noise reduction. This version identifies some of the stars used in Eddington's analysis.

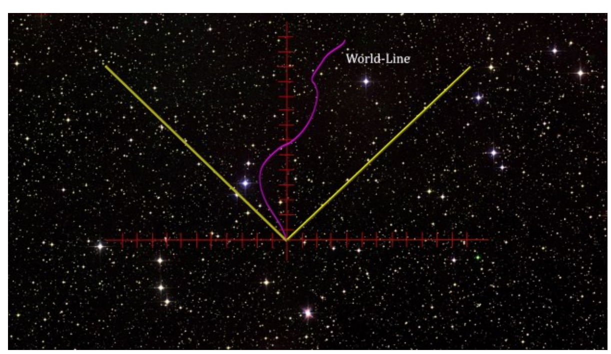


But the Sun's corona is strong. It interferes with all the measurements. It is estimated that errors as large as 20% are inherent in Eddington's and other visible starlight bending experiments around the Sun. But other tests have produced much more accurate results. For example, the European Space Agency's Hipparcos satellite (1989-93) designed to measure parallax distances to 100,000 stars charted the positions of stars so accurately that no eclipse was needed to see the effect of the Sun's gravity. They produced numbers with only a 0.1% error. In 2003, using radio frequency light and measuring techniques that eliminated the error producing impact of the Sun's corona, astronomers measured how much waves sent from the Earth to the Cassini satellite and back again were deflected by the Sun. Their error rates were around 0.03%. These and many more light bending experiments have confirmed that Einstein's equations are correct.

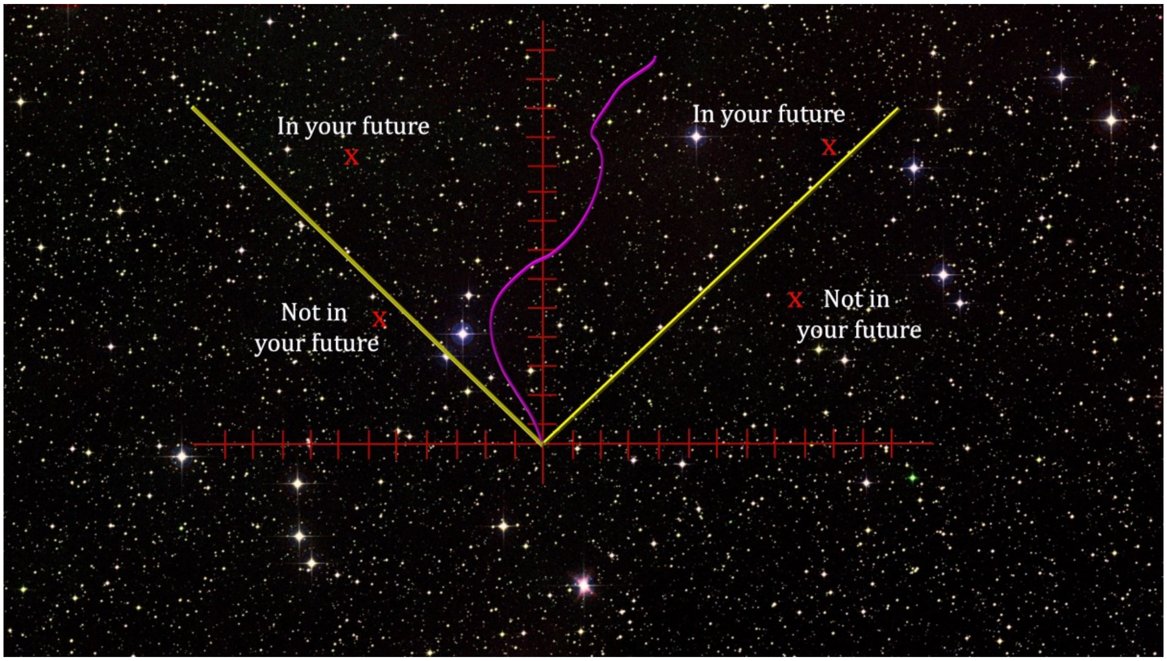


Light-cone Tipping
One of the key implications for the bending of light is its impact on what's physically possible in heavily curved space-time.

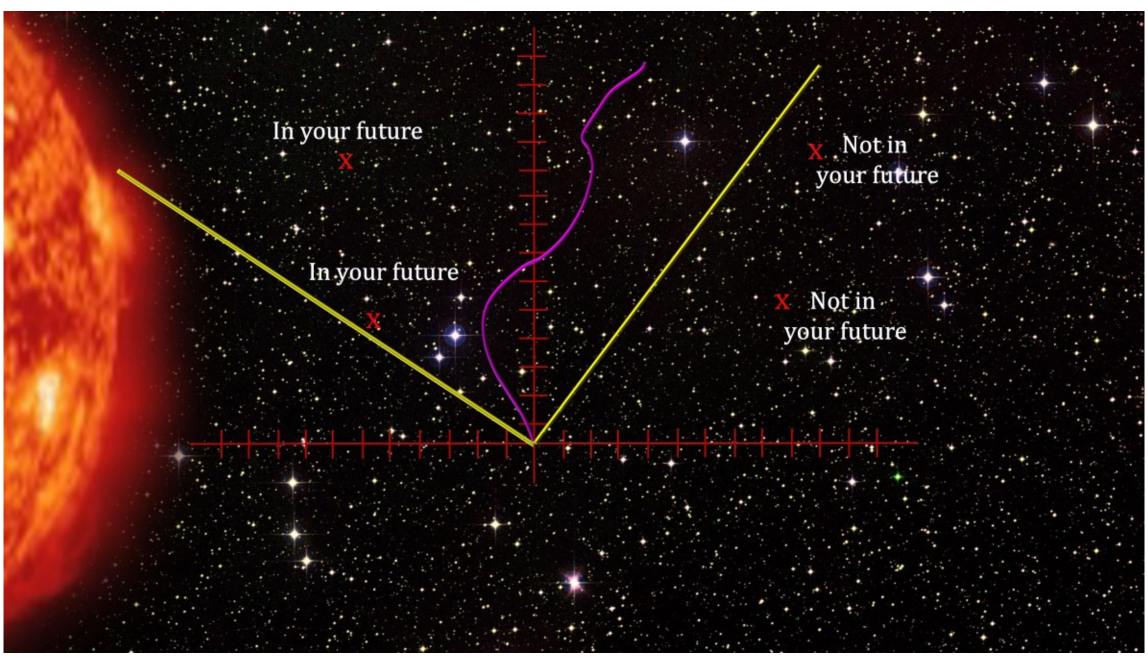
Here's a two-dimensional slice of the future light-cone that we developed in the previous segment on SR. This purple line represents a path taken by anything with mass. It's called the world-line and can be anywhere inside the light-cone. In this representation, world-lines have to remain between the two arms of the light cone because nothing can travel faster than the speed of light.



The speed of light lines are the divider between events that are in your future (if it's your light cone) and events that are not. By "in your future" I mean that you can be connected to them physically in some way.



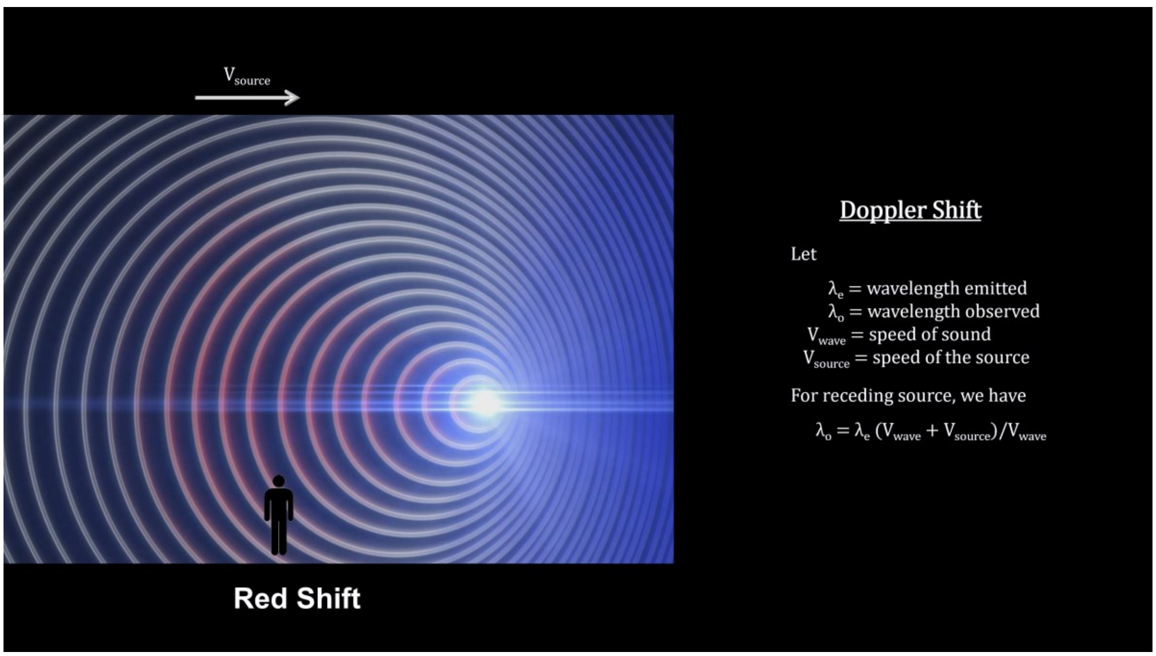
Now suppose there is a great mass-energy density to the left of the cone. The light would be bent in its direction. We see that points that were impossible to reach before, now fall inside the cone and are reachable. And we see that points that were reachable inside the cone now fall outside the cone and are unreachable. This is light-cone tipping. The closer we get to the source of the gravity, the greater the space-time curvature. And the larger the matter curving the space, the greater the curvature. We'll take another look at this when we get to black holes.



