Gravitational Lensing

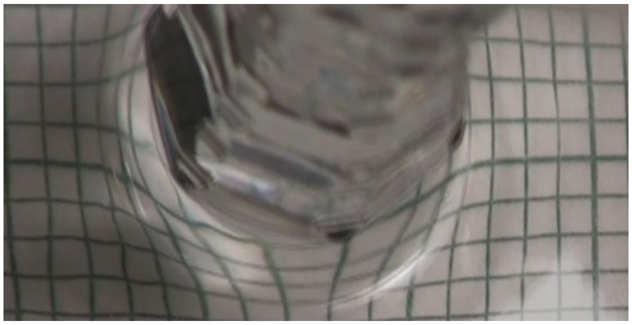
{**Abstract:** *In this segment of the "How Fast Is It" video book, we cover gravitational lensing. First, we illustrate how the light is bent, followed by some Einstein Ring examples. We then cover the lens itself: how it magnifies; how it distorts; and how images are mapped back to the source celestial object. We also cover critical curves that can magnify an object by thousands of times. We use Abell 68 and MACS 1206 as examples. We cover flickering quasars and how they can be used to calculate the Hubble constant. We follow that with multiple Type 1a supernovae image timings that can also be used to calculate the Hubble constant. We use the supernova Refsdal with its Einstein Cross as an example. We then cover lensing galaxies like Hamilton's Object, Starburst Arc and Abell 1689-zD1. We finish with lensing stars namely Icarus and Earendel.*}

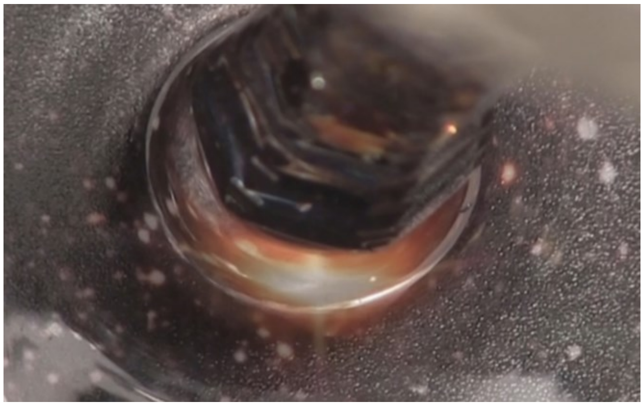
Introduction

We have seen how a mass like our Sun can bend light. It's not hard to extend this simple concept to what a galaxy with hundreds of billions of stars could do. Galaxies have a mass distribution with the maximum at the center trailing off to a minimum at the edges. This distribution acts like a lens for light passing through it.

Here's the bottom of a wine glass with a configuration similar to a galaxy.

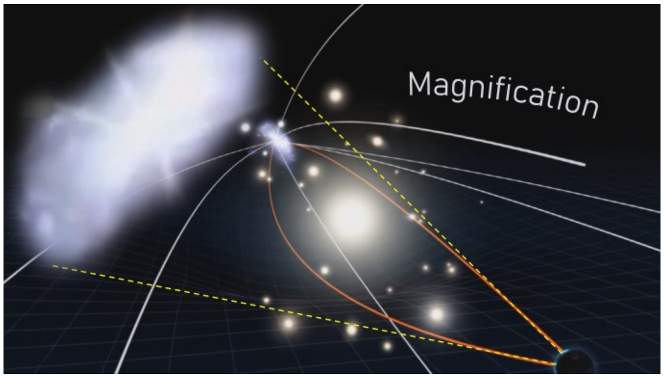
You can see what it does to the graph lines as it passes over them.





When we repeat the movement over a star, we see how it distorts the light into a circular pattern.

We call the bending of light through gravitational fields Gravitational Lensing. Here's how this lensing works on a galactic scale. A distant galaxy would be seen here on Earth directly if there were no intervening massive cluster to bend its light.

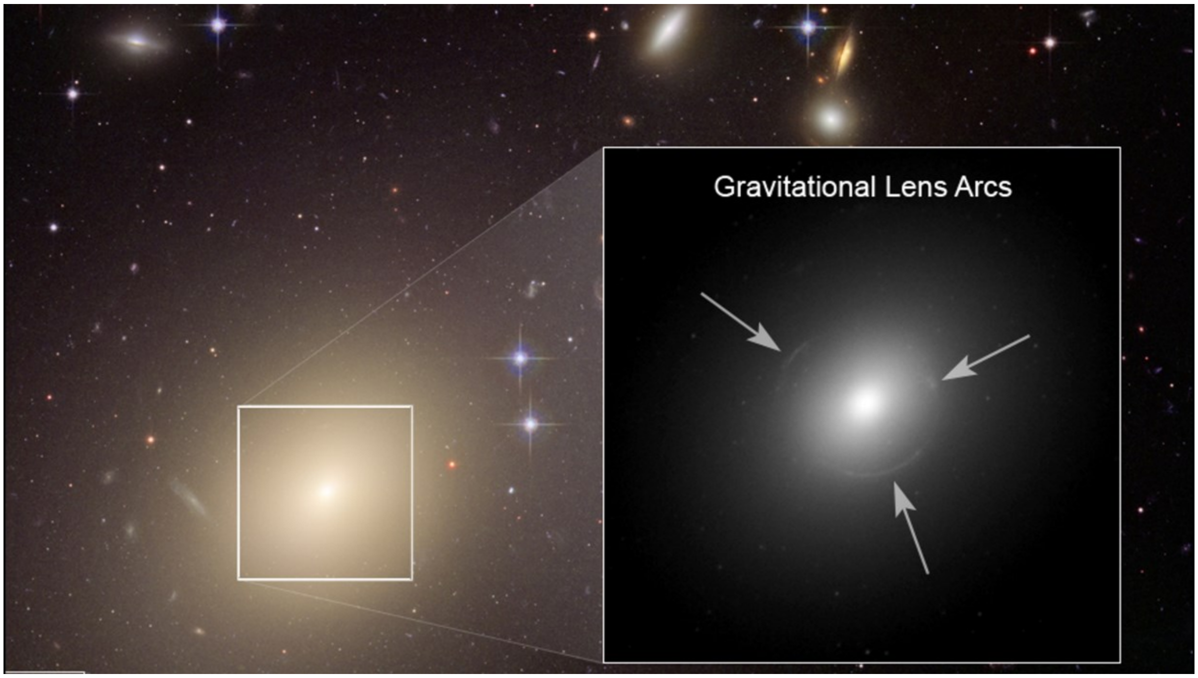


But with such a cluster, the light from the distant galaxy gets bent. Light bent in our direction will continue on to Earth. Working back from Earth's point of view, we see the distorted and magnified image. The amount of bending depends entirely on the structure of the lensing cluster. Magnifications can range from factors of 2 to 40 times the size of the distant object. We'll see that, under the right circumstances, an object can be magnified over a thousand times.

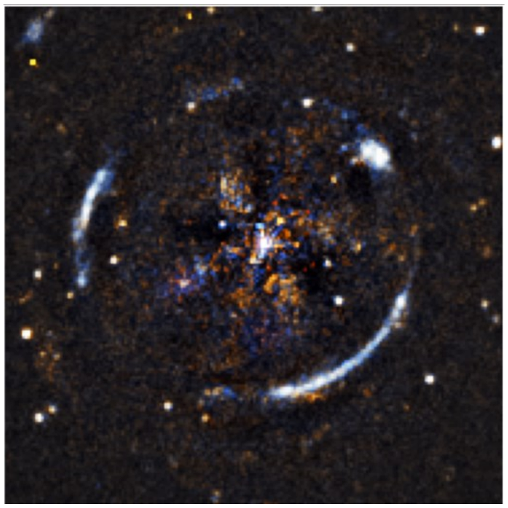
Einstein Rings

In the course of analyzing this Hubble image, astronomers discovered that ESO 325 G004, is actually a gravitational lens. This means that the focusing power of the enormous mass making up the galaxy caused the light from some background object, probably a distant "dwarf" galaxy, to be deflected and magnified.

As a result, the more distant galaxy appears brighter, and distorted into the shape of an arc, or ring, known as an "Einstein ring" because the phenomenon was first predicted by Albert Einstein. This is the closest known example of strong gravitational lensing.

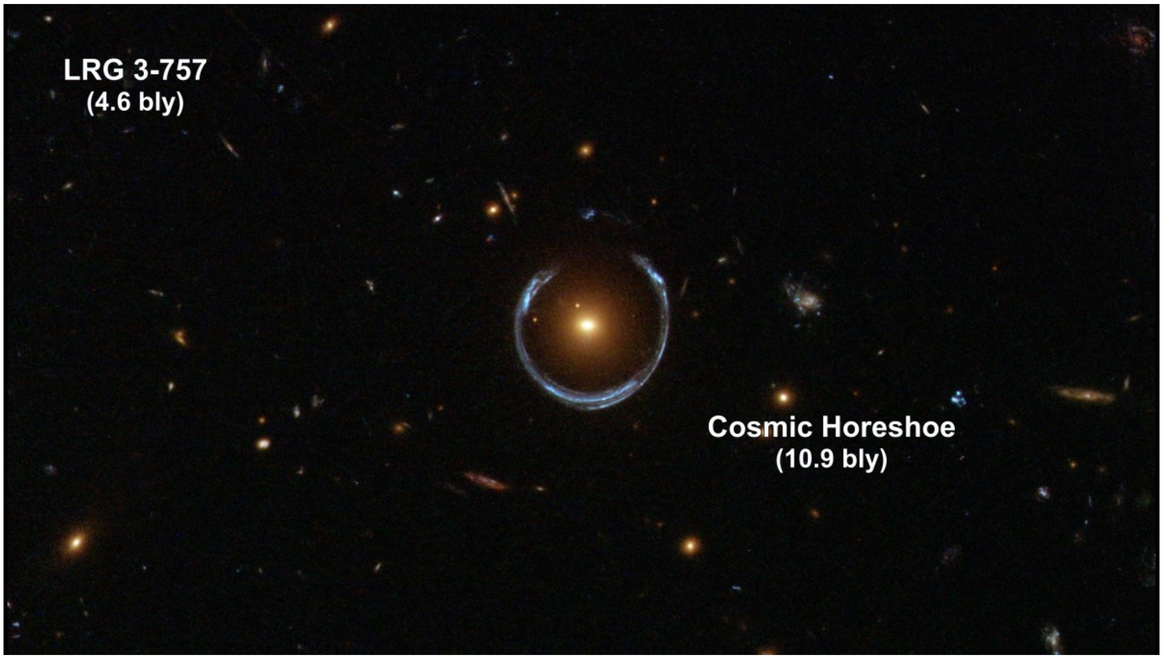


Here's an image created by adding data collected by the Multi Unit Spectroscopic Explorer (MUSE) instrument on the European Southern Observatory (ESO) Very Large Telescope (VLT). [The instrument measured the velocity of stars in the foreground galaxy to produce a velocity dispersion map. This enabled astronomers to infer ESO 325's mass. The inset shows the map, which becomes visible after subtracting the foreground lens light.]

