Gravitational Waves

{**Abstract:** *In this segment of the "How Fast Is It" video book, we cover Gravitational Waves. We examine just what a 'ripple in space-time' is. We'll cover the wave properties and how they contract objects it encounters. We'll examine binary star systems and the waves they create. This includes the properties of the binary encoded in the created gravitational wave including luminosity distance. We'll use stars the size and mass of our Sun to calculate the expected amplitude and wavelength magnitudes. We'll move from normal stars to neutron stars to stellar mass black holes. Along that progression, we'll see their signature waveforms and build the gravitational wave sensitivity graph. We'll cover the Hulse-Taylor pulsar (PSR B1913+16) and how it provided indirect evidence for the existence of gravitational waves. We'll then cover direct detection with Michelson Interferometers. We'll cover the LIGO technology and sensitivity and then examine the first detection event, and show how it fit the waveform and magnitudes for merging stellar black holes. We'll include a look at the future of gravitational wave observatories. We'll finish by covering the nature of gravitational waves created during the Big Bang and how they might be detected today.*}

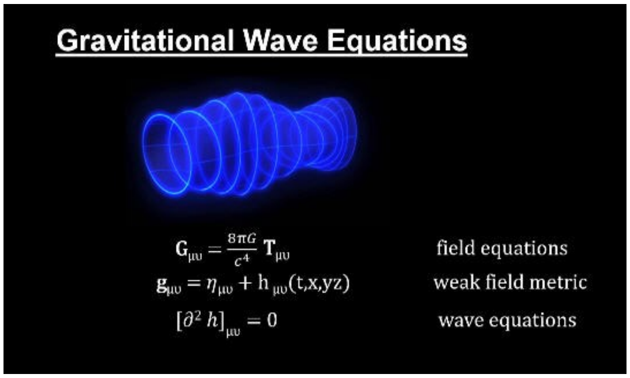
Introduction

At the end of the second segment on General Relativity, released at the end of 2015, I pointed out that gravitational waves had yet to be directly detected. But in January 2016, it was announced that gravitational waves had in fact been detected. So, I thought I should create this addition to the "How Fast Is It" video book to cover gravitational waves.

- We'll cover exactly what a gravitational wave (a ripple in space-time) is.
- We'll examine a top astronomical event that could create waves large enough to measure at great distances.
- We'll review how a Michelson Interferometer like the one we covered in "How Small Is It" can be used to detect gravitational waves.
- And we'll see how the Laser Interferometer Gravitational Wave Observatory (LIGO) actually detected the gravitational wave from a merging pair of black holes.

Gravitational Waves

Our first step is to examine just what a ripple in space-time means. Here, far from an event that could create a GW, we have a relatively flat space with a Euclidian metric **g** that isn't changing with time. A ripple represents small deviations from this flat space-time metric. We use **h** to represent these deviations. **[**In other words, **h** is the metric tensor of the gravitational wave.**]**



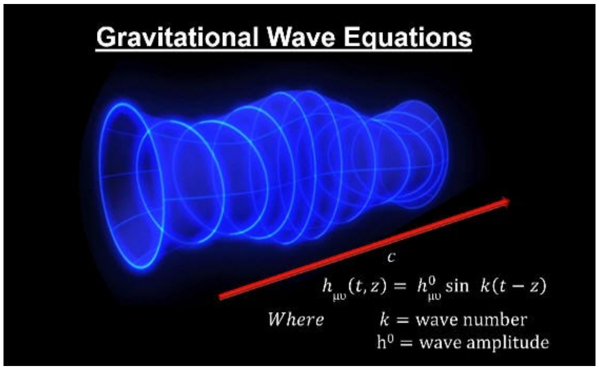
Gravitational Wave Equations

$G_{\mu\nu} = \frac{8\pi G}{c^4} \mathbf{T}_{\mu\nu}$ field equations

$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(t,x,yz)$ weak field metric

$[\partial^2 h]_{\mu\nu} = 0$ wave equations

Solutions to Einstein's equations show that **h** oscillates sinusoidally just like light. And it travels at the same speed as light.

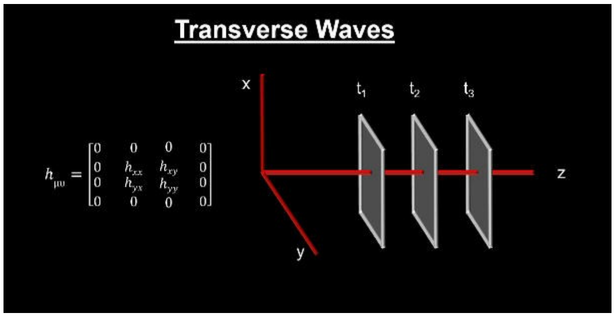


Gravitational Wave Equations

$$h_{\mu\upsilon}(t,z) = h^0_{\mu\upsilon} \sin k(t-z)$$

Where k = wave number
h^0 = wave amplitude

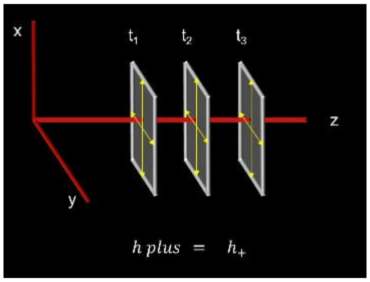
{It turns out that solving these equations in the weak limit also removes all components of **h** except for those that operate in the plane perpendicular to the direction of the wave.} As a wave moves down the z axis, planes at different times experience different values for the metric used to measure distance on the plane. This makes the wave a transvers wave just like light.



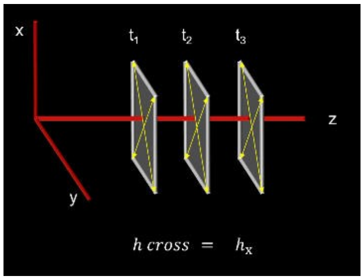
Transverse Waves

$$h_{\mu\upsilon} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & h_{xx} & h_{xy} & 0 \\ 0 & h_{yx} & h_{yy} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

We see two possible polarizations for a gravitational wave. We call one **h plus (h_+)** for action along the x and y axis. We call the other **h cross (h_x)** for action along the diagonal. This is again much like electromagnetic wave polarizations except that these two polarizations are 45 degrees apart and EM wave polarizations are 90 degrees apart.