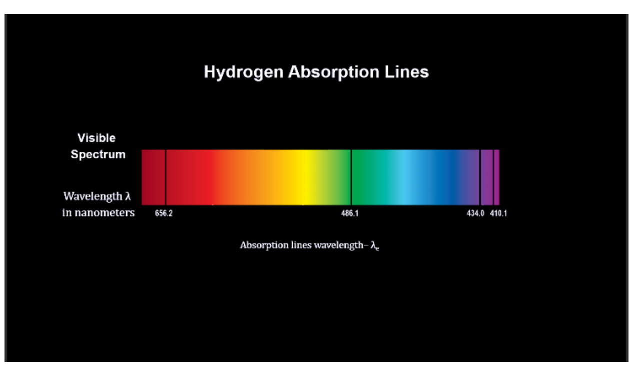
Gravitational Redshift Test

The third test of Einstein's relativity theory proposed by Einstein himself involved the shifting of light wavelengths to the red in the curved space of a gravitational field. This phenomenon is called Gravitational Redshift. To see how this works, we'll take a minute to review just what redshift is.

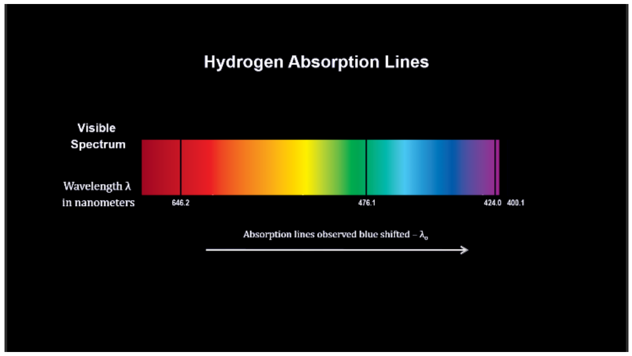
Most people have had the experience of hearing the pitch of a car horn, train whistle or ambulance siren drop as the source moved past. As the sound source moves toward the observer, the sound waves are compressed, making the pitch of the sound higher. As the sound source moves away from the observer, the sound waves are stretched out, making the pitch of the sound lower.



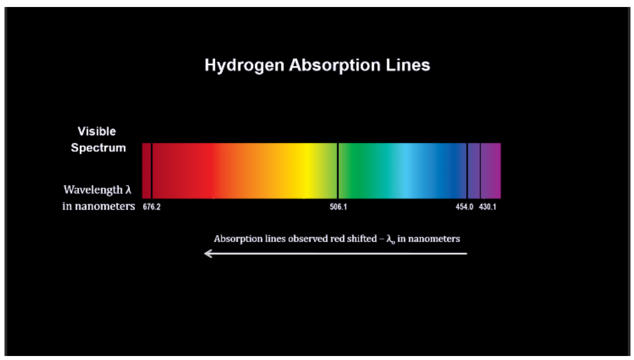
The same effect works for light. Here we have the visible spectrum from a star. Hydrogen in the star's atmosphere creates absorption lines with a unique pattern. Here's the pattern for a star at rest with respect to the observer.



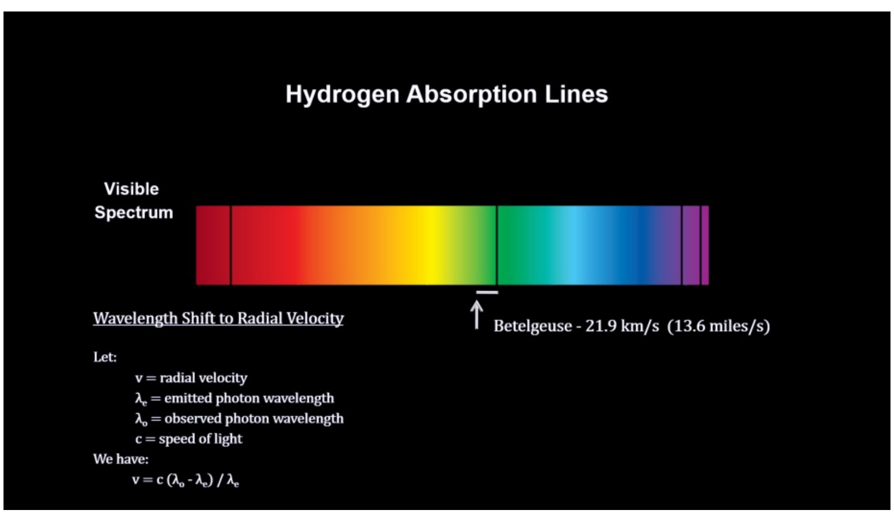
Light from an approaching star has its wavelengths shortened. We see that the lines shift to the blue. Their said to be 'blue shifted'.



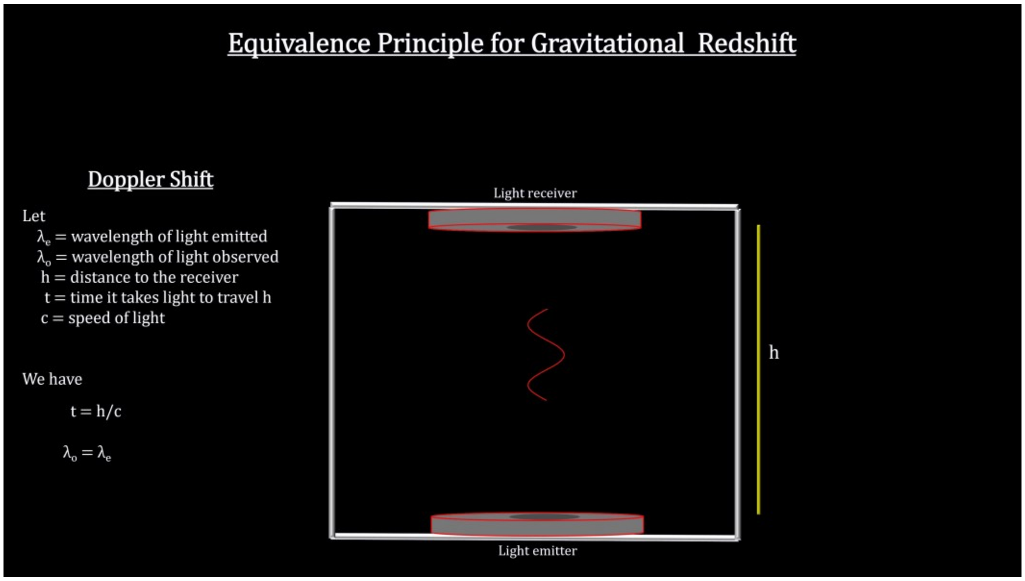
And light from a receding star has its wavelengths lengthened. We see that the lines shift to the red. Their said to be 'red shifted'.



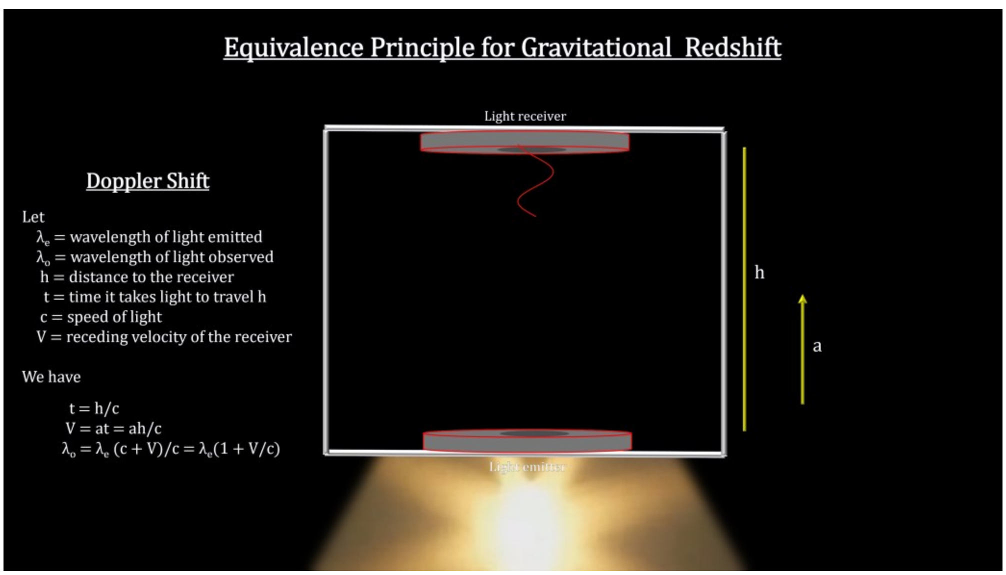
The key to measuring the Doppler Effect is to measure the change in position of the spectral lines. The further the shift, the faster the radial velocity.



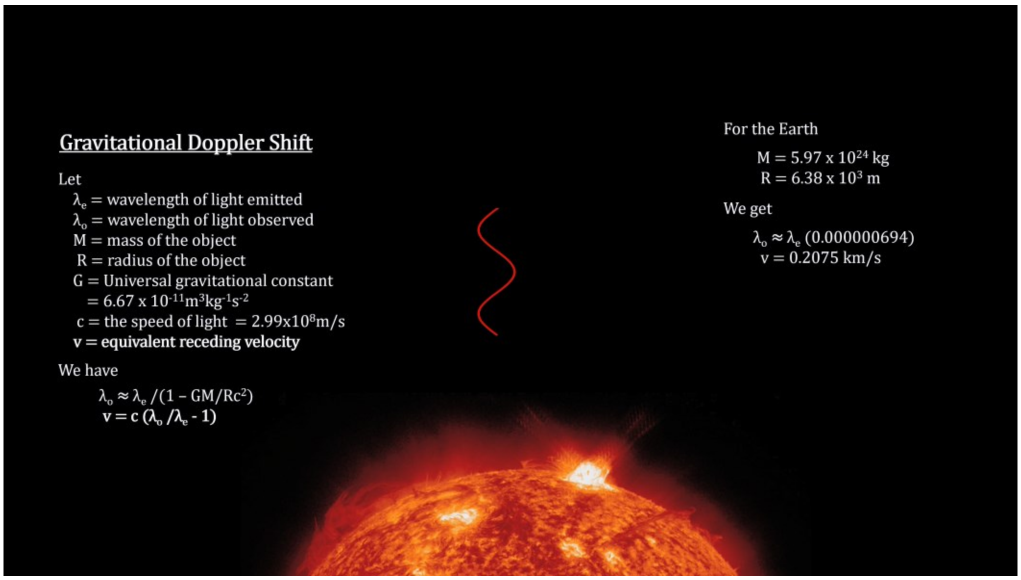
When the shift to the red is caused by gravity instead of receding velocity, the phenomenon is called Gravitational Redshift. Einstein developed the concept for this using the Elevator Thought Experiment. Consider the elevator at rest with a light emitter fixed to the floor and a receiver fixed directly above it on the ceiling at a known distance. The emitter sends photons with a controlled wavelength to the receiver where the arriving wavelength is measured. Here the measured wavelength of the light observed will be the same as the wavelength of the light transmitted.