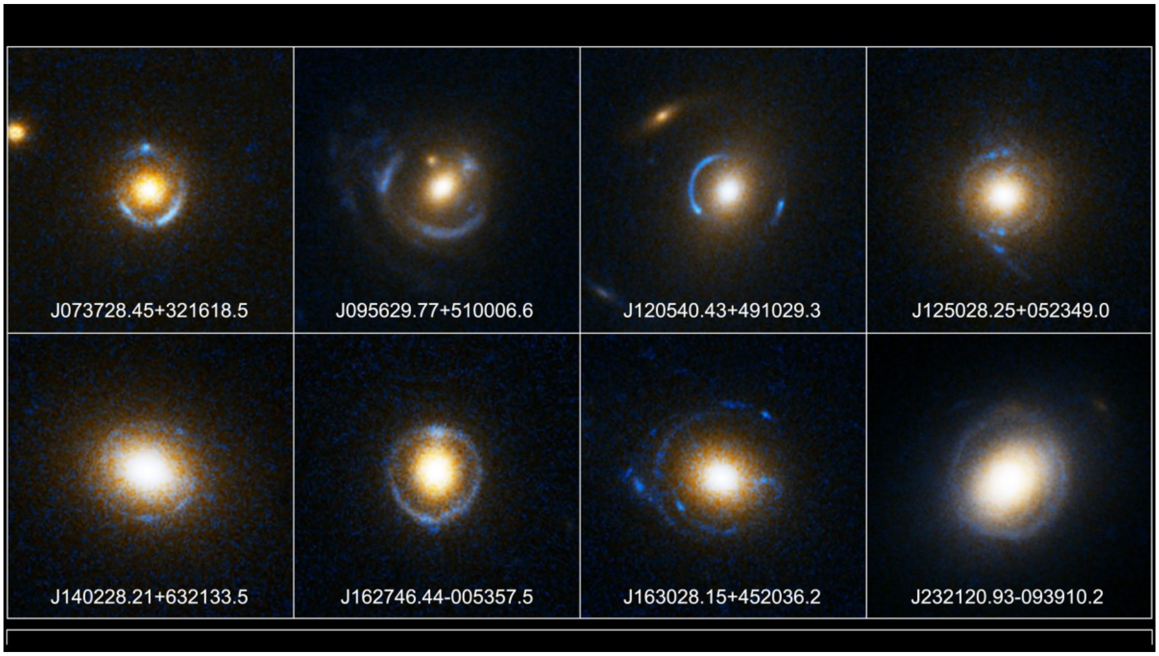


This was the first test of Einstein's light bending theory done outside of our own galaxy. It passed the test - demonstrating that the theory held across the universe and not just in the Milky Way.

Here's another one. The foreground Luminous Red galaxy 4.6 bly away has about ten times the mass of the Milky Way. The blue arc around it is the gravitationally lensed image of a more distant galaxy 10.9 bly away. The object's nickname is "the Cosmic Horseshoe". Astronomers first discovered it in 2007 using data from the Sloan Digital Sky Survey. But this Hubble image offers a much more detailed view.

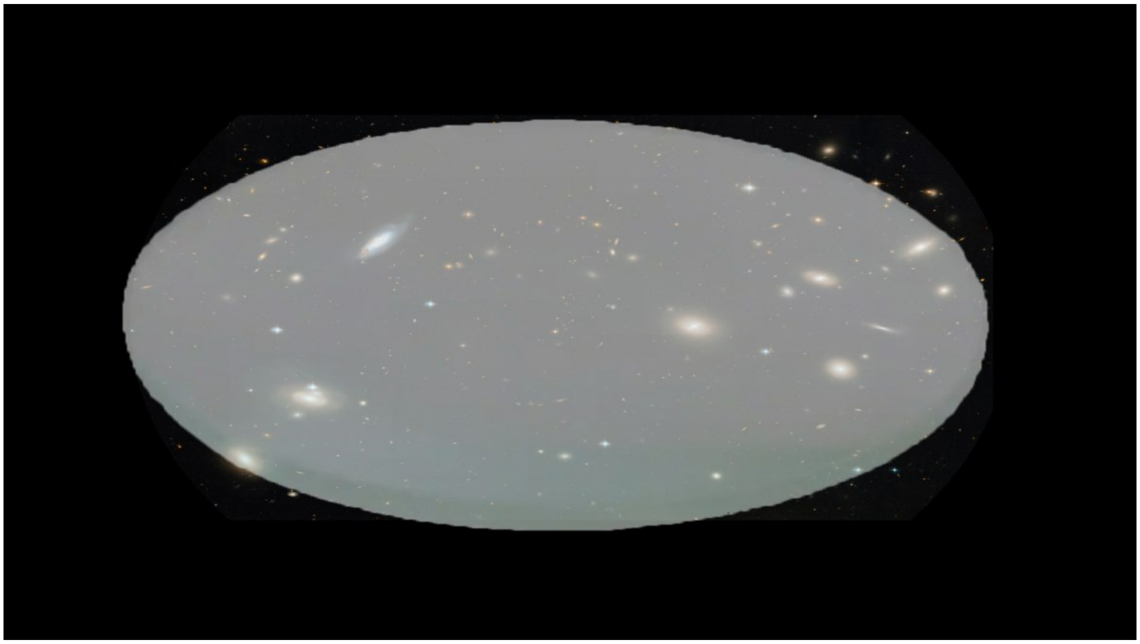


Although the universe is filled with galaxies, Einstein Rings are a rare occurrence because it requires an almost perfect alignment of a distant galaxy with an intervening one that has enough mass to gravitationally focus the light. But a number have been found. Here are 8 of them from the Sloan Digital Sky Survey (SDSS) and Hubble.

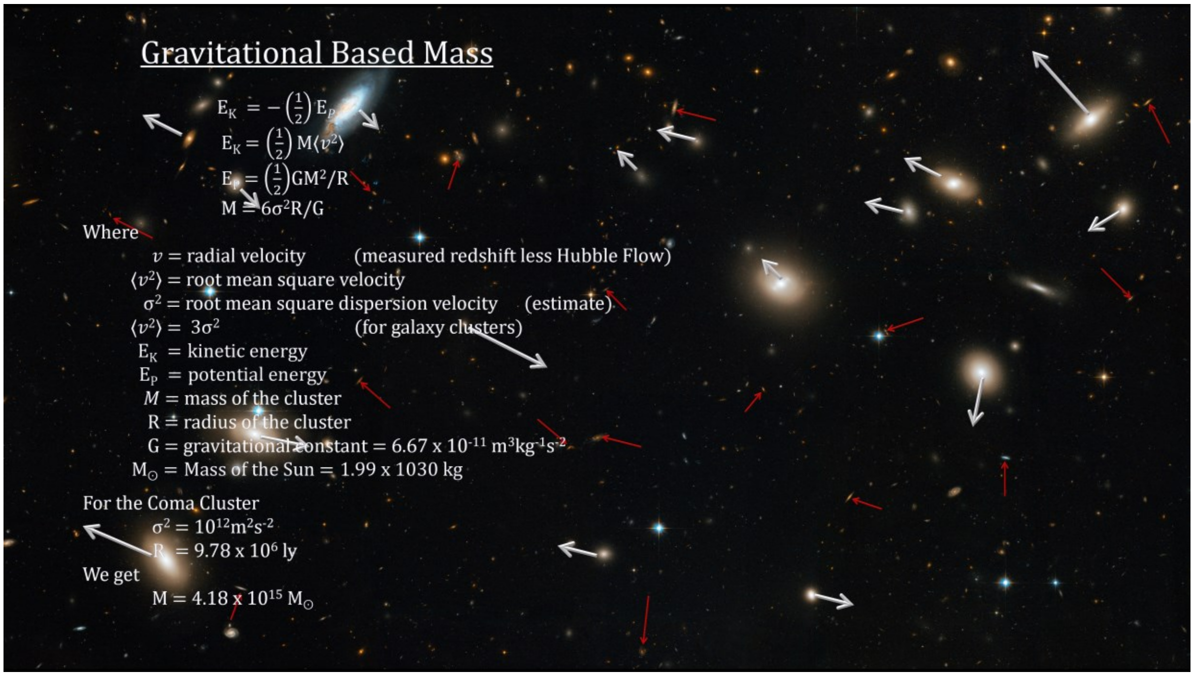


The Gravitational Lens Itself

We've seen how massive galaxies can act as gravitational lenses to create Einstein Rings. You can imagine how massive clusters with thousands of galaxies would produce much larger and more powerful gravitational lenses. But before a gravitational lens can provide accurate detailed information about the object being lensed, we need to know its light bending characteristics. Clusters of galaxies don't provide smooth lenses. They are quite complex and no two are alike. The source object's light is bent by the cluster's clumpy matter distribution. And to make it harder, most of the cluster is dark matter that we can't see directly. All this makes cluster lens analysis very difficult and time consuming. But with these 'magnifying glasses' in space being our only window into the early Universe, astronomers and astrophysicists have been analyzing galaxy cluster lensing for half a century now. We'll start with the Coma Cluster to illustrate how they do it.



The first step is to measure the radial velocities of every galaxy in the cluster from their doppler shifts. This is then generalized into their three-dimensional velocity dispersion statistical equivalents. This galaxy motion gives us the kinetic energy of the cluster. This allows us to solve for the mass of the cluster. In addition, each lensed object found in the cluster is analyzed to calculate the magnitude of the mass needed to create the observed distortion. The end result is a model for what the cluster will do with the light passing through it.