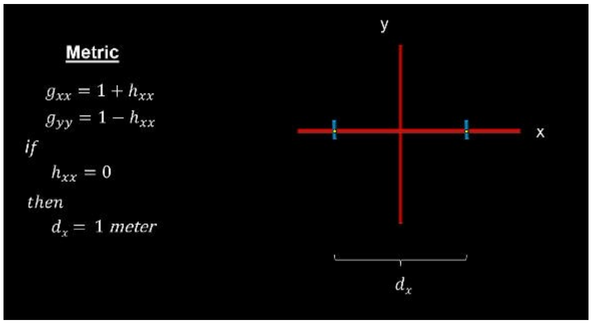


h plus = h_+



h cross = h_x

To see what an oscillating **h plus (h_+)** metric does, we'll measure the changes in the distance between points on the plane when a gravitational wave passes. Here we have an x-y plan with the wave passing into the page. We mark two points on the x axis 1 meter apart in Euclidian flat space where **h** is zero.



Metric

$g_{xx} = 1 + h_{xx}$

$g_{yy} = 1 - h_{xx}$

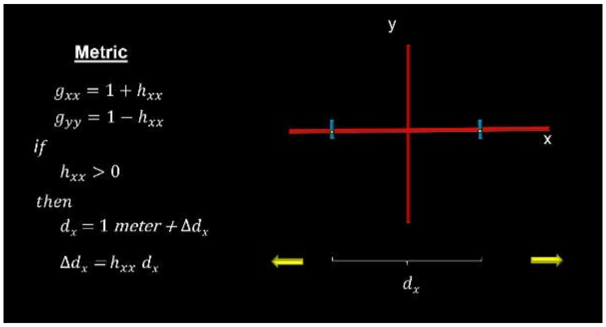
if

$h_{xx} = 0$

then

$d_x = 1\ meter$

When **h** is greater than zero, the distance between the two points on the x axis becomes longer than 1 meter by an amount equal to **h times the original distance**.



Metric

$g_{xx} = 1 + h_{xx}$

$g_{yy} = 1 - h_{xx}$

if

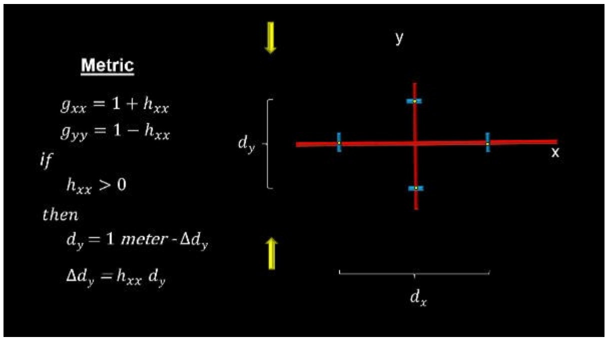
$h_{xx} > 0$

then

$d_x = 1\ meter + \Delta d_x$

$\Delta d_x = h_{xx}\ d_x$

At the same time, a one-meter distance on the y axis will shrink to less than 1 meter by the same amount.



Metric

$g_{xx} = 1 + h_{xx}$

$g_{yy} = 1 - h_{xx}$

if

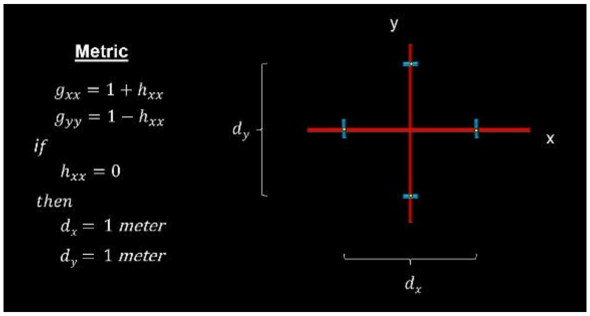
$h_{xx} > 0$

then

$d_y = 1\ meter - \Delta d_y$

$\Delta d_y = h_{xx}\ d_y$

When **h** returns to zero, the distances between these points returns to 1 meter.



Metric

$g_{xx} = 1 + h_{xx}$

$g_{yy} = 1 - h_{xx}$

if

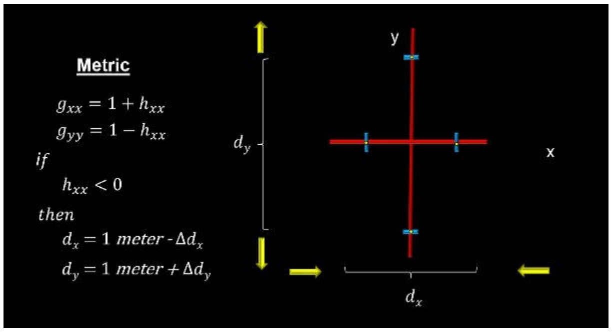
$h_{xx} = 0$

then

$d_x = 1 \; meter$

$d_y = 1 \; meter$

When **h** is less than zero, the distance between the two points on the x axis will become shorter than 1 meter, and the distance between the two points on the y axis will become longer than 1 meter.



Metric

$g_{xx} = 1 + h_{xx}$

$g_{yy} = 1 - h_{xx}$

if

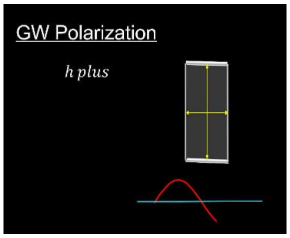
$h_{xx} < 0$

then

$d_x = 1 \; meter - \Delta d_x$

$d_y = 1 \; meter + \Delta d_y$

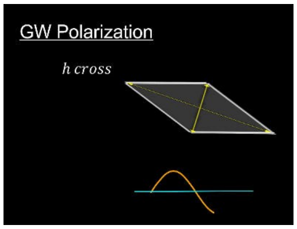
Here's an exaggerated look at what an oscillating **h plus (h_+)** polarized gravitational wave does to a square plate it passes through. Again, the wave is passing into the page.