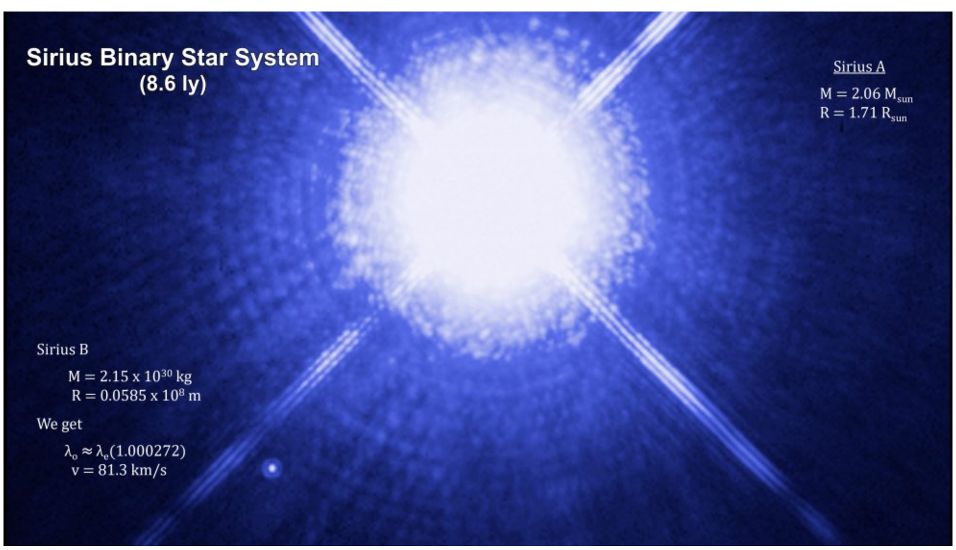
Now put the elevator into a constant acceleration. Note that the receiver, at the time the light is observed, is further away from the point where the light was transmitted than it was in the static case. In other words, the receiver has acquired a velocity with respect to the light. And like the train whistle moving away, its wavelength is increased – shifted to the red. By the Equivalence Principle, the same result must hold in a gravitational field.



But to calculate the effect as light moves away from a massive object, we need to take into account that the acceleration due to gravity is not constant. It decreases with distance as the light travels through the curved space around the object. The Schwarzschild metric that we used in the first two tests on Mercy's orbit and light bending around the Sun, gives us the equation. We see that the amount of gravitational redshift for light from the surface of a massive object reaching a distant observer is proportional to the object's mass divided by its radius. Here's the gravitational redshift for the Earth. And the Sun with triple the Earth's mass-to-radius ratio. These are very small hard to measure shifts on the order of a tenth of a nanometer. In fact, churning matter on the Sun's surface can have up to a thousand times the radial velocity equivalent to this redshift, making it impossible to measure gravitational redshift.

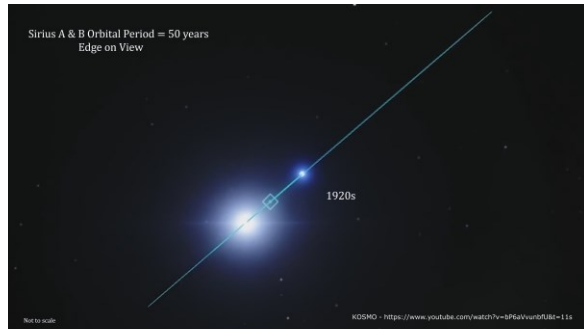


Astronomers concluded that, in order to measure this effect, they need a star with a calmer surface and larger mass to radius ratio. That would be a White Dwarf. For that reason, they focused on the nearby Sirius binary star system with its giant star Sirius A and orbiting white dwarf star Sirius B to test Einstein's theory.

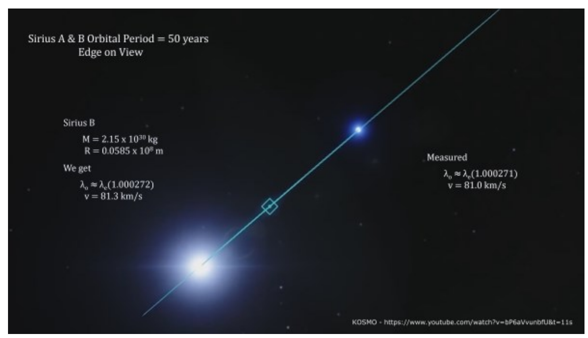


This binary system's orbital period is 50 years.

In the 1920s, when the first measurements of Sirius B's gravitational red shift were made, the two stars were close together on the sky and the results were said to be contaminated by light from Sirius A.



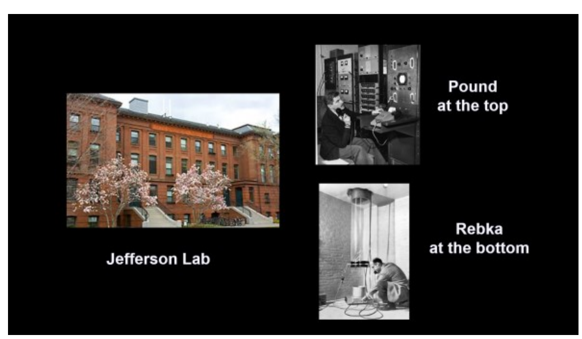
It wasn't until the 1960s that they were far enough apart to significantly reduce this contamination. At that time, astronomer Jesse Greenstein working out of the Mount Wilson Observatory measured the gravitational redshift effect to be 81 km/s, not far from the theoretical 81.3 km/s.



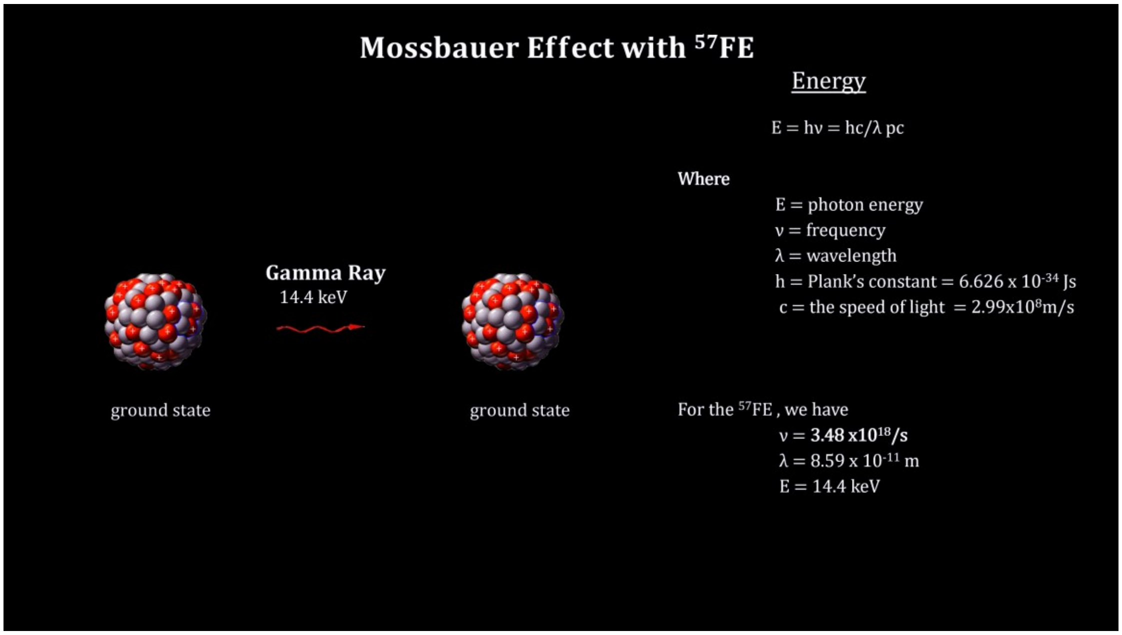
But the number of variables remain too large, and difficulties separating out shift due to actual receding velocity made the results less than conclusive for testing Einstein's theory. But two physicists in a lab did prove Einstein correct. We'll cover their experiment in the next segment.