Gravitational Based Mass

$$E_K = -\left(\frac{1}{2}\right)E_P$$

$$E_K = \left(\frac{1}{2}\right)M\langle v^2\rangle$$

$$E_P = \left(\frac{1}{2}\right)GM^2/R$$

$$M = 6\sigma^2 R/G$$

Where

v = radial velocity (measured redshift less Hubble Flow)

$\langle v^2\rangle$ = root mean square velocity

σ^2 = root mean square dispersion velocity (estimate)

$\langle v^2\rangle = 3\sigma^2$ (for galaxy clusters)

E_K = kinetic energy

E_P = potential energy

M = mass of the cluster

R = radius of the cluster

G = gravitational constant = 6.67×10^{-11} m^{3}kg^{-1}s^{-2}

$M_\odot$ = Mass of the Sun = 1.99 x 1030 kg

For the Coma Cluster

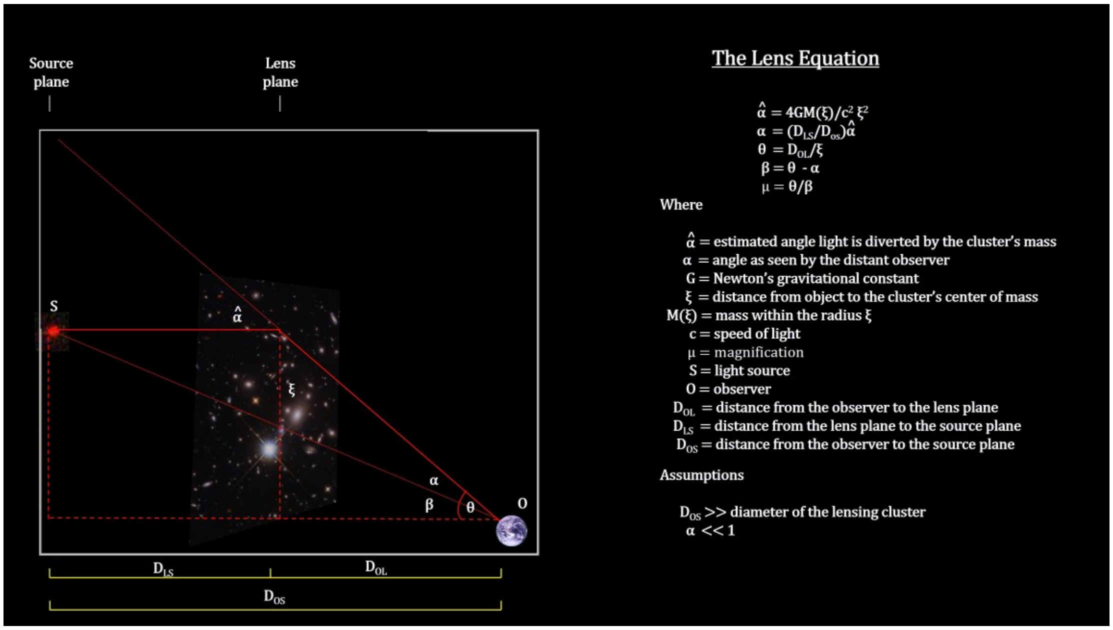
$\sigma^2 = 10^{12}$m^{2}s^{-2}

R = 9.78×10^6 ly

We get

$M = 4.18 \times 10^{15}\ M_\odot$

To illustrate the basic geometry of gravitational lensing, we'll use the giant galaxy cluster WHL0137-08's lensing of the Sunrise Arc. Here's the basic geometry of gravitational lensing. This is Sunrise Arc light on a direct path to us as if the galaxy cluster was not there. We could not detect this light. It is too dim for even our most powerful space telescopes. Here's its light headed in another direction. As it encounters the cluster, it is bent towards us by the cluster's mass. We can estimate the deflection angle once we know the mass and center of mass of the cluster. On Earth, we observe the image to be on a straight line at an angle from its actual direction. The Lens Equation gives us these angles. This geometry enables us to map points seen on the lens plane back to its position on the source plane.



The Lens Equation

$\hat{\alpha} = 4GM(\xi)/c^2\,\xi^2$
$\alpha = (D_{LS}/D_{os})\hat{\alpha}$
$\theta = D_{OL}/\xi$
$\beta = \theta - \alpha$
$\mu = \theta/\beta$

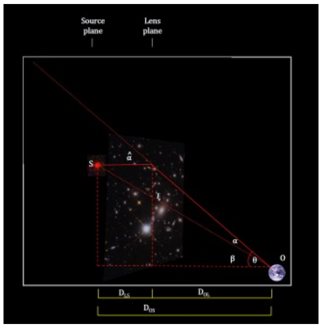
Where

$\hat{\alpha}$ = estimated angle light is diverted by the cluster's mass
α = angle as seen by the distant observer
G = Newton's gravitational constant
ξ = distance from object to the cluster's center of mass
$M(\xi)$ = mass within the radius ξ
c = speed of light
μ = magnification
S = light source
O = observer
D_{OL} = distance from the observer to the lens plane
D_{LS} = distance from the lens plane to the source plane
D_{OS} = distance from the observer to the source plane

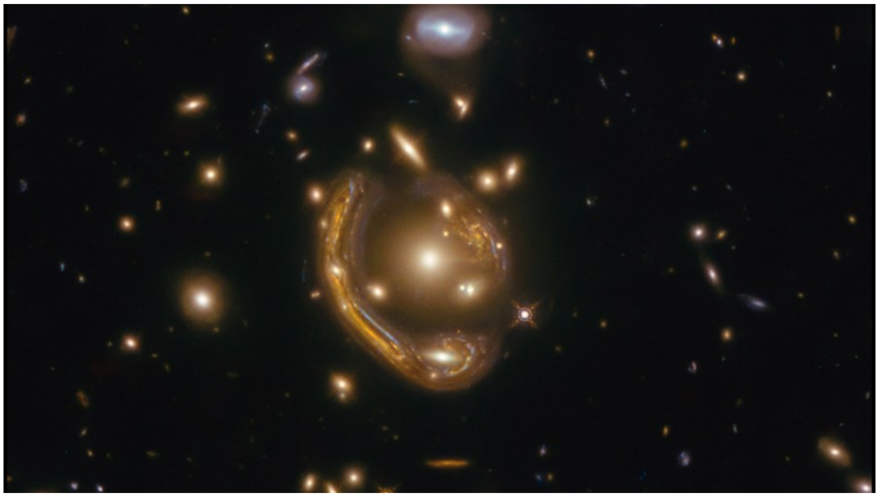
Assumptions

$D_{OS} >>$ diameter of the lensing cluster
$\alpha << 1$

Note that the magnitude of the light bending changes for object's closer to the lens. The lens' magnification will decrease as the ratio of the distance from the lens to the object over the distance of the lens to the observer decreases.

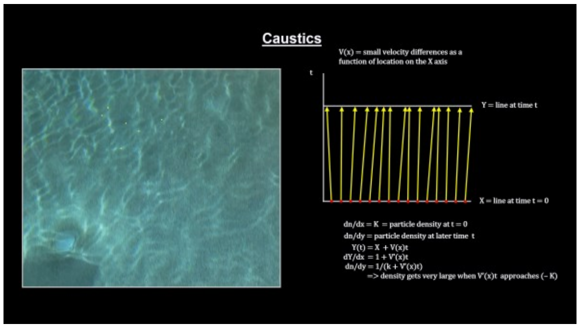


Here’s an example. In this image, a remote galaxy has been magnified by a factor of 20, and distorted by gravitational lensing. Lensing effects also created multiple apparitions around the curved arc of the single background magnified galaxy. The object, GAL-CLUS-022058s, was nicknamed the "Molten Ring" because of its appearance.



The typical magnification created by this lensing ranges from 20 to 40 times the source size. But there is one key additional optical process involved that can magnify an object ~~10s of~~ thousands of times over. Here’s how it works.-Picture a set of uniformly distributed photons on a line, each moving in a slightly different direction. They start out with a uniform particle density. But, because of the small velocity differences, the particle density will vary as time goes by. Areas of high and low density will develop. The density at a later time ‘t’ is described by an equation. The equation has hot spots when the denominator approaches zero.

Extending this to three dimensions, we get density peaks along curved lines that themselves intersect at points with maximum intensity. I see this phenomenon in my own back yard swimming pool. Sunlight is evenly distributed as it reaches the water’s surface. Small waves on the surface are creating small changes in the sunlight’s direction. It’s the caustic process that generates the lines at the bottom of the pool. These lines are not ‘ripples in space-time’. They are simply lines of intense light magnification.



Caustics

$V(x)$ = small velocity differences as a function of location on the X axis

$dn/dx = K$ = particle density at $t = 0$

dn/dy = particle density at later time t

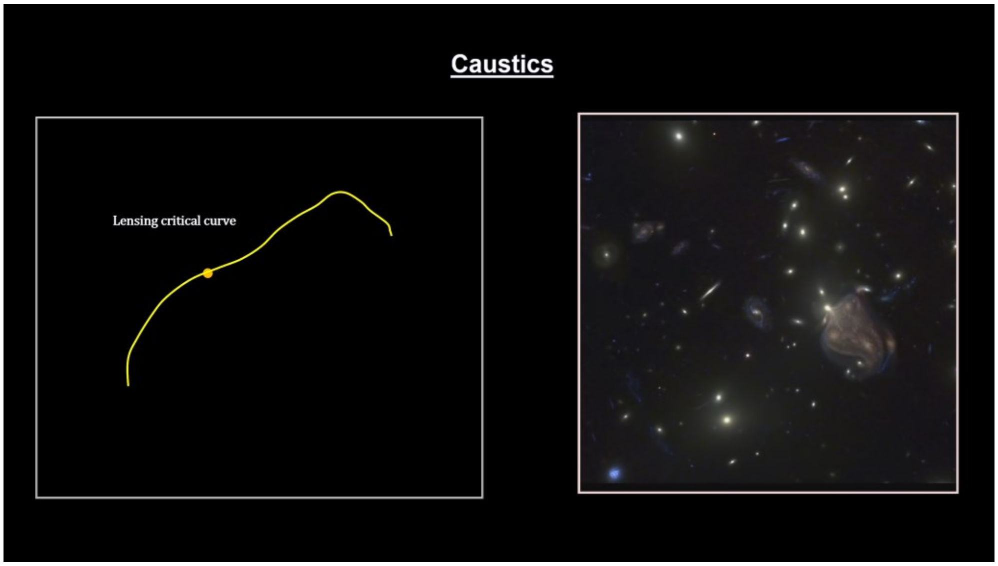
$Y(t) = X + V(x)t$

$dY/dx = 1 + V'(x)t$

$dn/dy = 1/(k + V'(x)t)$

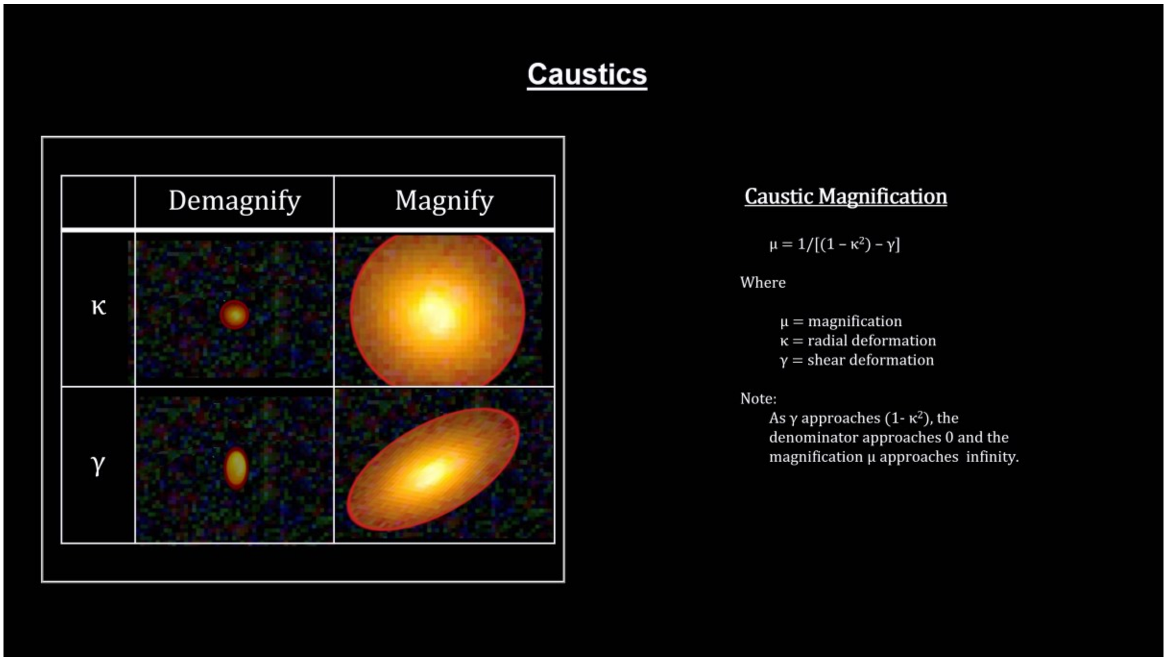
=> density gets very large when $V'(x)t$ approaches $(-K)$

Light passing through a galaxy cluster is impacted in exactly the same way. Lines of extreme magnification, referred to as lensing critical curves, are created by the caustic process. As a distant object approaches and passes over a critical curve, it's image will be duplicated side by side and under some circumstances will develop yet a third image.