GW Polarization

h plus

For an **h cross (h_x)** polarized wave, the effect would be similar but shifted 45 degrees.



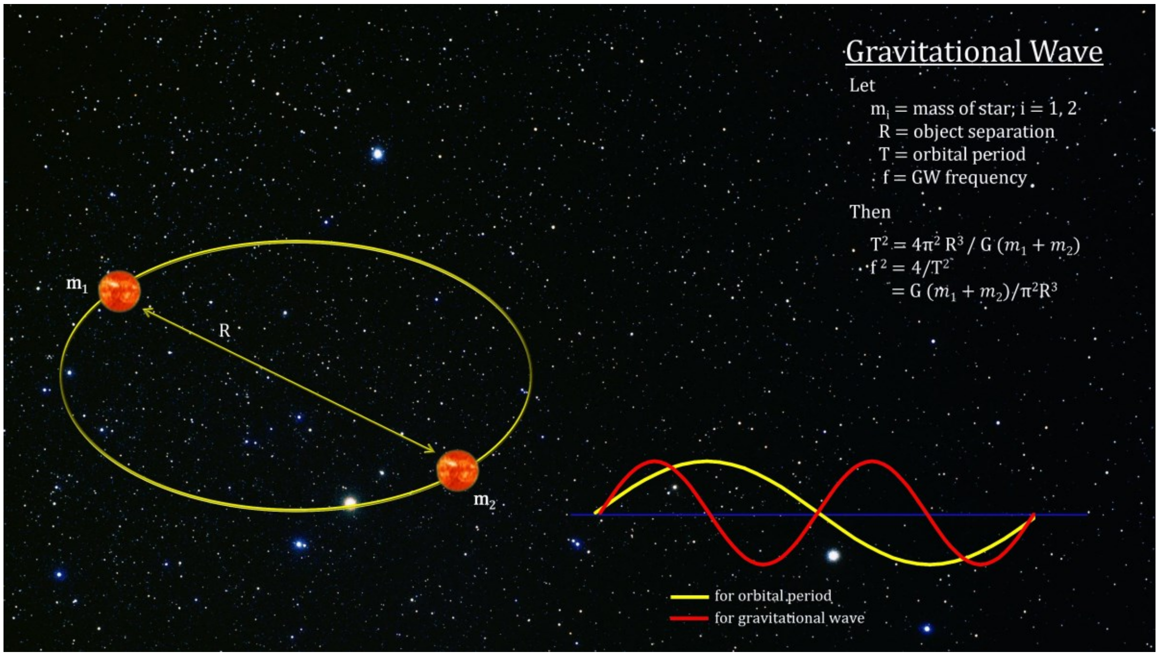
GW Polarization

h cross

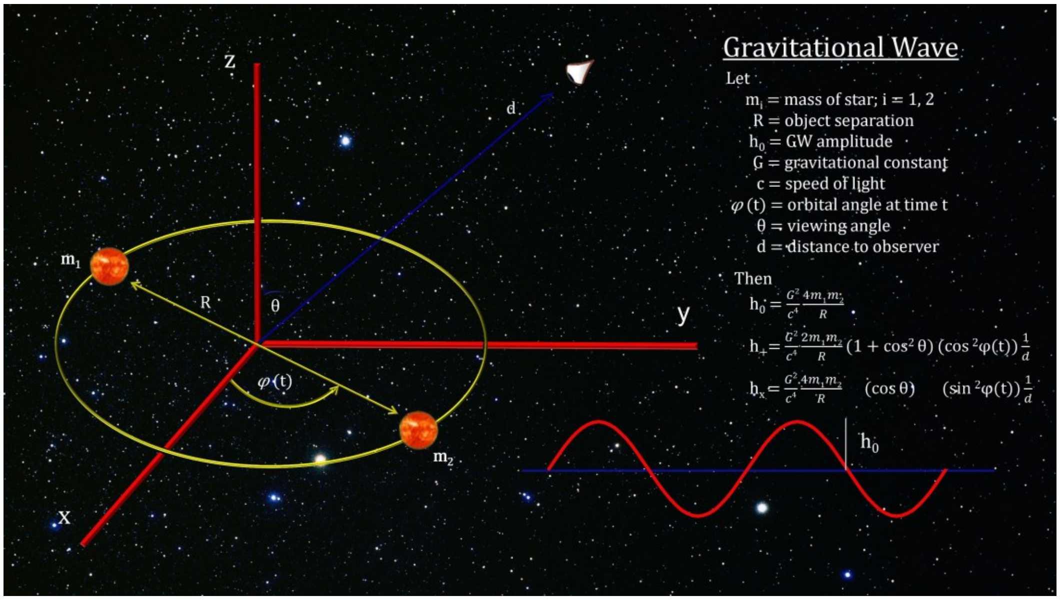
When describing a GW, we can now be more precise than "it's a ripple in space-time". A gravitational wave is an oscillating polarized metric that operates in the plane perpendicular to the direction of the wave as it moves through space at the speed of light. And we have seen what this means for the objects that encounter such a wave: they are stretched and squeezed in various directions.

Creating a Gravitational Wave

Here's one of the ways to create a significant Gravitational Wave. It's a binary star system with two masses revolving in a circular orbit around a common center of gravity. The stars' acceleration creates gravitational waves that travel out from the system in all directions - just like the light waves they are generating. The GW solutions show that the frequency of the created GW is twice the rotation rate of the binary system.



To understand the factors involved in the generation of the two polarizations and their amplitude, we have constructed a coordinate system with its center at the center of the orbital motion in the orbital plane. The amplitude of the gravitational wave depends on the masses of the two objects and the distance between them. In addition, we need to consider the angular rotation and the viewing angle to determine the strength of each polarity at any point in time.