Pound Rebka Experiment

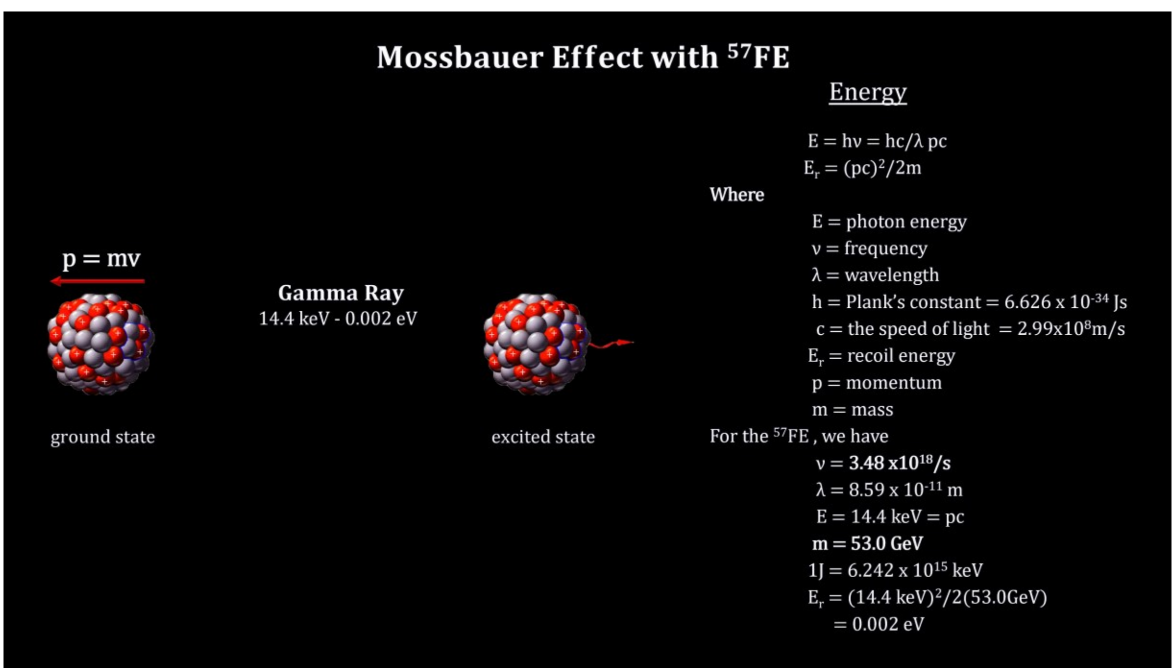
In 1959, physicists Robert Pound and Glen Rebka preformed an experiment in the Jefferson Physical Lab at Harvard to demonstrate gravitational redshift. It was based on physicist Rudolph Mossbauer's effect discovered two years earlier that involves the emission and absorption of gamma rays from the excited states of an iron nucleus.



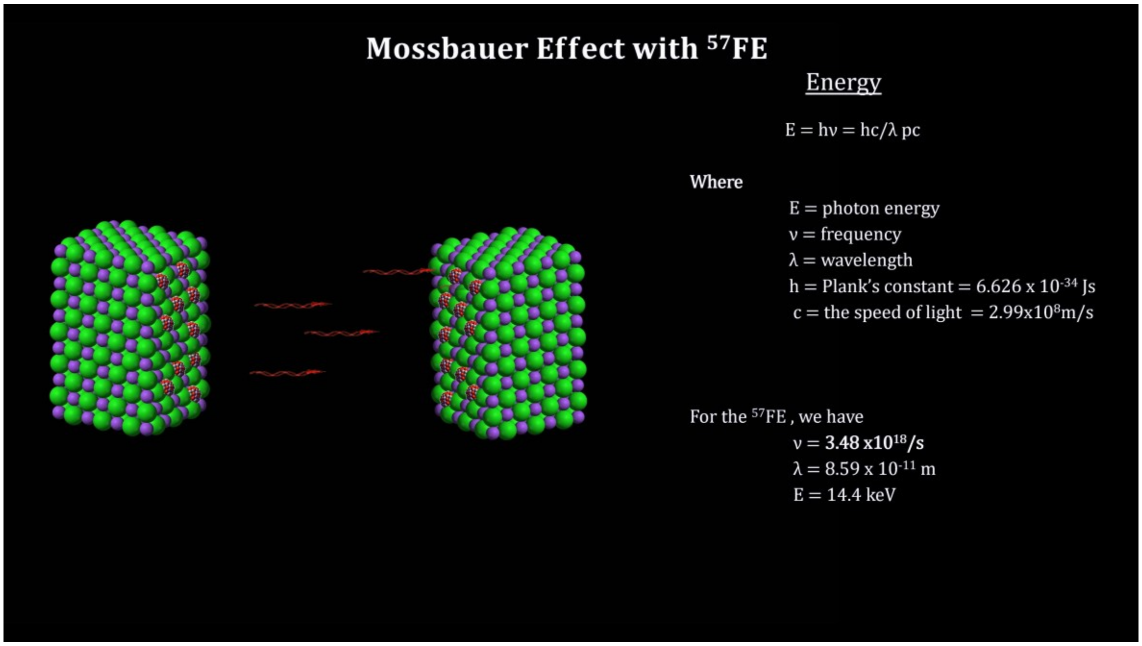
Here we have an iron atom's nucleus in an excited state. When it falls to a lower energy level, a gamma ray photon carrying the energy is emitted. Once this photon encounters a like atom, it will be absorbed – raising the energy level of the encountered atom's nucleus.



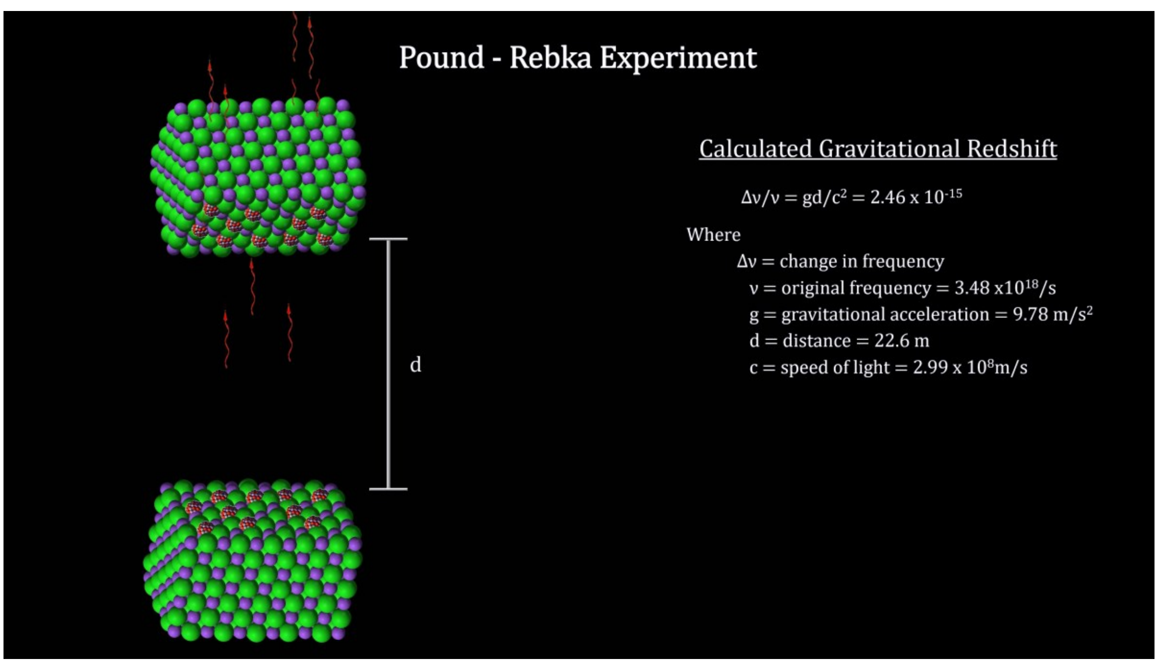
The problem is that when the gamma ray is ejected, the nucleus recoils. Because of energy momentum conservation, the recoil energy reduces the energy of the gamma ray. The gamma ray is no longer a match for the other nucleus and it moves right through. There is **no** absorption.



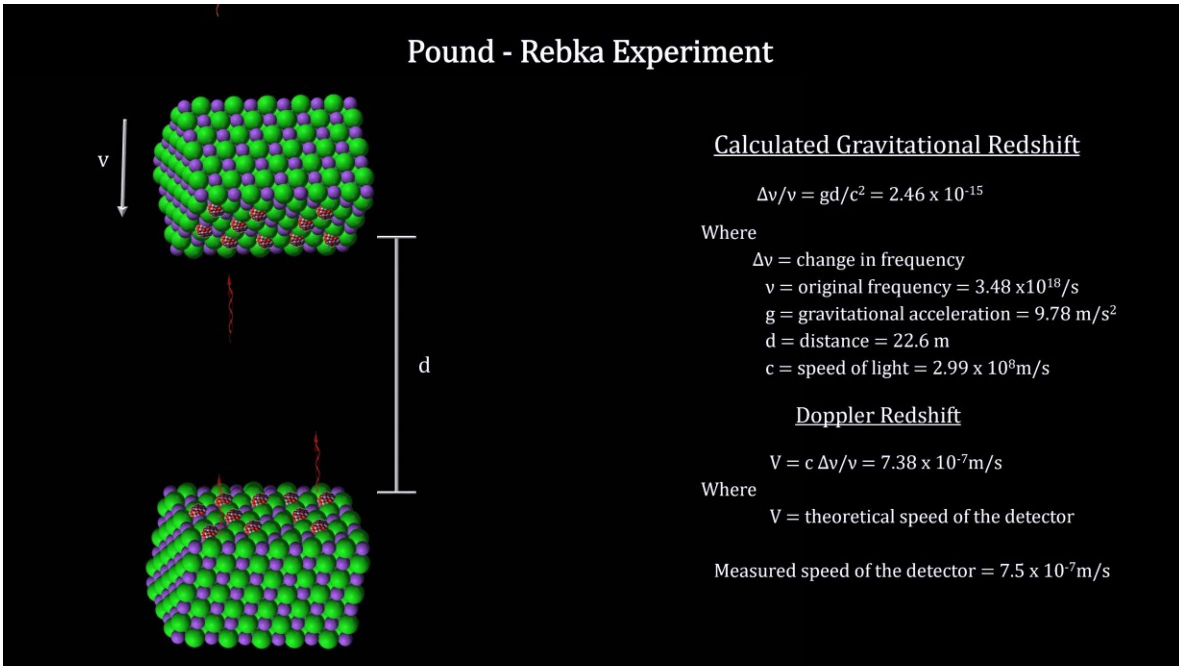
What Mossbauer discovered was that if he imbeds the atoms in a crystal, the recoil is reduced dramatically, and absorption can be established.