Caustics

Astrophysicists and astronomers model these lines, and use the lens equations to map them back to the source. They can then reconstruct the shape and dimensions of an object, determine its deformations, calculate its observed magnification and deduce its intrinsic luminosity.



Caustics

Caustic Magnification

$\mu = 1/[(1 - \kappa^2) - \gamma]$

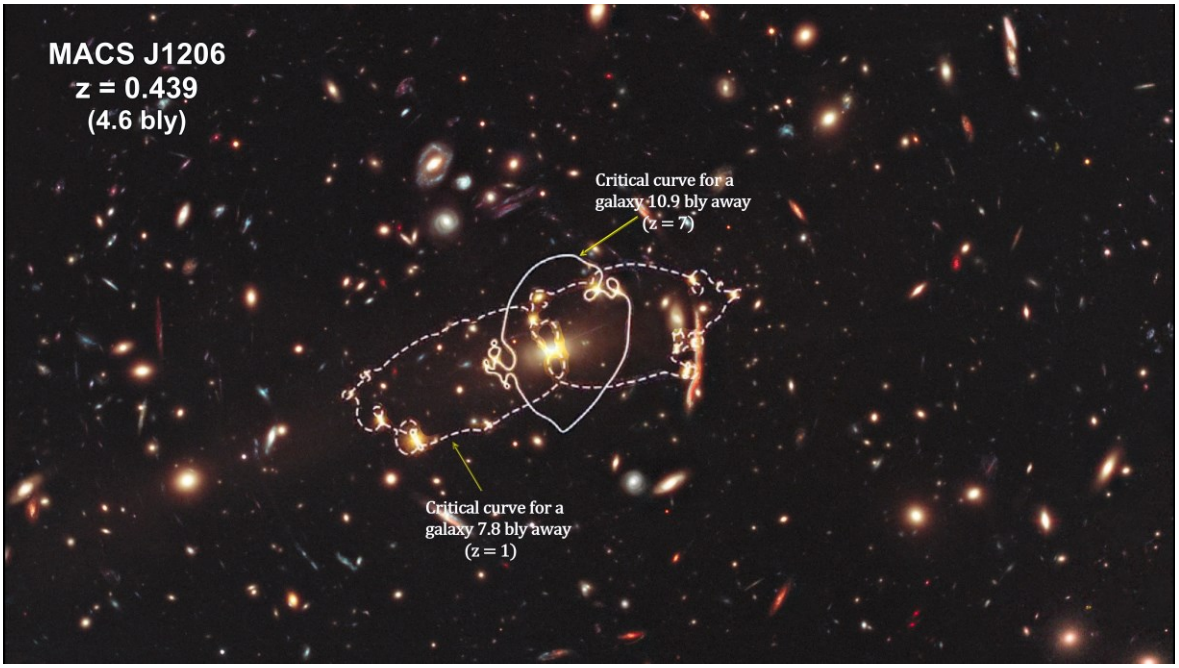
Where

μ = magnification
κ = radial deformation
γ = shear deformation

Note:
As γ approaches $(1 - \kappa^2)$, the denominator approaches 0 and the magnification μ approaches infinity.

Here’s an example. This galaxy is visible twice, because its light through a critical curve or sometimes referred to as a ripple of dark matter in Abell 68.

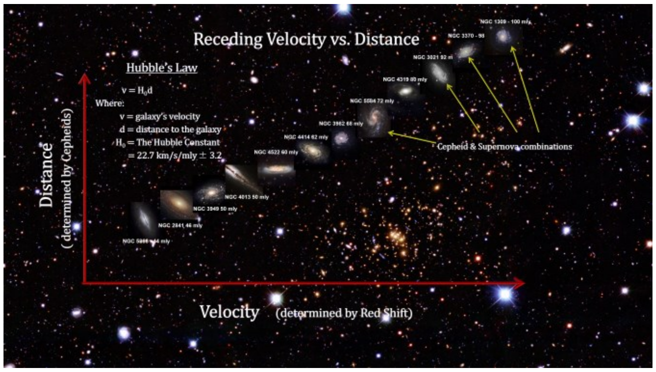
The galaxy cluster MACS 1206, 4.6 bly away, has produced 47 multiple images of 12 newly identified more distant galaxies, as seen in this Hubble picture. This dashed line identifies a calculated critical curve for a galaxy (z=1) 3.2 bly behind the cluster. This solid line identifies a calculated critical curve for a galaxy (z=7) 8.3 bly behind the cluster. You can see how the gravitational lens characteristics changes with the distance to the far away sources.



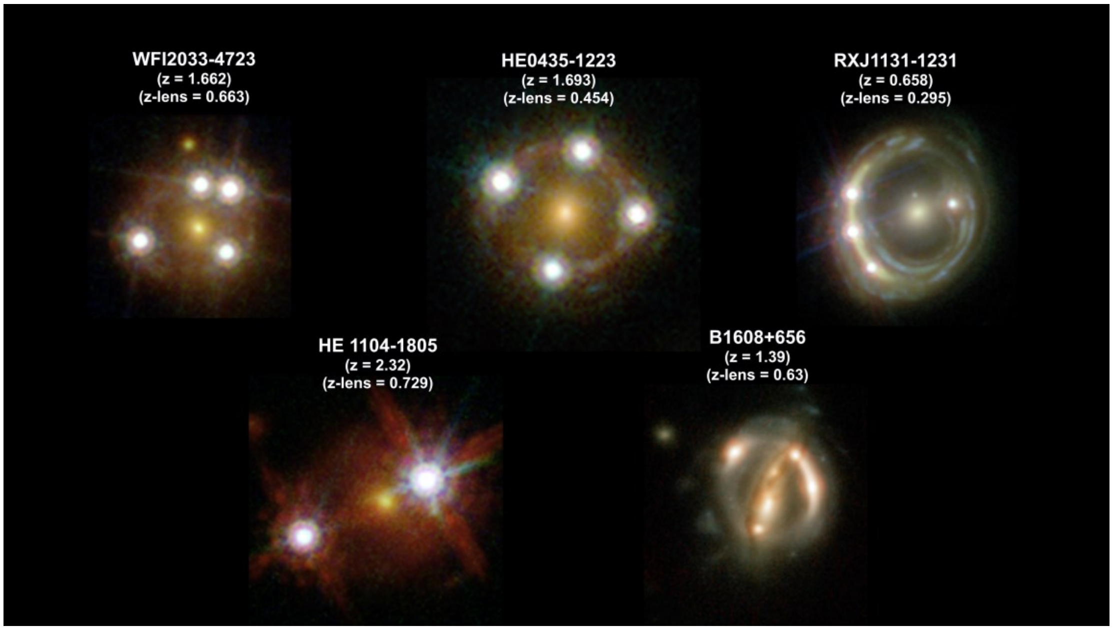
Flickering Quasars and the Hubble Constant

We covered the Hubble constant in depth in the "How Old Is It" video book. It was developed by Edwin Hubble in the late 1920s with his studies of nearby galaxies.

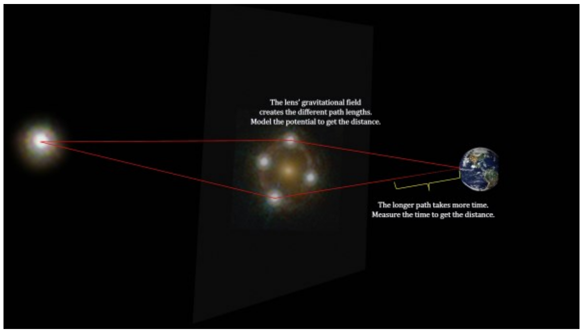
He used the Cosmic Distant Ladder cepheid variables and Type 1a supernova standard candles inside distant galaxies to determine receding velocities. His constant gives us the rate at which the Universe is expanding. Today's current value from distance ladder measurements is 22.7 km/s for every million light years further away a celestial object is.



In 2016, group of astronomers used the Hubble Space Telescope and others to study five galaxies gravitationally lensing five quasars in order to arrive at an independent measurement of the Hubble constant. Quasars are incredibly luminous galaxy cores. It is thought that they are massive black holes actively accreting huge amount of matter. The strong gravitational lensing creates multiple images of the background quasars. Some are smeared into extended arcs. Some of these quasars flicker. While the relative time between two flickers is correctly represented in this animation, in reality, the delays are in the range of days to two weeks.



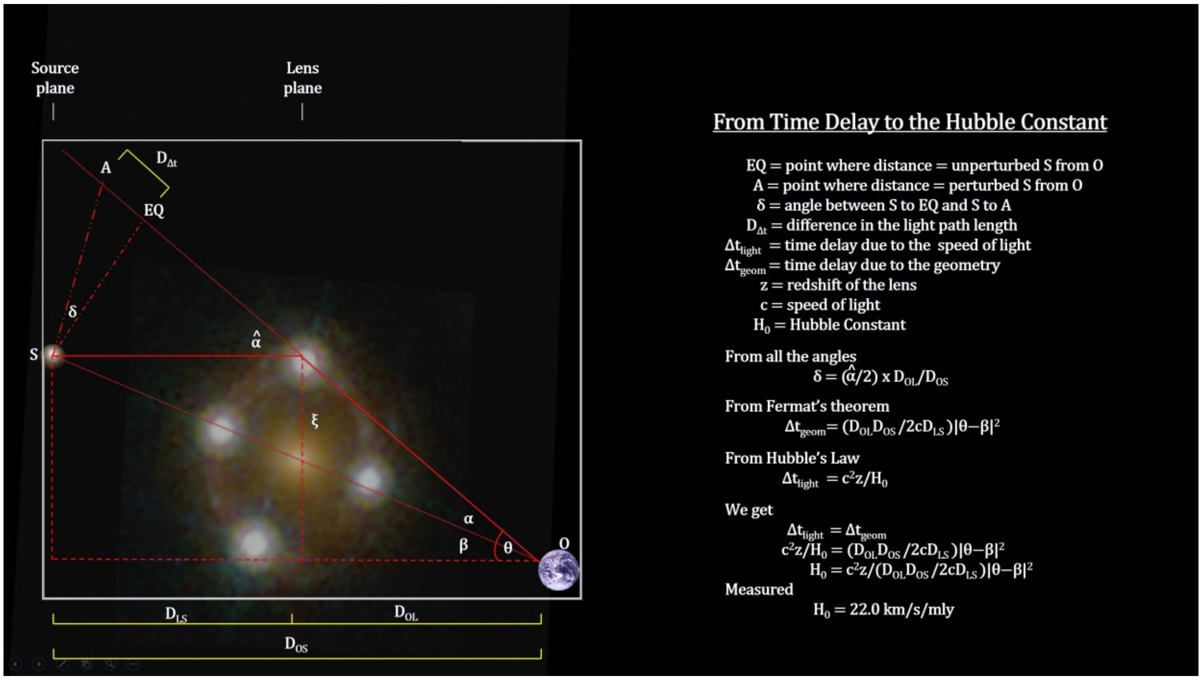
Because the lensed quasar is not perfectly aligned with the lensing galaxy, and the lensing galaxy is not perfectly spherical, the light from the different images of the background quasar follows paths which have different lengths. The delays between them depends on the lengths of the paths the light has taken. It is possible to determine the Hubble Constant from the flicker delay time.