Gravitational Wave

Let

m_i = mass of star; i = 1, 2
R = object separation
h_0 = GW amplitude
G = gravitational constant
c = speed of light
φ(t) = orbital angle at time t
θ = viewing angle
d = distance to observer

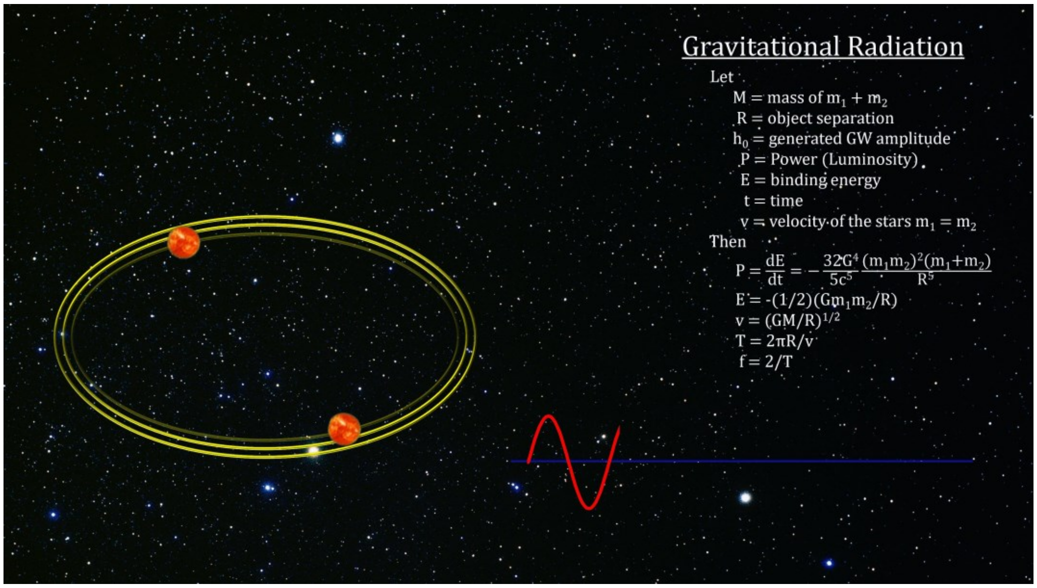
Then

$$h_0 = \frac{G^2}{c^4}\frac{4m_1m_2}{R}$$

$$h_+ = \frac{G^2}{c^4}\frac{2m_1m_2}{R}(1+\cos^2\theta)(\cos 2\varphi(t))\frac{1}{d}$$

$$h_x = \frac{G^2}{c^4}\frac{4m_1m_2}{R}\quad(\cos\theta)\quad(\sin 2\varphi(t))\frac{1}{d}$$

There is one more key factor to consider when it comes to binary systems: namely that GWs carry energy and momentum away from the system. We call this "**gravitational luminosity**". Newton and Kepler provided the mechanics for understanding what happens to the orbit when gravitational energy is lost. Because binding energy is negative, a loss of energy will make it a larger negative. This has the effect of reducing the distance between the two objects. This in turn increases their velocity. A shorter circumference and faster velocity reduce the time it takes for a full orbit and therefore increases the frequency of rotation and therefore the frequency of the GW.



Gravitational Radiation

Let

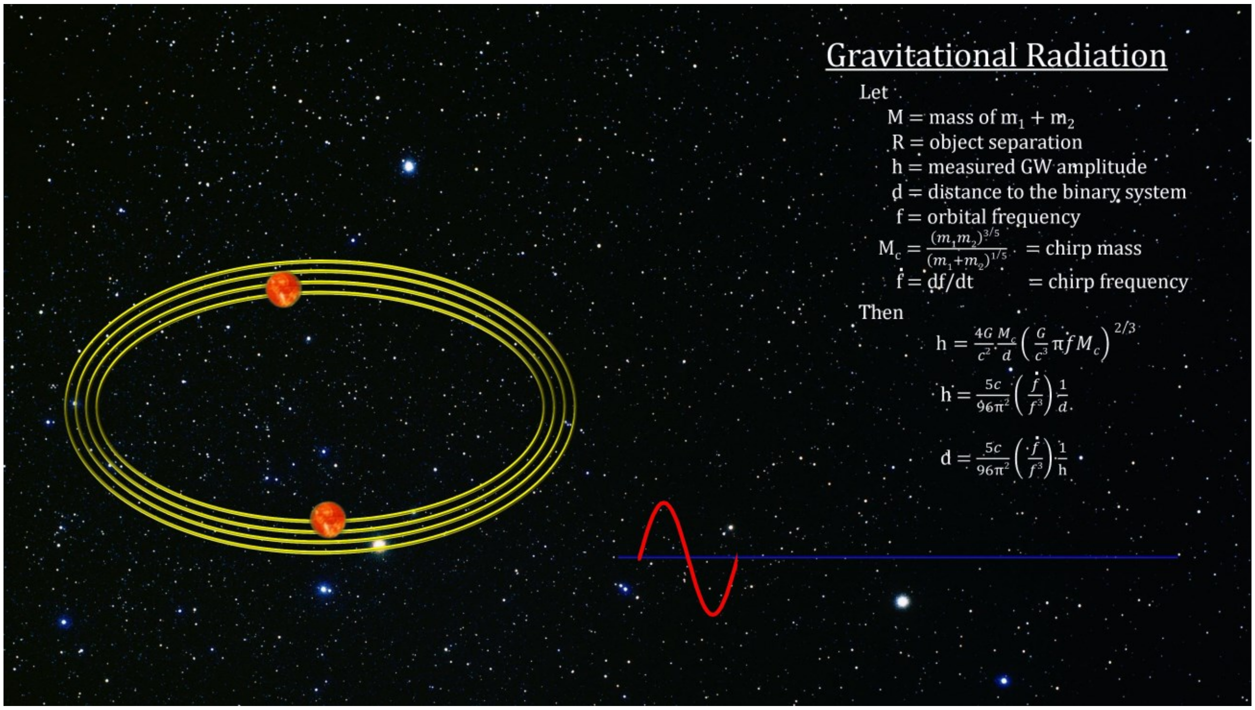
M = mass of $m_1 + m_2$
R = object separation
h_0 = generated GW amplitude
P = Power (Luminosity)
E = binding energy
t = time
v = velocity of the stars $m_1 = m_2$

Then

$$P = \frac{dE}{dt} = -\frac{32G^4}{5c^5}\frac{(m_1m_2)^2(m_1+m_2)}{R^5}$$

$E = -(1/2)(Gm_1m_2/R)$
$v = (GM/R)^{1/2}$
$T = 2\pi R/v$
$f = 2/T$

And the wave equations show that the amplitude of the GW will increase with the frequency. The rate that the frequency is changing is called the **chirp**. It gives us the ability to express the amplitude of the GW in terms of the frequency and the rate that the frequency is changing, instead of the masses and the distance between the masses. This is critical, because for most cases, we will have no way of knowing directly what the masses are or how far apart they are. But measuring the frequencies might be possible. If we can also measure the amplitude, we can even calculate the distance to the binary system. Because this distance is based on GW luminosity, it is called the "Luminosity Distance". For most all GW sources, this will be the **only** way to figure out how far away they are.