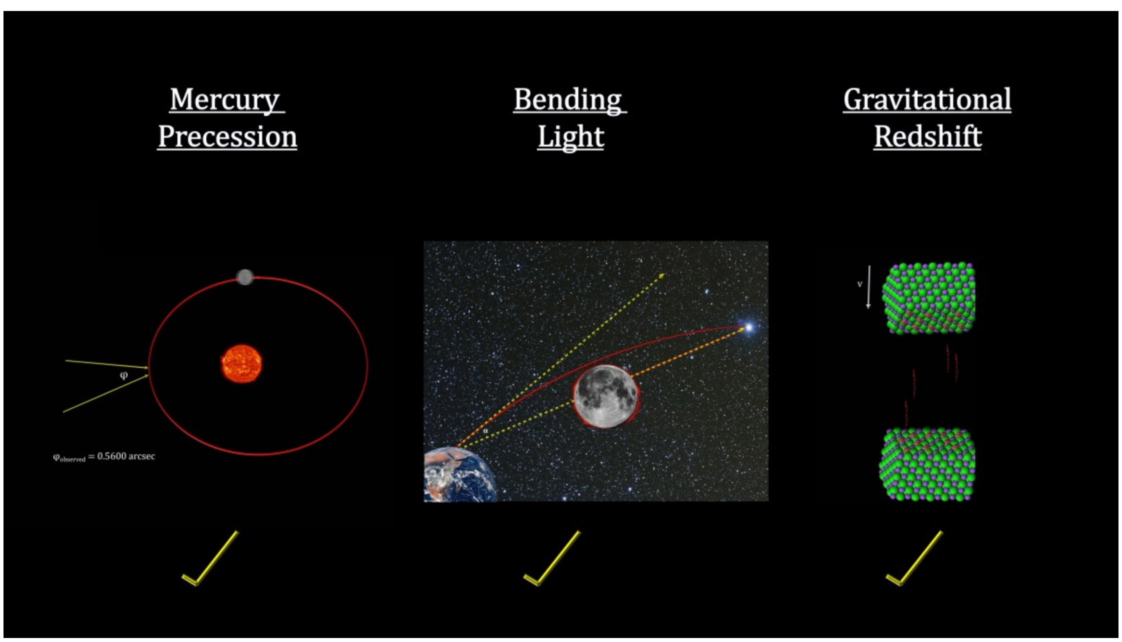
Pound and Rebka use this Mossbauer Effect. They placed an emitter at the bottom of a tower in the Laboratory and installed a detector 22.6 meters above. No absorption was detected because gravitational redshift changed the frequency of the emitted gamma rays so no energy match existed in the detector. The calculated shift was extremely small, but the Mossbauer Effect is sensitive enough to measure it.



They then adjusted the detector's velocity down until absorption occurred. We get the amount the frequency changed using the well understood relativistic Doppler redshift equation just like the Doppler shift in starlight. Their results came to within 1.6% of the value predicted by Einstein's field equations using Schwarzschild's metric.

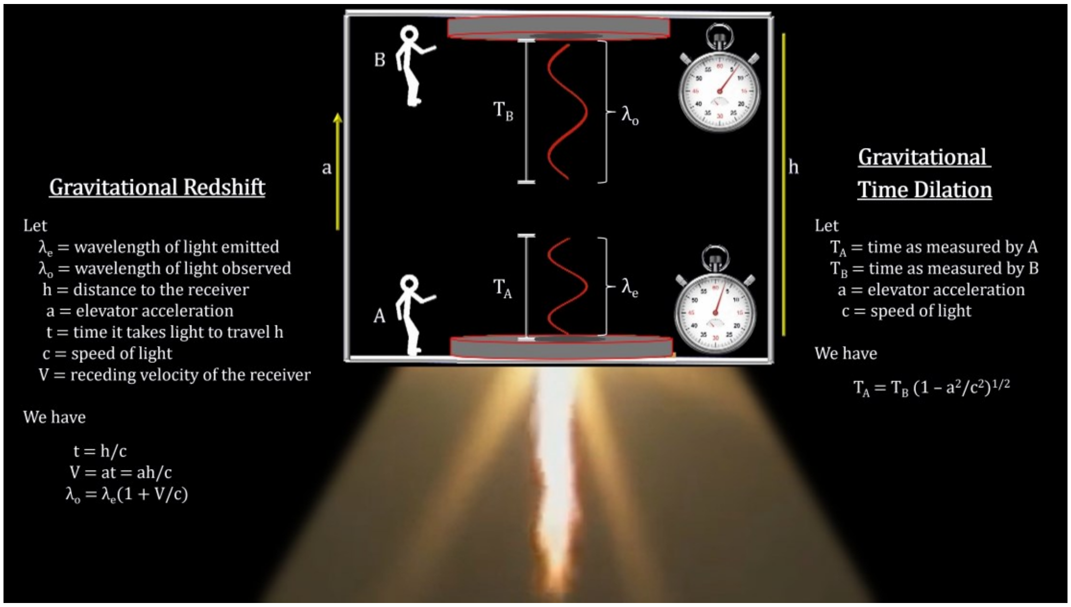


Although this experiment did not produce new results, it showed that gravitational redshift, one of GR's most significant findings, was consistent with all physical conservation laws. This gave the GTR 3 successes out of 3 tests.

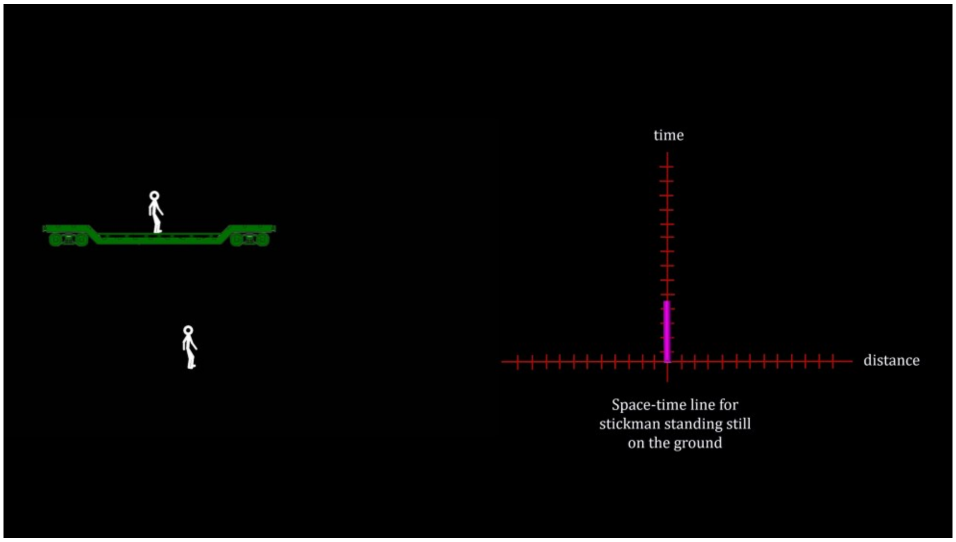


Gravitational Time Dilation

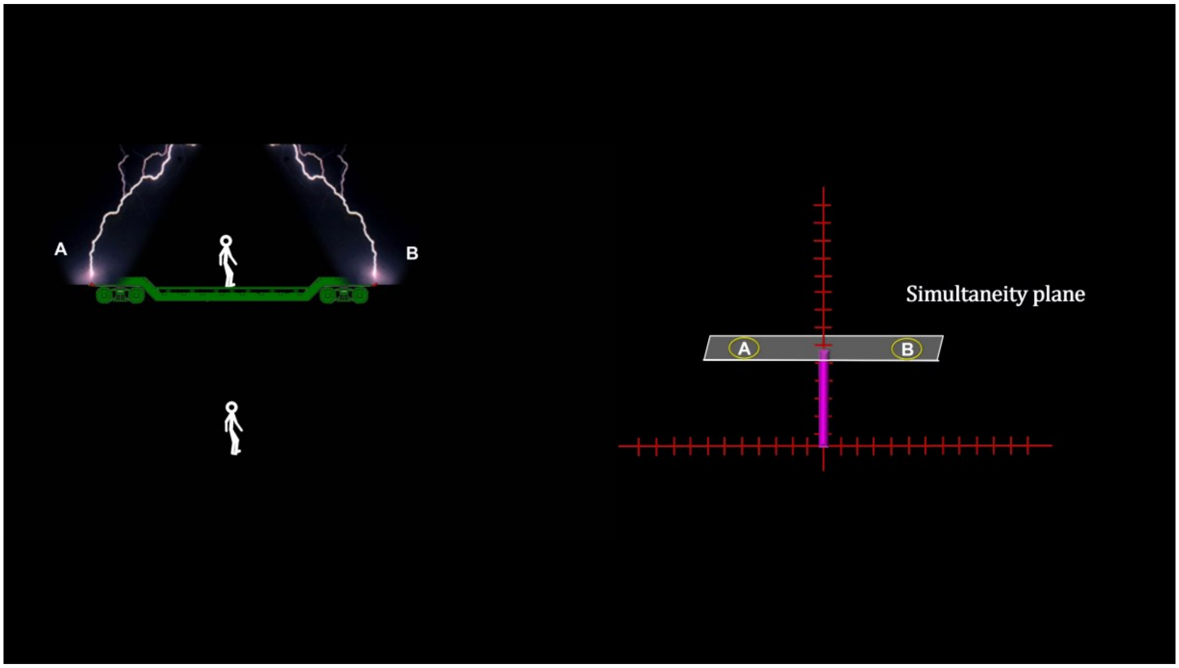
One of the most dramatic consequences of GR is how gravitational redshift leads directly to the conclusion that a gravitational field slows time. We'll use the elevator thought experiment to illustrate how clocks closer to the source of the gravity run slower than those further away. Picture a wave sent from the bottom to the top. Let the leading edge of the wave mark the start of a time interval, and let the trailing edge of the wave mark the end of the time interval. At the receiving end, the viewer sees that, because the length of the wave has been stretched due to gravitational redshift, the length of time observed is slower than the viewer's clock. The lower clock's time is dilated.