The lens' gravitational field creates the different path lengths. Model the potential to get the distance.

The longer path takes more time. Measure the time to get the distance.

Here's the lens equation we covered earlier. If we mark the length from the observer to the first flicker, and mark the length from the observer to the second flicker, we create an angle. The distance between these two points will be equal to the light travel time delay multiplied by the speed of light adjusted for the redshift of the lensing galaxy. Analysis of the lens itself also gives us a measure of the distance based on the lens mass distribution geometry. It's based on an optics principle first discovered by the French mathematician Pierre Farmat in 1662. In fact, Fermat's principle holds in the general theory of relativity where gravitational lenses replace glass. These two distance quantities are responsible for the measured time delay. So, they are equal. You can now solve for the Hubble constant. With accurate measurements of the time delays between the multiple images, as well as computer models for the lensing galaxy, the team determine the Hubble constant. It is in agreement with the type 1a supernovae method.



From Time Delay to the Hubble Constant

EQ = point where distance = unperturbed S from O
A = point where distance = perturbed S from O
δ = angle between S to EQ and S to A
$D_{\Delta t}$ = difference in the light path length
Δt_{light} = time delay due to the speed of light
Δt_{geom} = time delay due to the geometry
z = redshift of the lens
c = speed of light
H_0 = Hubble Constant

From all the angles

$$\delta = (\hat{\alpha}/2) \times D_{OL}/D_{OS}$$

From Fermat's theorem

$$\Delta t_{geom} = (D_{OL}D_{OS}/2cD_{LS})|\theta - \beta|^2$$

From Hubble's Law

$$\Delta t_{light} = c^2 z/H_0$$

We get

$$\Delta t_{light} = \Delta t_{geom}$$
$$c^2 z/H_0 = (D_{OL}D_{OS}/2cD_{LS})|\theta - \beta|^2$$
$$H_0 = c^2 z/(D_{OL}D_{OS}/2cD_{LS})|\theta - \beta|^2$$

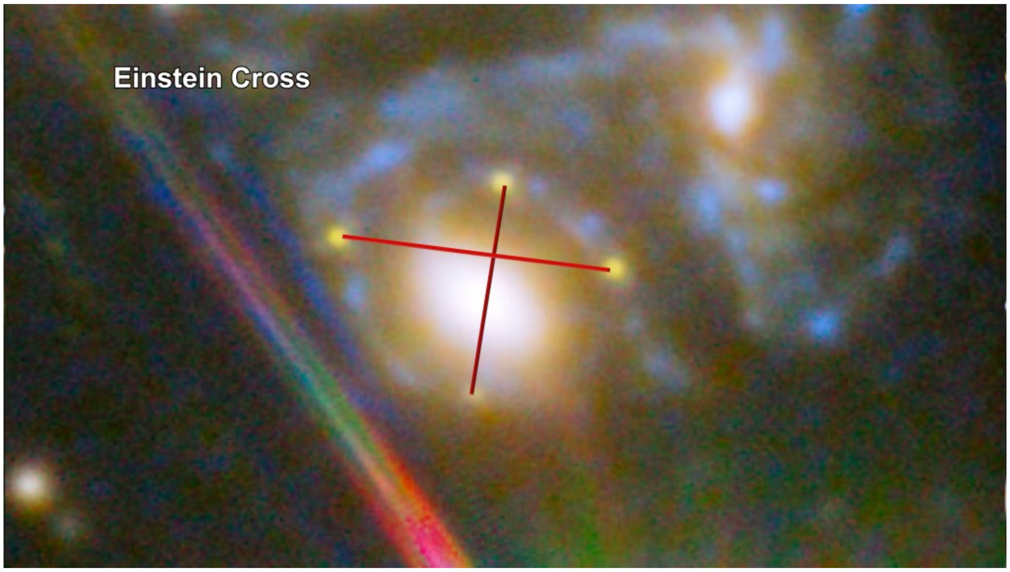
Measured

$$H_0 = 22.0 \text{ km/s/mly}$$

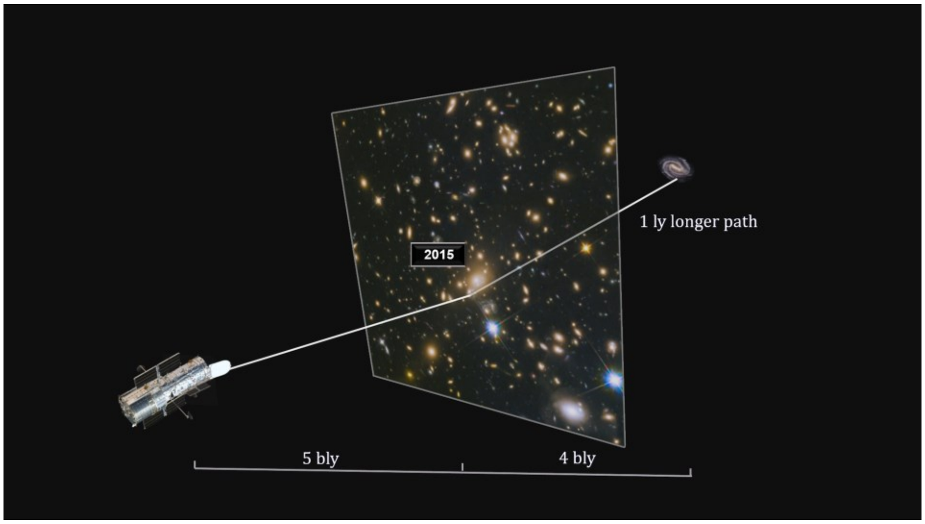
Gravitationally Lensed Supernovae

One of the most interesting uses of strong galaxy cluster gravitational lensing involves distant supernovae. Multiple light paths through the cluster produce multiple arrival times here on Earth. Here are a few examples. In 2014, a team of astronomers found a supernova in this galaxy cluster [MACS J1149+2223] over 5 billion light years away. [The supernova's nickname is Refsdal.]

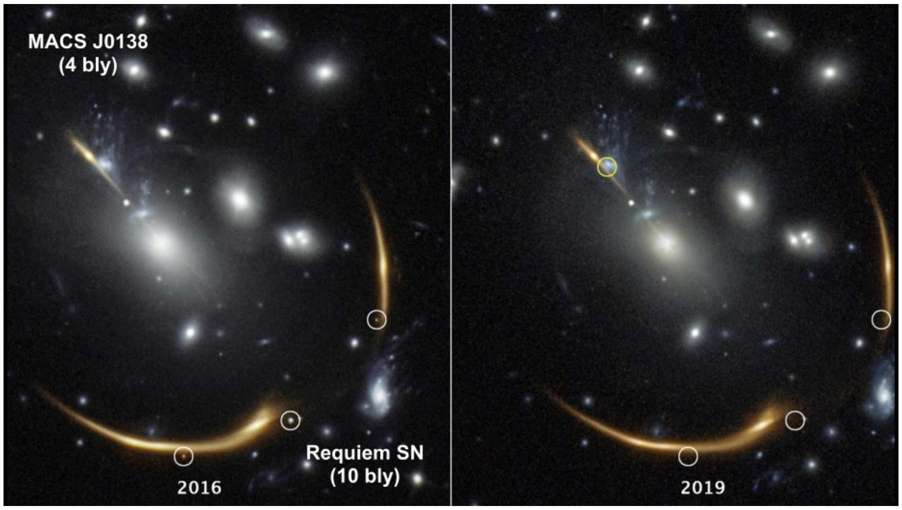
The supernova actually happened in a galaxy 4 billion light years behind this cluster – making it 9 billion light years away. The huge mass of the foreground galaxy and the cluster bent the light from the distant supernova - creating four separate images of the same explosion. The images are arranged around an elliptical galaxy in a formation known as an Einstein cross – because he was the one who predicted these phenomena.



Following this discovery, astronomers modeled several possible gas and dark matter distributions in the galaxy cluster. Each model predicted that another image of this supernova will appear in the cluster, but they had different time estimates ranging from 2015 through 2025. In December 2015 it appeared. For the first time in history, the time and location of a supernova was accurately predicted. We actually saw the supernova happen. Instead of detecting a flash in the sky and turning telescopes to its location, we had the telescopes already focused on the correct area and recorded the event from beginning to end. This was powerful evidence for dark matter. This also constituted a measure of the speed of light without the use of mirrors.



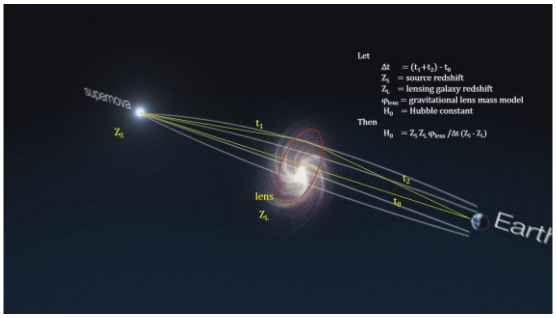
Here's another one. Three views of the same supernova appear in the 2016 image on the left, taken by the Hubble Space Telescope. But they're gone in the 2019 image. The distant supernova, named Requiem, is embedded in the giant galaxy cluster MACS J0138 4 bly away. The cluster gravitationally lensed the light from the supernova located in a galaxy far behind it 10 bly away. It also split the supernova's light into multiple mirror images, highlighted by the white circles in the 2016 image. Based on the foreground galaxy's dark matter distribution, researchers predict that a reappearance of the same supernova will happen in 2037. The predicted location of that fourth image is highlighted by the yellow circle at top left.