Gravitational Radiation

Let

M = mass of $m_1 + m_2$

R = object separation

h = measured GW amplitude

d = distance to the binary system

f = orbital frequency

$M_c = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$ = chirp mass

$\dot{f} = df/dt$ = chirp frequency

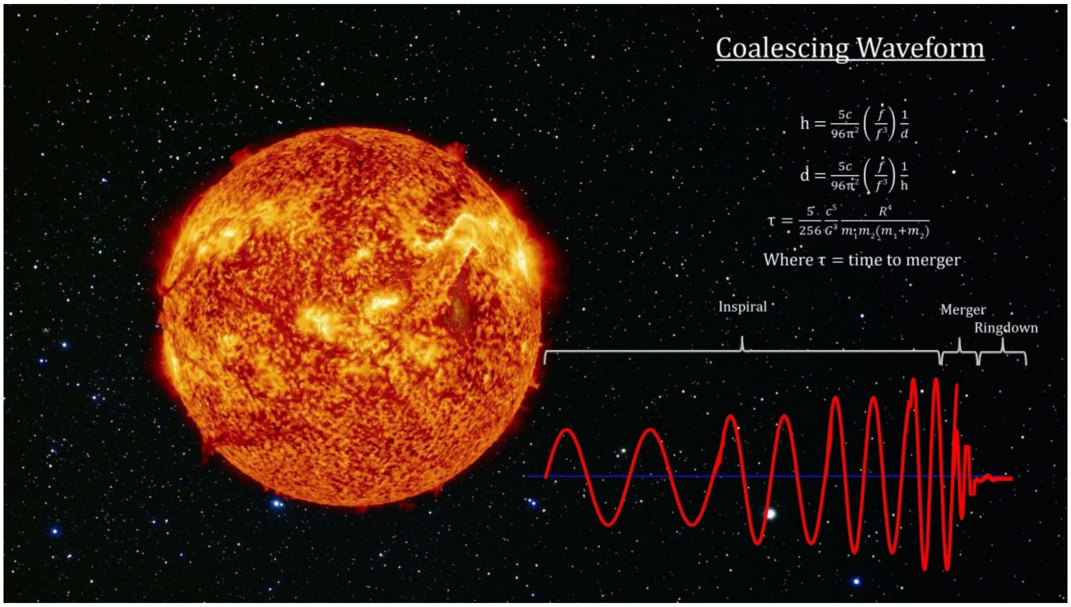
Then

$$h = \frac{4G}{c^2}\frac{M_c}{d}\left(\frac{G}{c^3}\pi f M_c\right)^{2/3}$$

$$h = \frac{5c}{96\pi^2}\left(\frac{\dot{f}}{f^3}\right)\frac{1}{d}$$

$$d = \frac{5c}{96\pi^2}\left(\frac{\dot{f}}{f^3}\right)\frac{1}{h}$$

With a decaying orbit, the objects will eventually collide and coalesce. We can even calculate how long that would take. The resulting waveform (called a coalescing waveform) serves as a signature for this kind of GW source. It has three phases: the in-spiral; the merger; and the ring-down to an object that is no longer asymmetric and therefore no longer radiating gravitational waves.



Coalescing Waveform

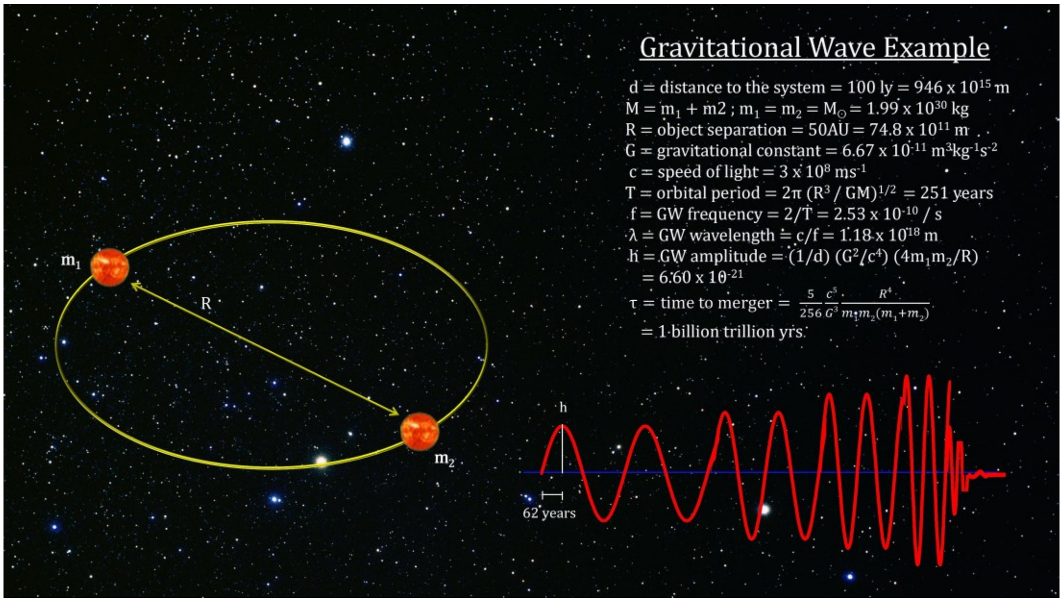
$$h = \frac{5c}{96\pi^2}\left(\frac{\dot{f}}{f^3}\right)\frac{1}{d}$$