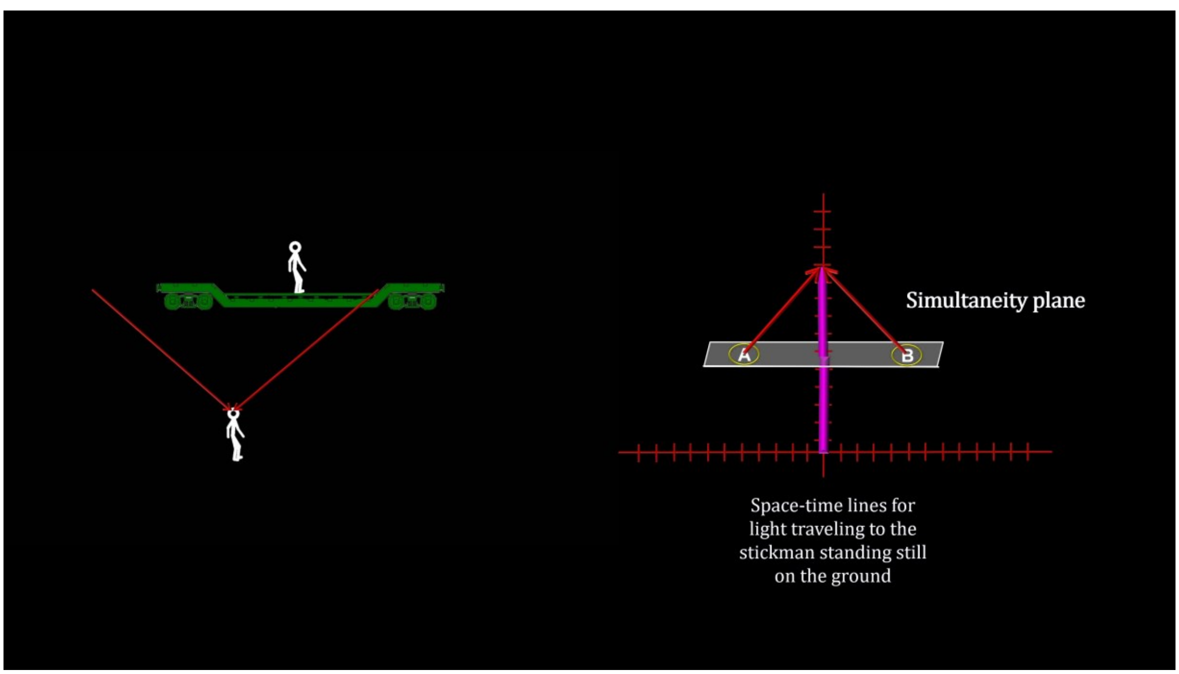
To help see how this works, we'll take another look at the lightning strike for the person on the train, and the person on the ground that we used in our segment on SR. Only this time, we'll map the events to our space-time graph. The world-line for the person standing on the ground is shown in purple.



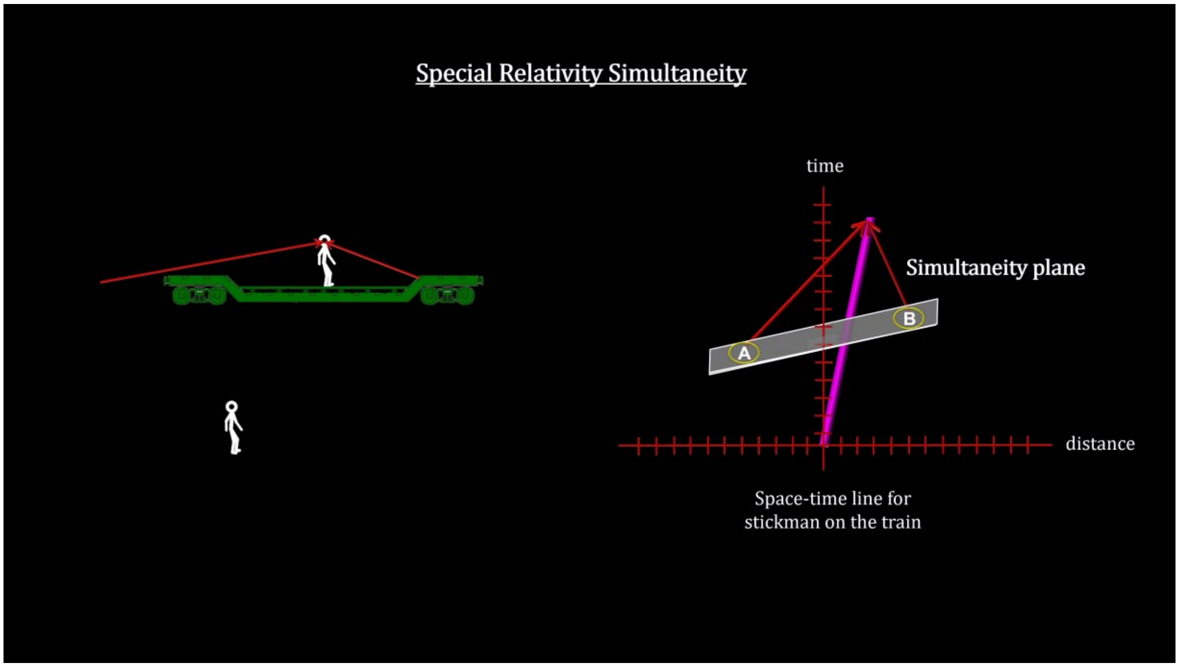
We label the lightning strikes A and B, and place the two events on the space-time graph with A to the left of the person on the ground and B to the right. The plane containing A and B contains all the points that are simultaneous for the person on the ground at the time of the two strikes. We call this the simultaneity plan.



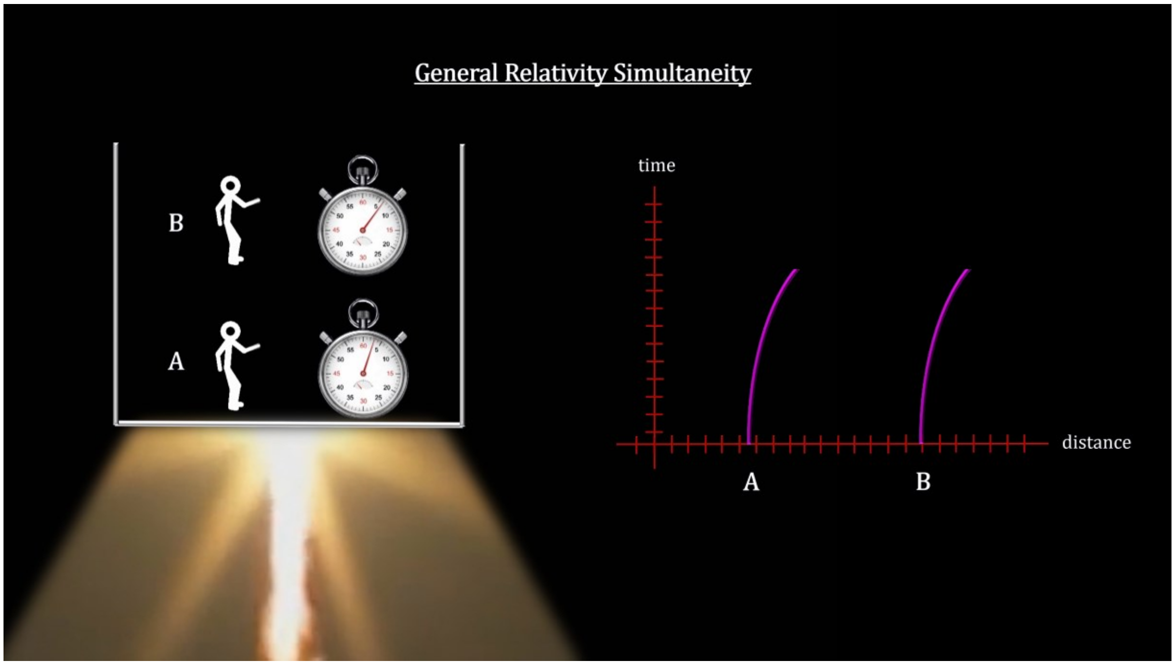
The light from both events travels at the speed of light so their world-line always moves at a 45-degree angle. They reach the person on the ground at the same time. This of course is what makes them simultaneous from the point of view of the person on the ground.



Now let's repeat the lightning strikes so that, from the point of view of the person on the moving train, they strike at the same time. In order for the light to reach the person on the train at the same time, the strike behind him will need to hit first from the person on the ground's point of view, because it will have to travel further to get to the moving person than the light from the strike that hits in front of him. So, we see that the Simultaneity plan, for the moving person, is necessarily tilted up on the right.