And like we did with flickering quasars, time delayed supernova images can also be used to calculate the Hubble constant. In 2017, a Swedish-led team of astronomers used the Hubble Space Telescope to analyze the multiple images of a gravitationally lensed type 1a supernova. This had never been done before. Here we see the lensing galaxy in the middle frame. It's over 2 billion light years away. The four images of the supernova can be seen in the right most frame. It originated over 4 billion light years away. **[**iPTF16geu was initially observed by the Palomar Transient Factory collaboration with the Palomar Observatory.**]**

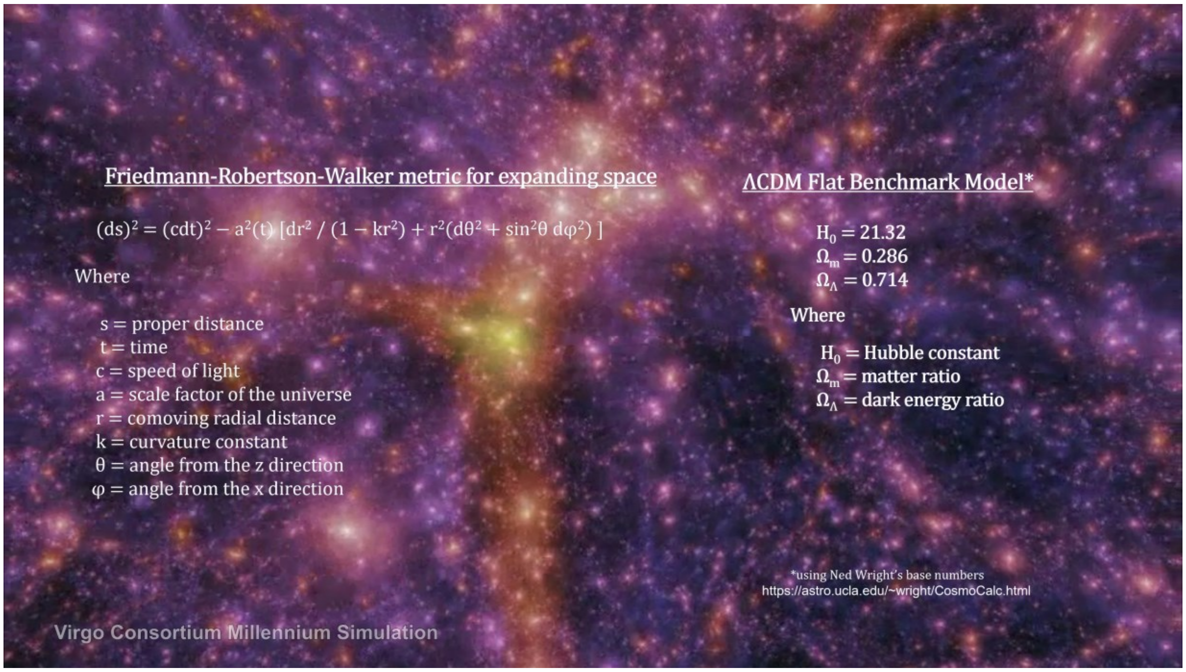
These four images of the exploding star and the time difference in their light profiles can be used to measure the Hubble constant in a completely different way. Since the light travel times for the various images are unequal, intrinsic variations of the source would be observed at different times in the images.

The time delay between the images is proportional to the difference in the light path lengths through the lensing galaxy's space-time, which in turn is proportional to one over the Hubble constant. So, by measuring redshifts and time delays, and by producing an accurate model for the lensing galaxy, the Hubble constant can be calculated.



Gravitationally Lensed Galaxies

Distant giant galaxy clusters around 4 bly away have been able to lens galaxies twice that far away. Without gravitational lensing, we would never have detected these more distant galaxies. At these distances, we need to take into account the expansion of the Universe in order to determine distance. For that, we use the Freeman - - metric. With this and a stated set of coefficients for the flat lambda cold dark matter model, we get the galaxy's distance from us when the light we see started it journey; the distance the light traveled to get here; and the distance from us the object is now.



Friedmann-Robertson-Walker metric for expanding space

$$(ds)^2 = (cdt)^2 - a^2(t)\,[dr^2 / (1 - kr^2) + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\,]$$

Where

s = proper distance
t = time
c = speed of light
a = scale factor of the universe
r = comoving radial distance
k = curvature constant
θ = angle from the z direction
φ = angle from the x direction

ΛCDM Flat Benchmark Model*

$H_0 = 21.32$
$\Omega_m = 0.286$
$\Omega_\Lambda = 0.714$

Where

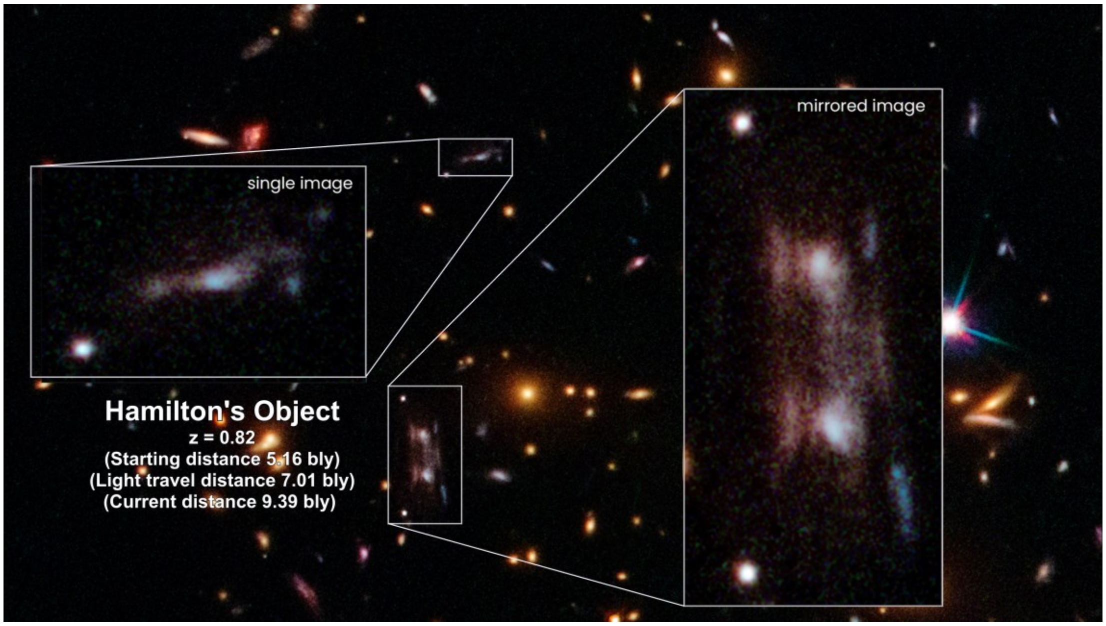
H_0 = Hubble constant
Ω_m = matter ratio
Ω_Λ = dark energy ratio

*using Ned Wright's base numbers
https://astro.ucla.edu/~wright/CosmoCalc.html

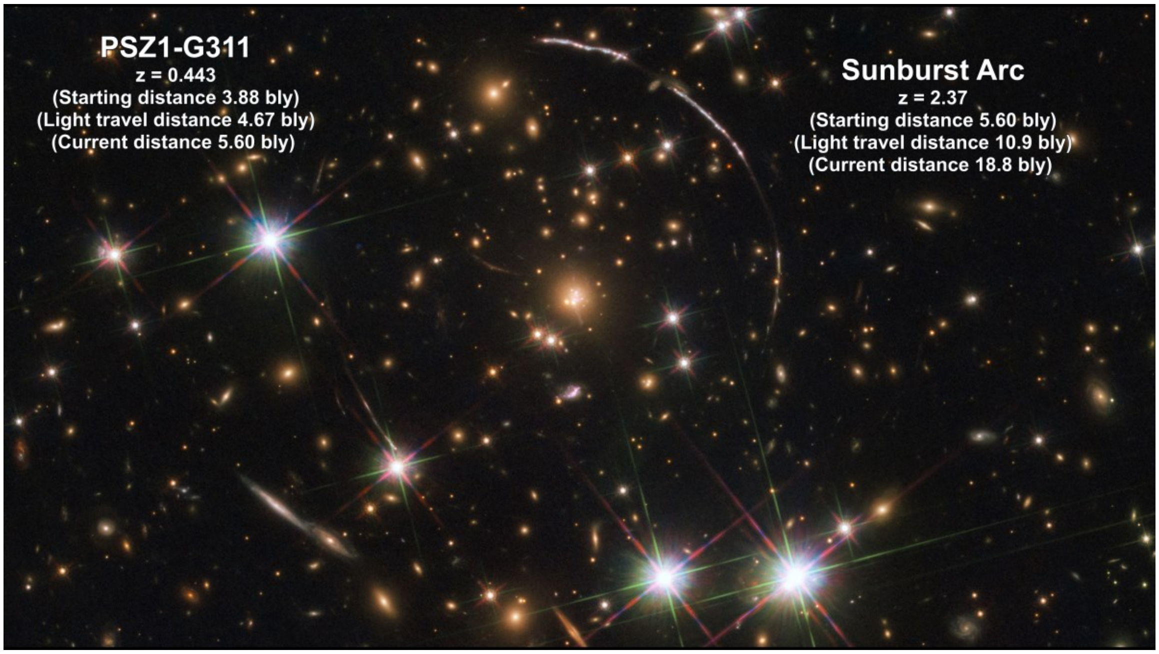
Virgo Consortium Millennium Simulation

Here are a few examples.

Astronomer Timothy Hamilton, using the Hubble Space Telescope, discovered these unusual objects, now named after him. The objects are the stretched images of a gravitationally lensed distant galaxy, located more than 11 billion light-years away. One appears to be a mirror image. In this case, a precise alignment between a background galaxy and a foreground galaxy cluster 7 billion light-years away produced twin magnified copies of the same image of the remote galaxy. This rare phenomenon occurs because the background galaxy straddles a ripple of dark matter in the foreground galaxy. As the faraway galaxy light passes through the cluster along this ripple, two mirror images are produced, along with a third image that can be seen off to the side.