$$d = \frac{5c}{96\pi^2}\left(\frac{\dot{f}}{f^3}\right)\frac{1}{h}$$

$$\tau = \frac{5}{256}\frac{c^5}{G^3}\frac{R^4}{m_1 m_2 (m_1+m_2)}$$

Where τ = time to merger

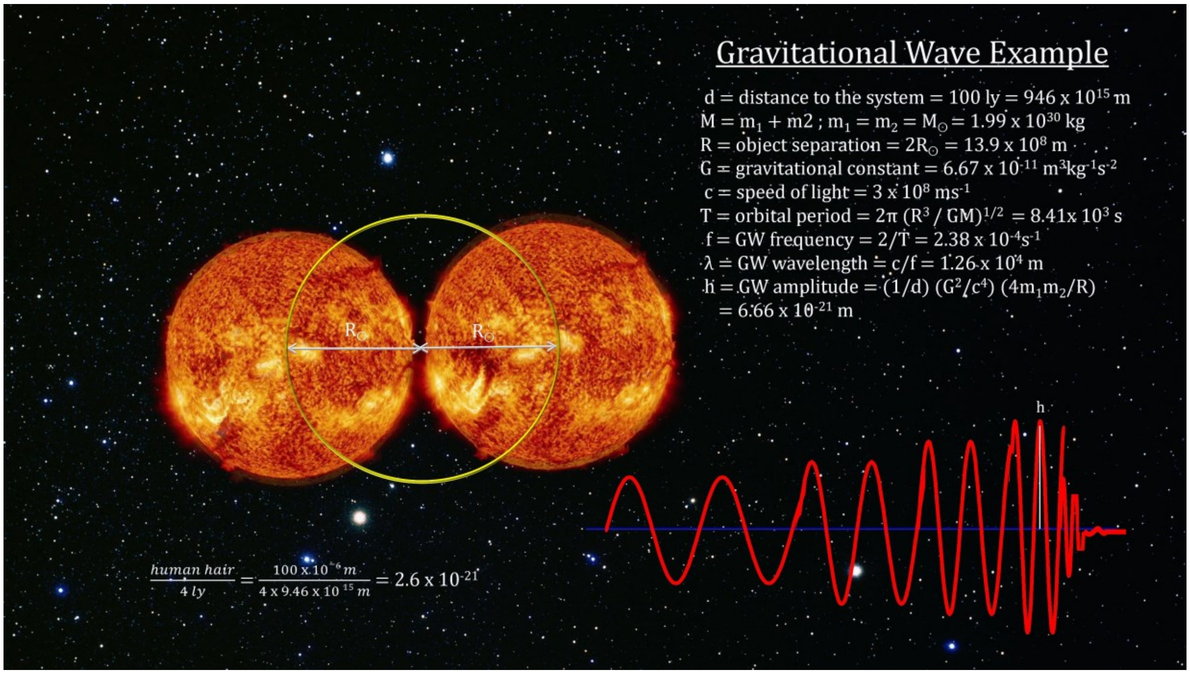
To get an idea on the expected amplitudes and frequencies for GWs created by a system like this one, let's put in some numbers. Suppose this system is 100 light years away and each star is the mass and size of our sun; and the distance between them is 50 times the distance between the Earth and the Sun (50 Astronomical Units). From the masses and the distance between them, we can calculate the stars' orbital period. And with that, we can calculate the orbital frequency; which gives us the frequency and wavelength of the resulting GW. And with the masses and frequency, we can calculate the amplitude of the resulting GW. Here we have a very small number. It would add around a hundredth of the diameter of an electron to a meter stick. What's more, it would take over 62 years to reach this miniscule stretched length! Not only that, it will take trillions of years to merger. The Universe itself is only around 13 billion years old.



Gravitational Wave Example

d = distance to the system = 100 ly = 946 x 10^{15} m
M = m_1 + m2 ; $m_1 = m_2 = M_\odot$ = 1.99 x 10^{30} kg
R = object separation = 50AU = 74.8 x 10^{11} m
G = gravitational constant = 6.67 x 10^{-11} $m^3kg^{-1}s^{-2}$
c = speed of light = 3 x 10^8 ms^{-1}
T = orbital period = $2\pi\ (R^3 / GM)^{1/2}$ = 251 years
f = GW frequency = 2/T = 2.53 x 10^{-10} / s
λ = GW wavelength = c/f = 1.18 x 10^{18} m
h = GW amplitude = $(1/d)\ (G^2/c^4)\ (4m_1m_2/R)$
= 6.60 x 10^{-21}
τ = time to merger = $\frac{5}{256}\frac{c^5}{G^3}\frac{R^4}{m_1m_2(m_1+m_2)}$
= 1 billion trillion yrs.

Should this system ever reach the point where it is close to merging, we'd get the maximum GW amplitude. We find that the distance between their centers of mass is still way too large to produce a significant amplitude. This one is approximately the ratio of the width of a human hair to the distance to Alpha Centauri - 4 lightyears away!