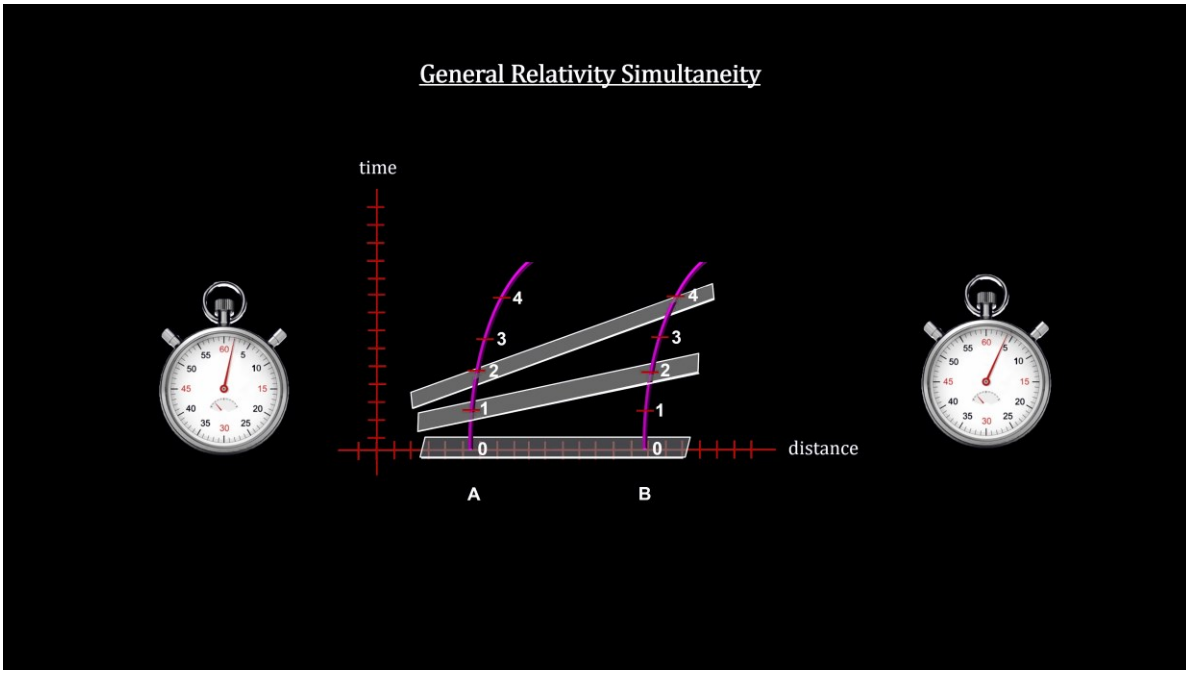


Now we can map the movements of A and B in the accelerating elevator to the space-time graph. The center is the source of the acceleration (aka gravity). A is to the right of it and B is a bit further to the right reflecting their distances from the source of the gravity. As the elevator accelerates, the world-lines on the space-time graph are not straight lines. They curve outwards because their velocity increases with every second.

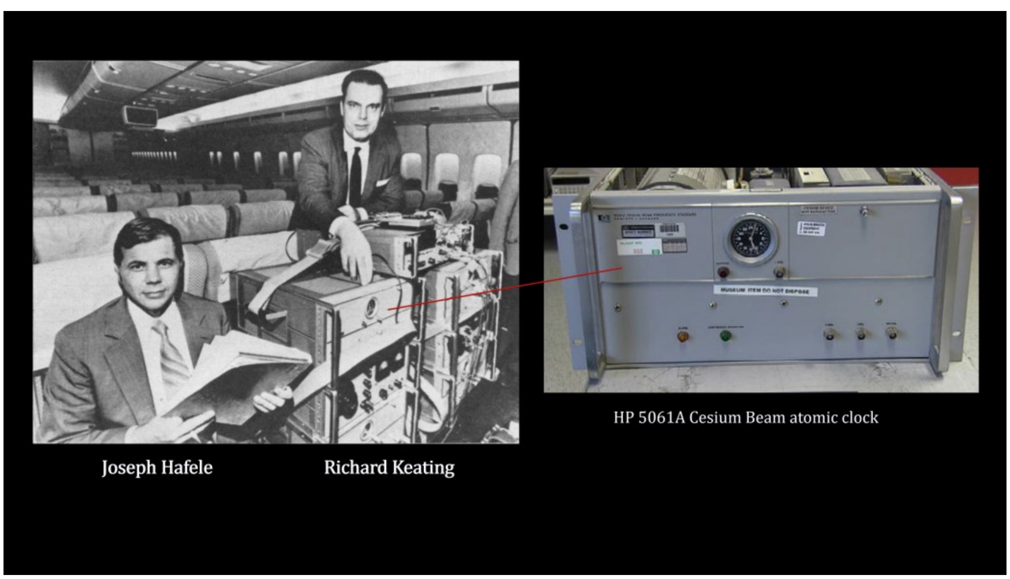


Here we have clocks that measure the proper time elapsed along each person's world-line. They mark the time in their own reference frame. At the start, they were both at rest, so their simultaneity plan is horizontal and they see each other's clocks reading zero. In this example, we see that after 2 seconds, we have a slightly tilted simultaneity plan. B sees that 'at the same time' his clock ticks 2, A's clock ticks 1. 'A' also sees his own clock reading 1 when B's clock reads 2. Continuing to a higher velocity, with the steeper slope for the simultaneity plan, B sees A's clock reading 2 when his own clock reads 4 seconds. 'A' also sees his own clock reading 2 when B's clock reads 4.

A and B **both** agree that A's clock is ticking slower than B's clock. [We are skipping the SR effects of time dilation and space contraction here. They play a big role as the velocities approach the speed of light.] The equivalence principle tells us that the same thing will happen near a massive body. Gravity slows down time. Newton's gravitation has no such implication.

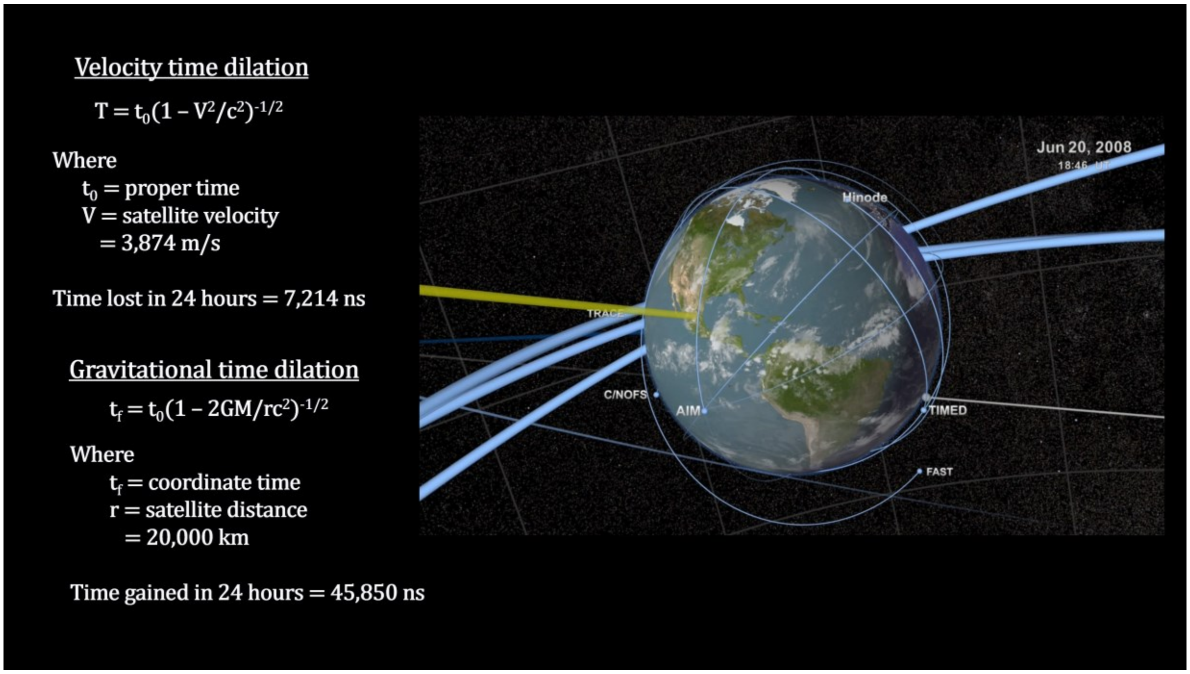


In 1971, Joseph Hafele, a physicist, and Richard Keating, an astronomer, took four cesium-beam atomic clocks aboard commercial airliners. They flew twice around the world, first eastward, then westward, and compared the clocks against others that remained at the United States Naval Observatory. When reunited, the three sets of clocks were found to disagree with one another, and their differences were consistent with gravitational time dilation.



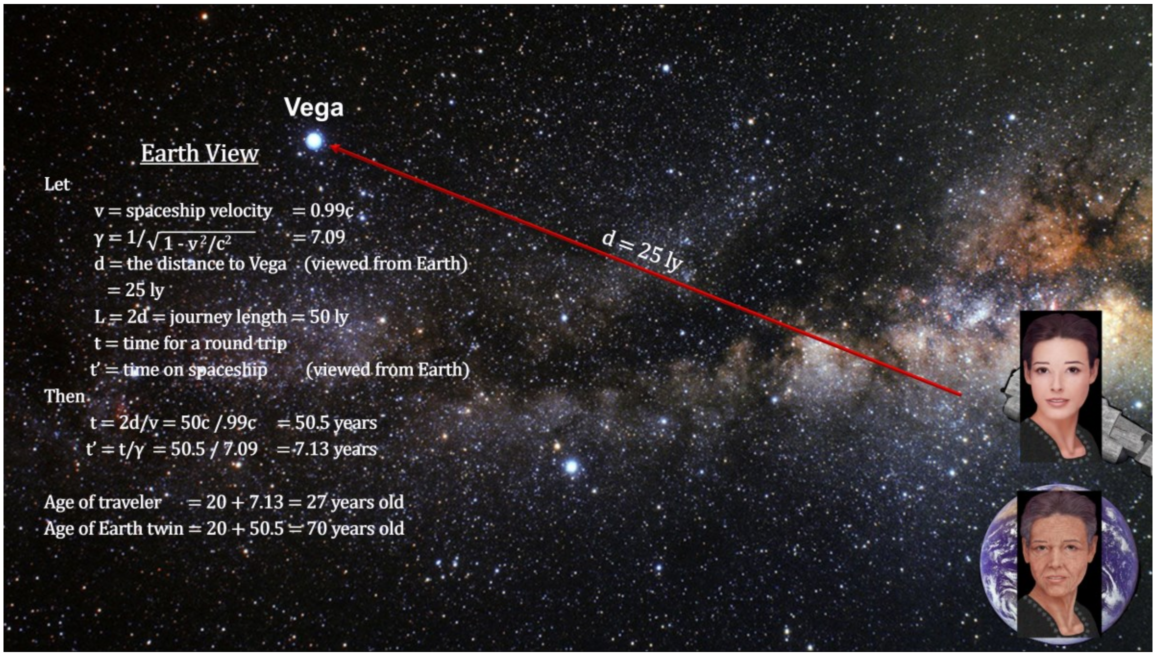
Today, we see this with our GPS systems. In our segment on SR, we saw that time dilation due to velocity differences have GPS satellites losing every day. Time that must be corrected for to get the right positions on the surface of the Earth.