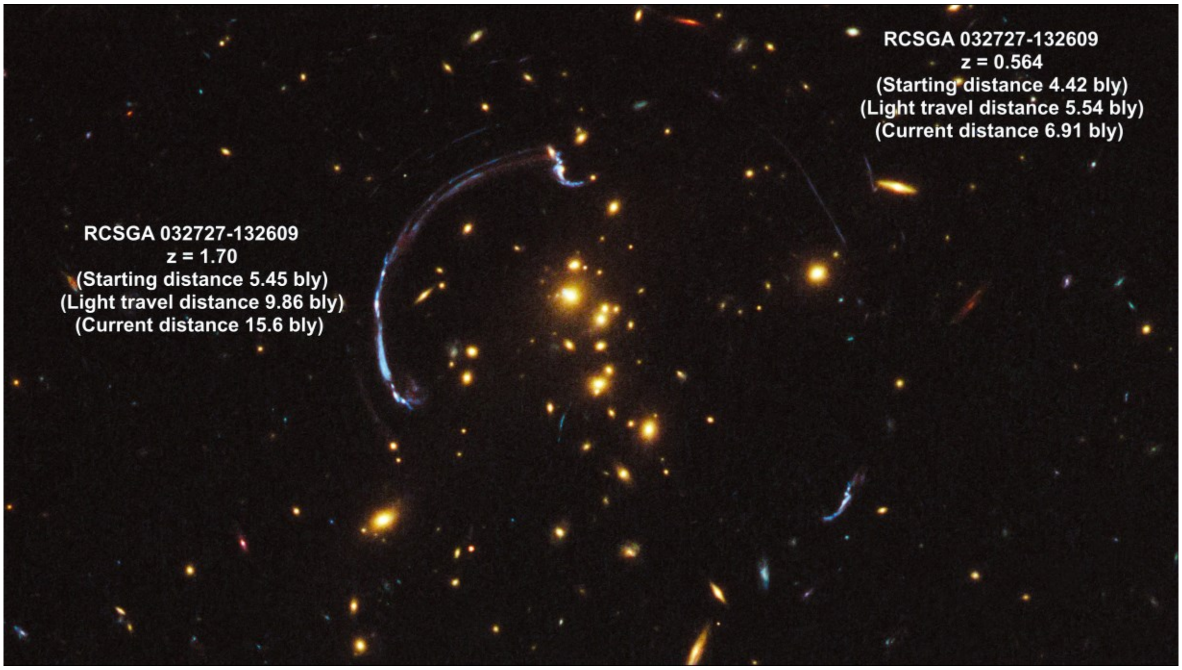


This Hubble image shows a massive galaxy cluster, about 4.6 billion light years away. Along its borders, four bright arcs are visible; these are copies of the same distant galaxy, nicknamed the Sunburst Arc. It's almost 11 billion light-years away. Its light is being lensed into multiple images by strong gravitational lensing. The Sunburst Arc is among the brightest lensed galaxies known and its image is visible at least 12 times within the four arcs.

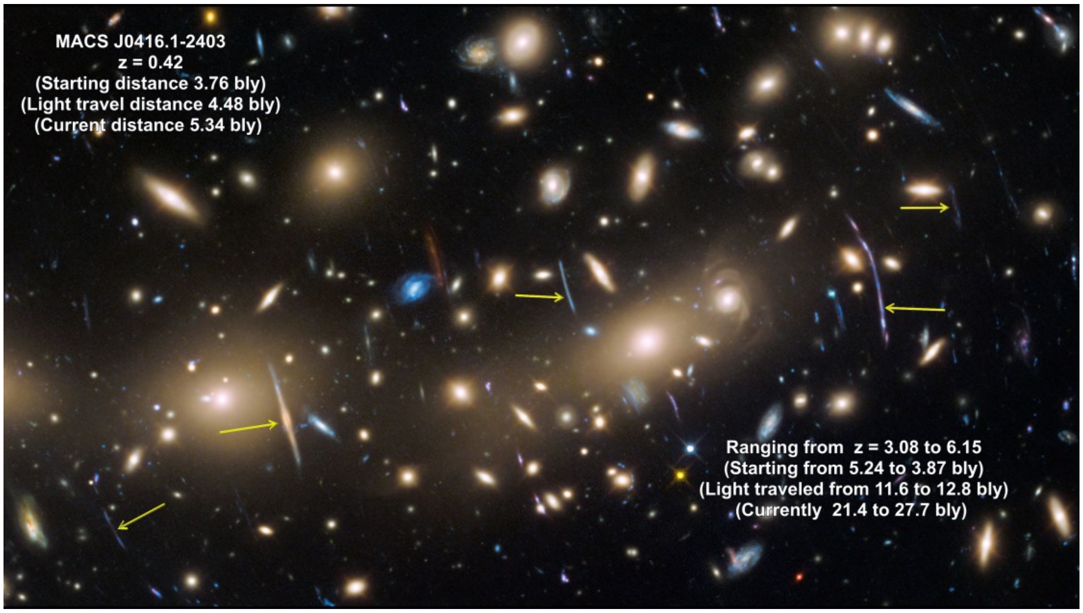


Here's a closer look at three of them. The lens makes various images from 10 and 30 times brighter. This allows Hubble to view structures as small as 520 light-years across — a rare detailed observation for an object that far away.

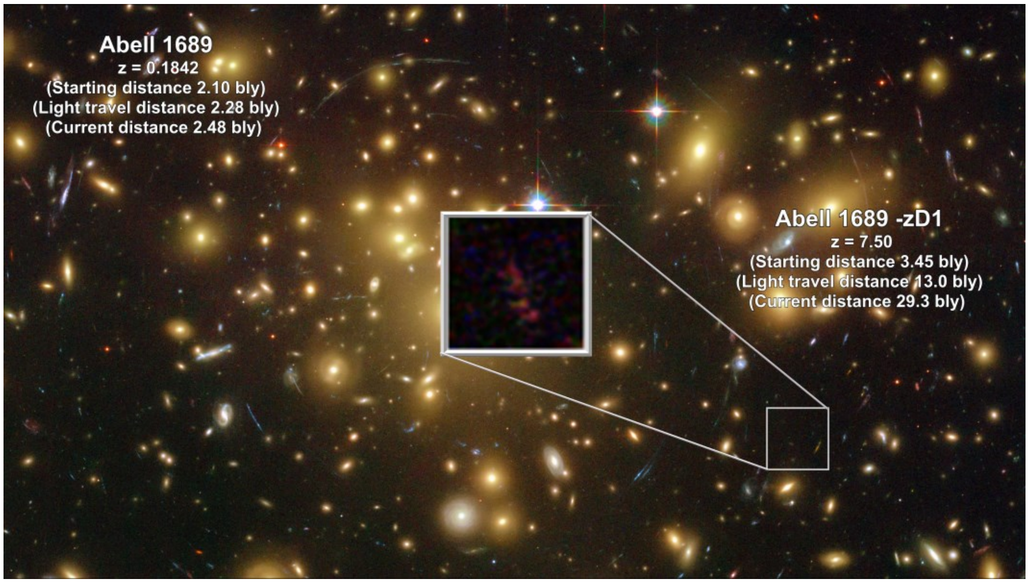
This is a close-up look at the brightest distant "magnified" galaxy in the universe known to date. It is one of the most striking examples of gravitational lensing. In this image the light from a distant galaxy, nearly 10 billion light-years away, has been warped into a nearly 90-degree arc of light in the galaxy cluster. The galaxy cluster that is bending the light lies 5 billion light-years away.



Here's another one. The light from this galaxy traveled 4.48 billion light years to get here. These foreground galaxy clusters are magnifying the light from the faint galaxies that lie far behind it. The faint light from these lensed galaxies traveled up to 12.8 billion light years. It's the gravitational lensing that allows us to see that far back in time. Without the magnification, these galaxies would be invisible for us.



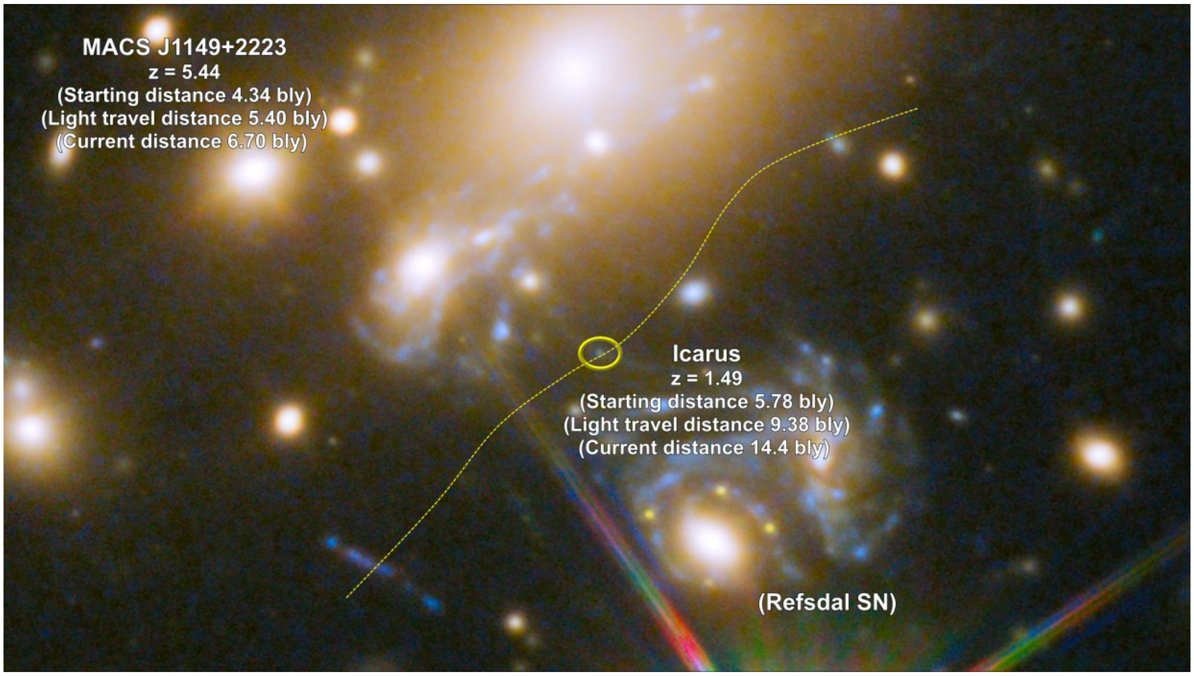
This is Abell 1689. It's one of the most massive galaxy clusters known. The gravity of its trillion stars, plus dark matter, acts like a 2-million-light-year-wide "lens" in space. Here's gravitationally lensed galaxy A1689-zD1. It is one of the most distant spectroscopically confirmed sources with a redshift of 7.5. We are seeing what zD1 looked light when the Universe was only about 700 million years old. It is the earliest known galaxy where dust was detected in its interstellar medium. And, surprisingly, it has the same ratio of dust to total mass as very mature galaxies such as our own Milky Way.



Gravitationally Lensed Stars

Gravitational lensing of objects on or near critical caustic curves can actually identify individual stars in a lensed galaxy. This is referred to as microlensing. To illustrate this ability, we'll cover two examples: Icarus and Earendel.