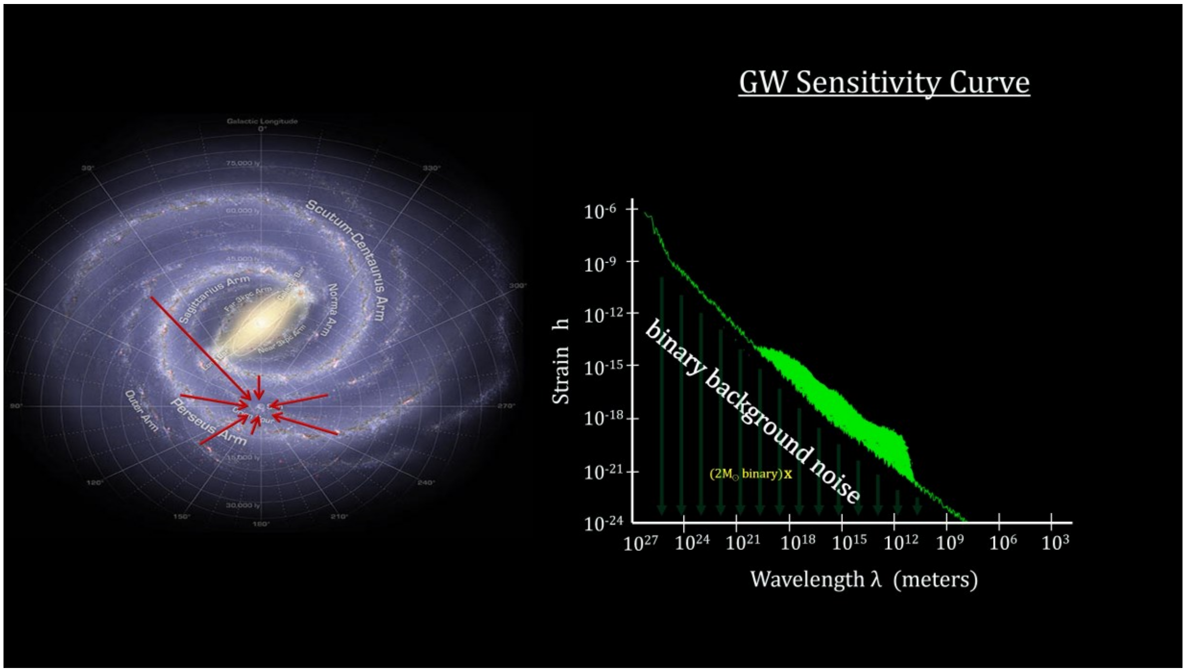
Gravitational Wave Example

d = distance to the system = 100 ly = 946 x 10^{15} m
M = m_1 + m2 ; $m_1 = m_2 = M_\odot$ = 1.99 x 10^{30} kg
R = object separation = $2R_\odot$ = 13.9 x 10^8 m
G = gravitational constant = 6.67 x 10^{-11} $m^3kg^{-1}s^{-2}$
c = speed of light = 3 x 10^8 ms^{-1}
T = orbital period = $2\pi\ (R^3 / GM)^{1/2}$ = 8.41x 10^3 s
f = GW frequency = 2/T = 2.38 x $10^{-4}s^{-1}$
λ = GW wavelength = c/f = 1.26 x 10^4 m
h = GW amplitude = $(1/d)\ (G^2/c^4)\ (4m_1m_2/R)$
= 6.66 x 10^{-21} m

$$\frac{human\ hair}{4\ ly} = \frac{100 \text{ x } 10^{-6}\ m}{4 \text{ x } 9.46 \text{ x } 10^{15}\ m} = 2.6 \text{ x } 10^{-21}$$

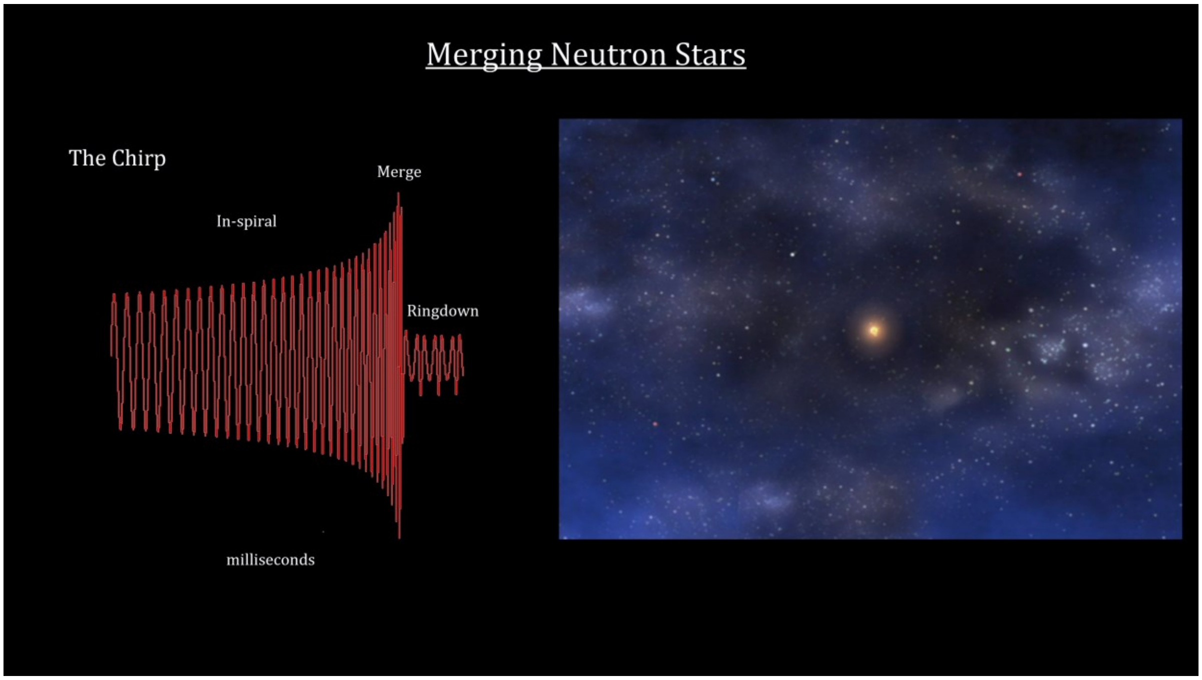
Here is where this data point fits on a graph with wavelength decreasing along the x axis and amplitude increasing along the y axis. Binary systems like this one are plentiful and all around us.

There are literally billions of them sending gravitational waves our way from every direction. But the gravitational waves they create are weak and totally indisguisable from one another. They just wind up contributing to a background noise level. In our GW sensitivity graph, we see that in order to detect a gravitational wave, a binary system will have to create waves with greater amplitudes and higher frequencies to generate smaller wavelengths than the noise level marked in green. To stand out, a binary system is needed that can achieve much higher velocities. And as we have seen from our example, the large diameters of stars prevent them from ever getting close enough to reach the needed velocities. But rotating neutron stars might be small enough to achieve the needed speed.