They must also take into account gravitational time dilation due to their being further away from the Earth than clocks on the ground. Based on the Schwarzschild metric, calculations show that the satellites' clocks will gain over 45,000 nanoseconds a day due to this general relativity effect. The accuracy of our GPS system is strong evidence for the correctness of the GTR. [So, the total relativity effect is the difference between the two (45850 – 7214) of 38636 ns per day.]

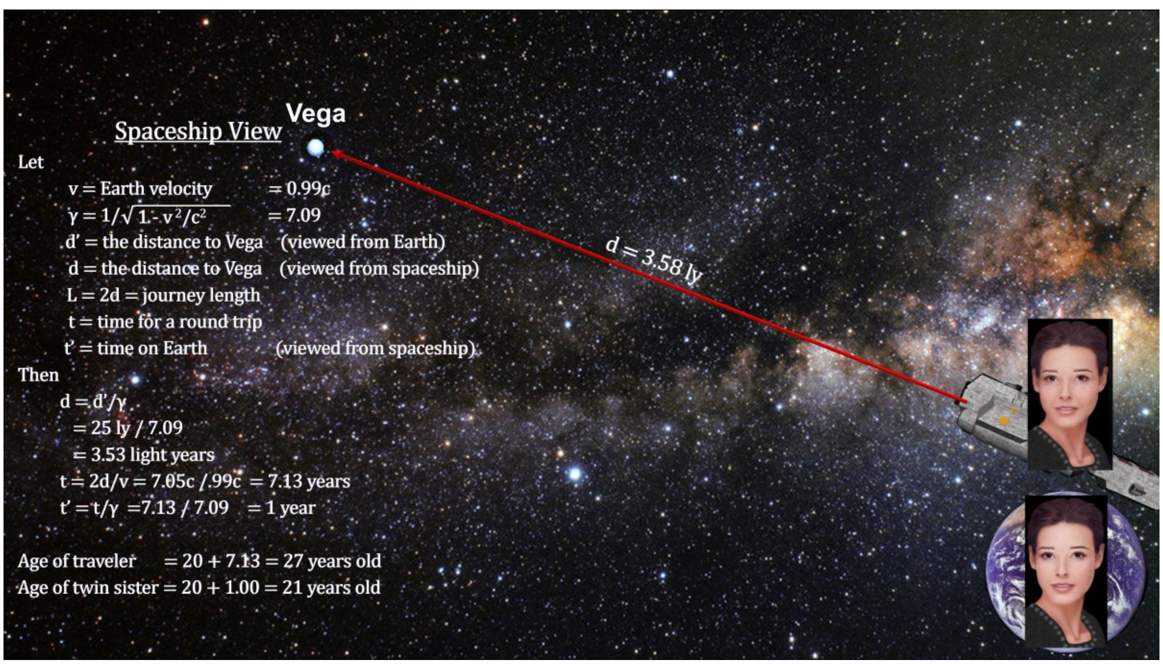


Twin Paradox Resolved

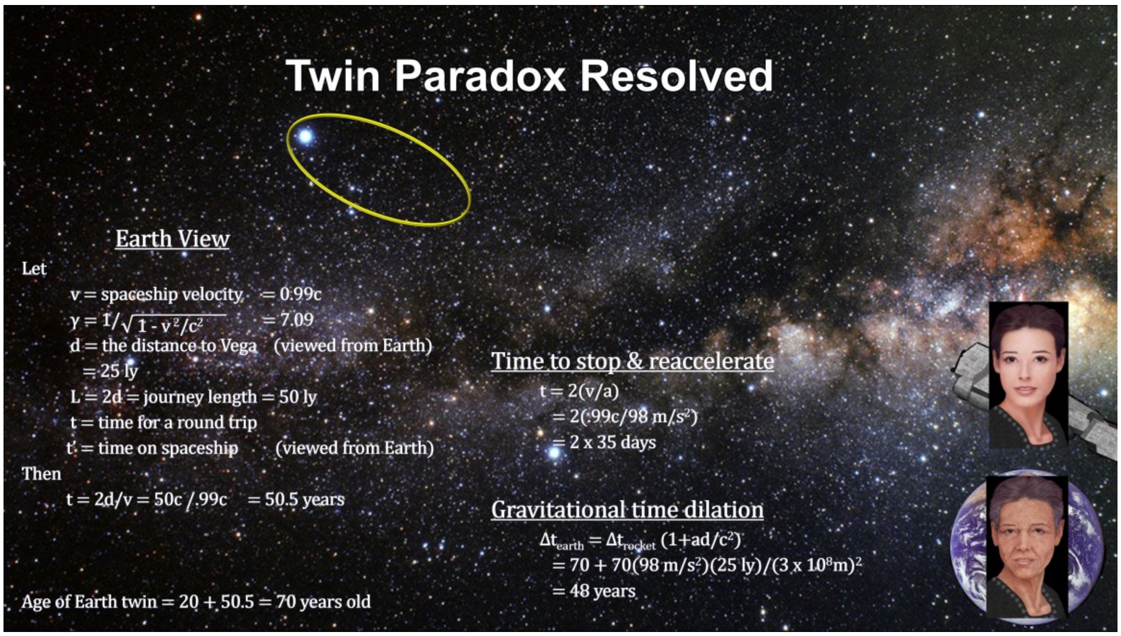
You may recall from our segment on Special Relativity that the time dilation due to velocity creates a paradox. It goes like this. Suppose two 20-year-old twins start out together on the Earth. One of them gets into a spaceship for a trip to Vega traveling at 99% of the speed of light. The person on the Earth sees the trip there taking just over 25 years, and the trip back taking the same amount of time. She is over 70 years old when the ship carrying her twin sister arrives back on Earth. But she also observes that her twin's clock ran a good deal slower than hers during the trip. Her twin is aging more slowly than she is. At 99% of the speed of light, time dilation would have the twin at just over 27 years old on her return - young enough to be her daughter rather than her twin.



But, from the point of view of the twin on the spaceship, she is motionless in her own reference frame, and the twin on the Earth is moving away and back. In addition, she sees the distance to Vega at only 3.5 light years due to space contraction. She also sees the twin on the ground aging slower than her over the 7-year journey. By her observations, her sister will be only 1 year older on her return due to time dilation. That's 6 years younger than she is – not 27 years older! How can it be that they are both older than the other? This is the paradox.

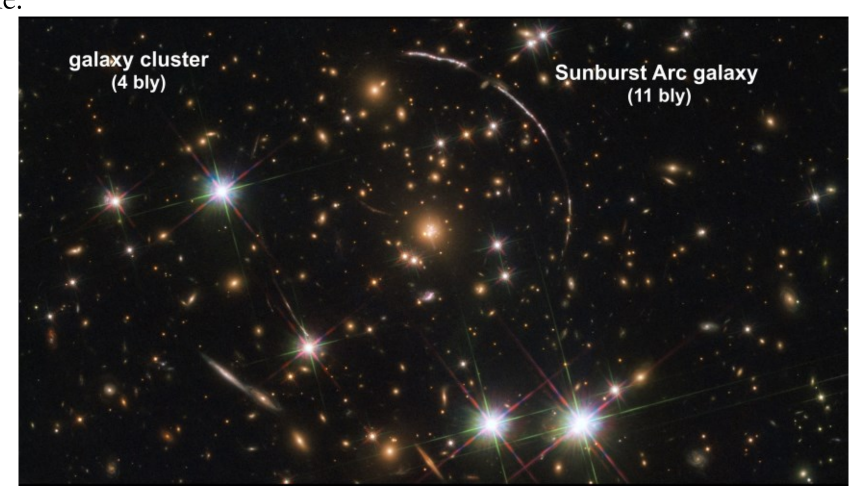


But there is at least one point where the twin in the rocket is not in an inertial reference frame. As the spaceship approaches Vega, it decelerates to a stop and then re-accelerates back to Earth. The traveling twin finds that she is in a gravitational field, and gravitational time dilation needs to be taken into account. Let's say her acceleration is 10 g's or 98 m/s^2. At this rate it will take her 35 days to decelerate to 0 and another 35 days to reaccelerate back to 99% of the speed of light. Gravitational time dilation shows that, as her clock ticks 70 days, her twin's clock on Earth will have ticked 18,134 days. That's 48 years. The twin on Earth agrees. So instead of both twins thinking the other should be younger, they both agree that the twin on the rocket to Vega and back is younger. No contradiction is involved and the paradox is resolved.



Conclusion

The GTR is now 100 years old. All the basic tests have shown it to be an accurate description of nature as we find it. The implications for astronomy have been enormous. In the next chapter, we'll cover Gravitational Lensing and how it enables us to see deeper into space than anyone ever thought possible.



Music

Mozart - Flute Concerto No 2 - Composed in the spring or summer of 1777 as an Oboe concerto.

Grieg - Holberg Suite, Sarabande (Andante). Based on eighteenth century dance forms, this was written in 1884 to celebrate the 200th anniversary of the birth of Danish Norwegian humanist playwright Ludvig Holberg.

Korsakov - Capriccio Espagnol. Written in 1887, this is an orchestral suite, based on Spanish folk melodies.

Greek letters:
- α β γ δ ε ζ η θ ι κ λ μ ν ξ ο π ρ σ τ υ φ χ ψ ω
- Α Β Γ Δ Ε Ζ Η Θ Ι Κ Λ Μ Ν Ξ Ο Π Ρ Σ Τ Υ Φ Χ Ψ Ω

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