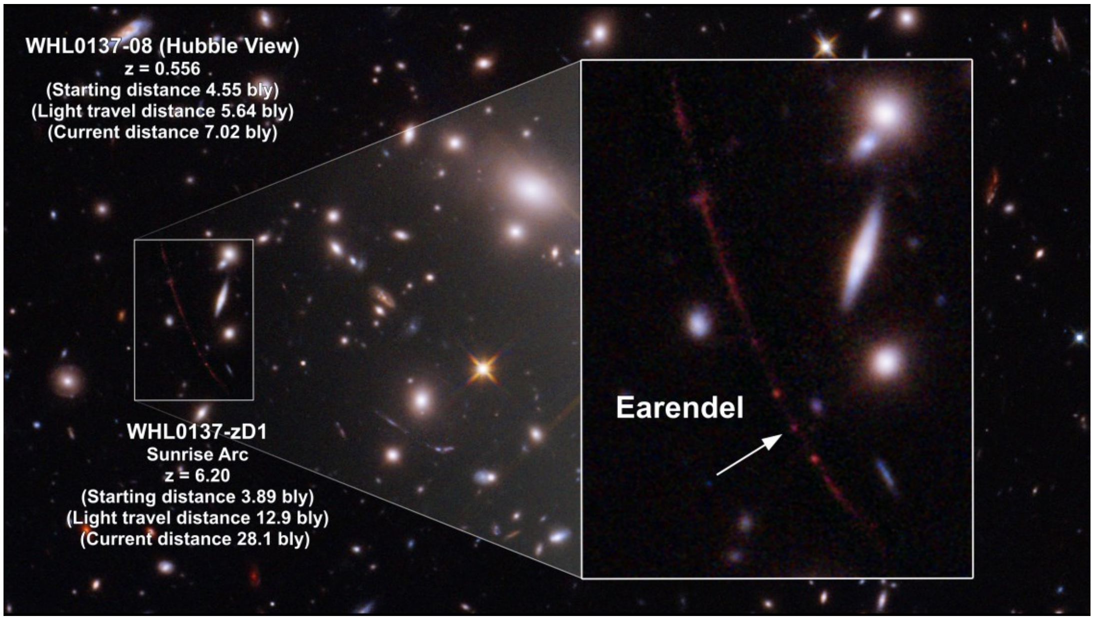
Here's MACS J1149, a foreground galaxy cluster is 5 billion light years away. We used this lens when we covered the Refsdal supernova. Here we lens the single star nicknamed Icarus in a galaxy 9 billion light years away. In 2018, this star was astride a critical curve. It was magnified around 2,000 times its actual size. At that time, it was the farthest individual star ever seen. The colors of the light coming from this object, showed that it was a blue supergiant star. This type of star is much larger, more massive, hotter, and possibly hundreds of thousands of times intrinsically brighter than our Sun. By 2022 the star had moved off the critical curve and is no longer visible.



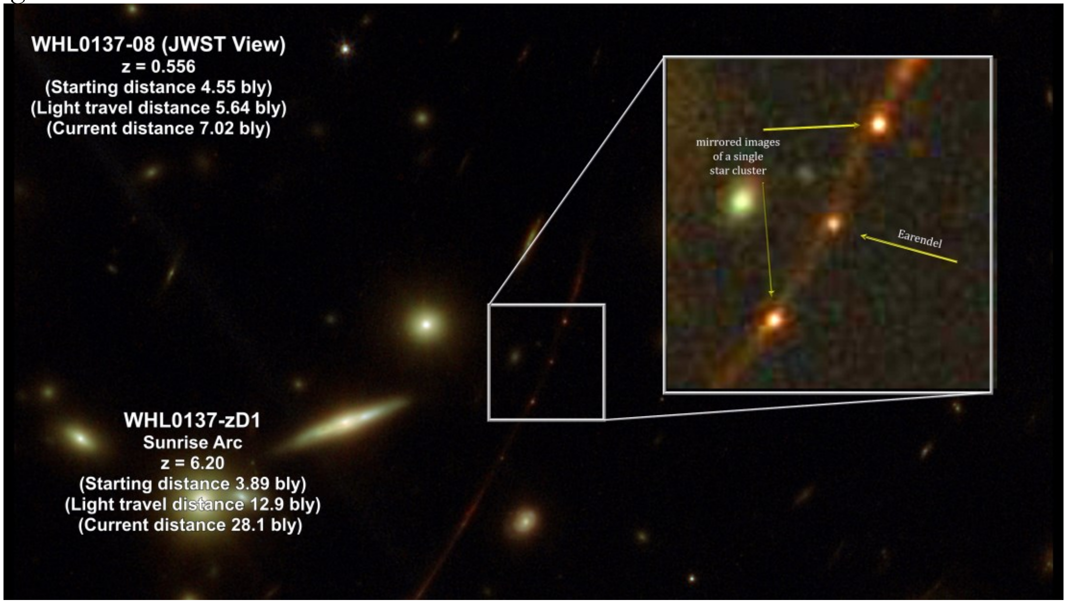
Here we have a massive galaxy cluster [WHL0137-08]. It has been studied for over 5 years since Hubble first captured the image in 2016. It has a redshift of 0.556. The galaxy was 4.5 bly away from us when the light we see started its journey. The light traveled 5.6 bly to get here, and it is currently 7 bly away. In this cluster, Hubble discovered a gravitationally lensed galaxy [WHL0137-zD1] nicknamed the "Sunrise Arc'. Its redshift is 6.2 with an angular size on the sky exceeding 15 arcseconds. That makes it 410,000 lightyears long.

The galaxy was only 3.9 bly away from the Milky Way when the light we see started its journey. The light traveled 12.9 bly to get here, and it is currently 28 bly away receding faster than the speed of light and beyond the visible horizon. No light leaving that galaxy now will ever reach the Milky Way.

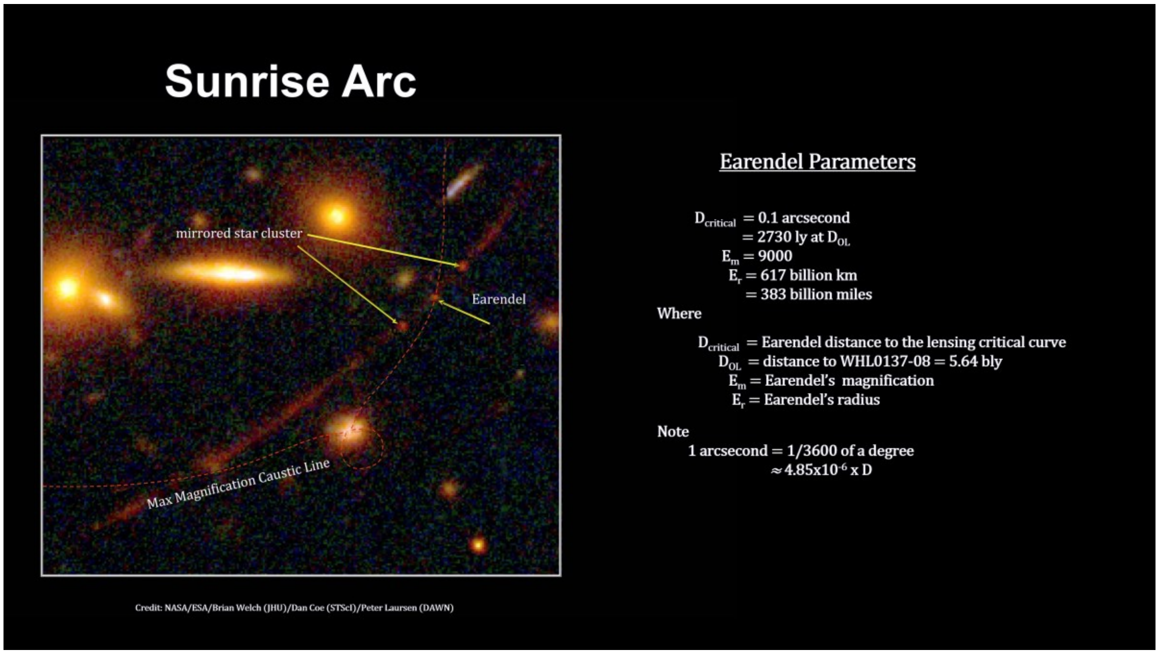
In this galaxy, Hubble discovered a single star. The star, LSz6 is nicknamed Earendel. The light we see from this star began its journey 900 mly into the Universe's expansion. This makes it the oldest most distant individual star ever seen. In addition, it is at least 50 times the mass of our Sun and millions of times brighter. This makes it one of the most massive stars known. Stars that massive only last for a few million years. So Earendel is long gone, having spewed the heaver elements it created into zD1 to become a part of the galaxy's next generation stars.



On July 30, 2022, a few months after Hubble's discovery, a team of astronomers called Cosmic Spring worked with Webb to train its Near-Infrared Imaging camera on the Sunrise Arc for over 4 hours as part of a survey of the lensing cluster [WHL0137-08]. It also managed to capture the red dot corresponding to Earendel. Webb's analysis of the image has already confirmed that this is indeed a single star system, and not a group of several stars. The red dots on either side of Earendel are a single mirrored star cluster.



Here's a higher resolution image of the area around Earendel produced by the JWST team. Objects close to a critical curve get mirrored into multiple images like these two images of a star cluster inside the Sunrise Arc galaxy. The critical curve responsible for this will pass through the midpoint between the two images. An object found at this midpoint like Earendel, would be so close to it that its multiple images cannot be resolved – it will appear as a single object. Four different models were used to locate the lensing critical curves. Here's one of them. The others are quite different, but they all pass through Earendel. The magnification drops off rapidly as the distance from the line increases. Earendel's distance from the line is within 0.1 arcseconds. That's a very small angle, but at these distances it represents 2730 ly. This distance along with its shape puts its magnification between 1,000 and 40,000 with 9,000 being the most likely. With this and the size of the image, we get a source object that has a radius less than 617 billion km. (That's 383 billion miles) This is a hundred times smaller than known small star clusters, leading to the conclusion that it is a single gigantic star or binary star system.



Sunrise Arc

Earendel Parameters

$D_{critical}$ = 0.1 arcsecond
= 2730 ly at D_{OL}
E_m = 9000
E_r = 617 billion km
= 383 billion miles

Where

$D_{critical}$ = Earendel distance to the lensing critical curve
D_{OL} = distance to WHL0137-08 = 5.64 bly
E_m = Earendel's magnification
E_r = Earendel's radius

Note

1 arcsecond = 1/3600 of a degree
$\approx 4.85 \times 10^{-6} \times D$

Credit: NASA/ESA/Brian Welch (JHU)/Dan Coe (STScI)/Peter Laursen (DAWN)

Now that we can examine stars this far away with gravitational lensing and the James Webb Space Telescope, I'm hoping that we will even get to the point where light fluctuations from a star like Earendel will someday tell us something about the earliest planets.

Music

@00:00 Rachmaninoff - Symphony No. 2 Adagio - Sofia Philharmonic Orchestra; from the album "Sergie Rachmaninoff Symphony No. 2", 2011

@14:37 Rachmaninoff - Piano Concerto No 2 in C minor – from the album "The Most Relaxing Classical Music Ever", 1993

@23:30 Rachmaninoff - Rhapsody on a Theme of Paganini - Variation 18 - from the album "The Most Relaxing Classical Music Ever", 1997

Greek letters:
- α β γ δ ε ζ η θ ι κ λ μ ν ξ ο π ρ σ τ υ φ χ ψ ω
- Α Β Γ Δ Ε Ζ Η Θ Ι Κ Λ Μ Ν Ξ Ο Π Ρ Σ Τ Υ Φ Χ Ψ Ω

⇒ → ± ⊙ ∞ ↛ ∃ ∄ ∈ ∉ ∬ ∫ ≅ ≥ ≤ ≈ ≠ ≡ √ ∛ ~ ∝ ħ ÷ ∂ ⊥