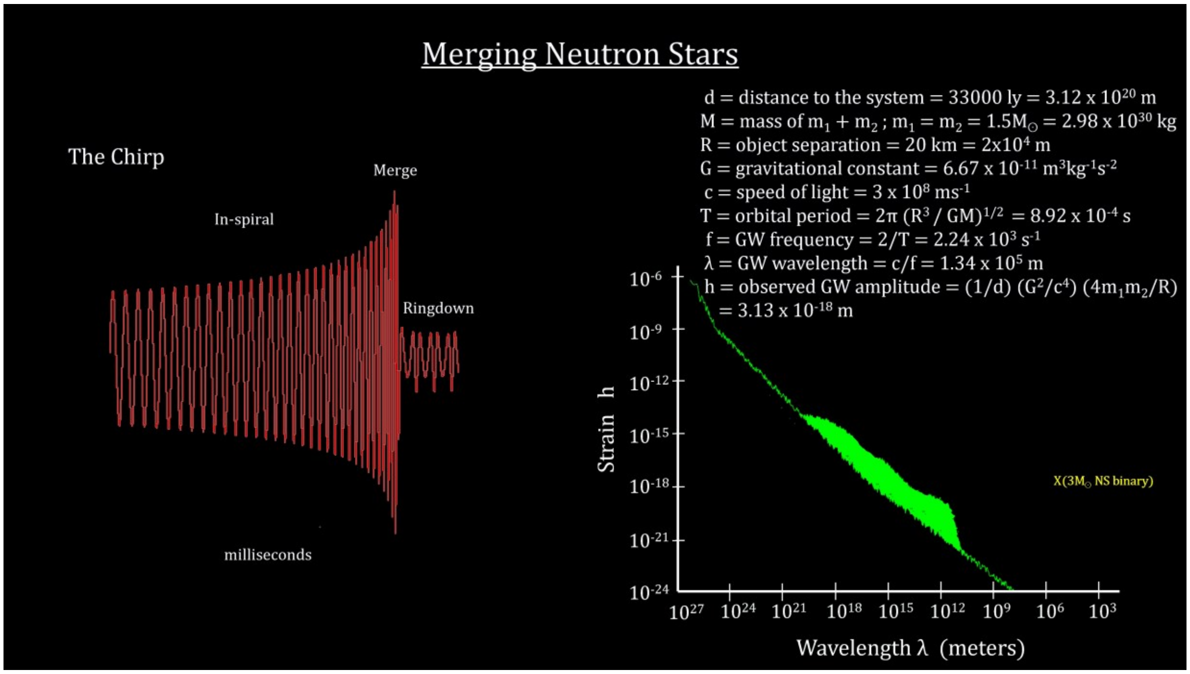
Coalescing Neutron Stars

We covered neutron stars in the “Globular clusters and Supernova” chapter of our “How far away is it” video book. Here’s a system with two equal-mass neutron stars that have reached the point where they are whirling around each other 10,000 times a second. The stars merge in a few milliseconds, sending out a burst of gravitational waves and a brief, intense gamma-ray burst. Here we have the signature curve for this coalescing binary neutron star scenario. You can see the three phases: the in-spiral; the coalesce or merger; and the ring-down to an object – most likely a Kerr Black Hole - that is no longer asymmetric and therefore no longer radiating gravitational waves. If we fed the waveform into an audio generator, it would sound like this {chirp}. We call it the chirp.



Merging Neutron Stars

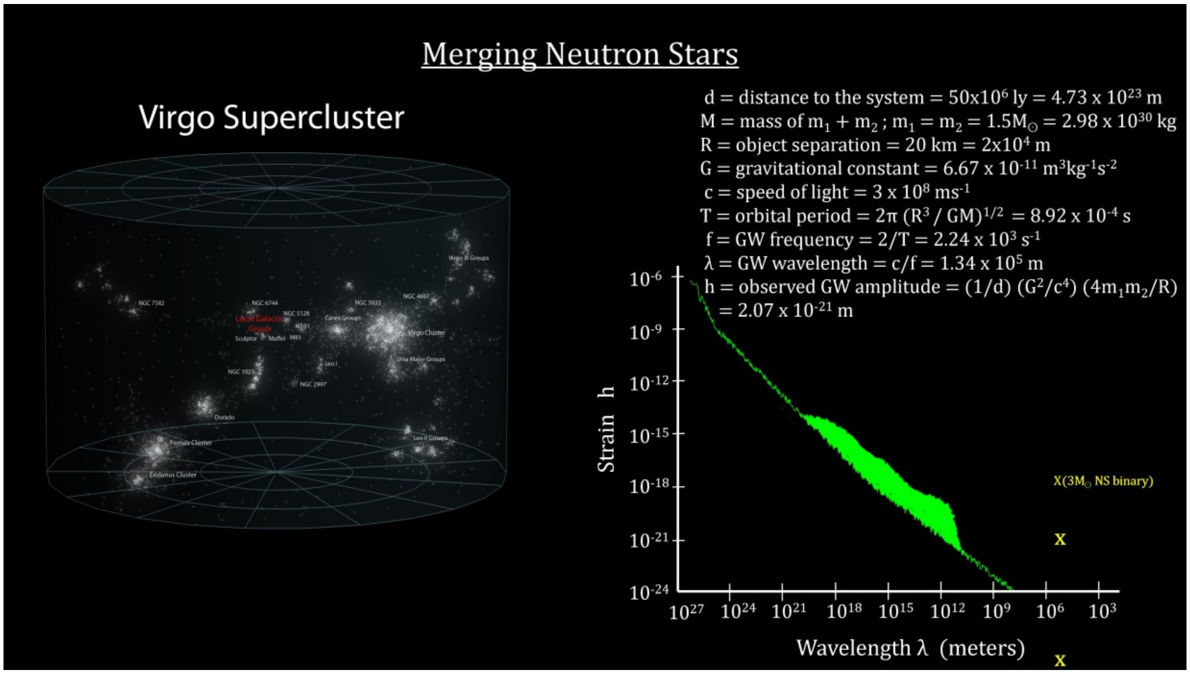
The mass of a typical neutron star is 1.5 times the mass of the sun with a radius of only 10 km. If the system is 33 thousand light years away (an average distance for a Milky Way object), it would give us a theoretically detectable wavelength and amplitude. Here we mark its position on the GW sensitivity graph.



Merging Neutron Stars

d = distance to the system = 33000 ly = 3.12×10^{20} m
M = mass of $m_1 + m_2$; $m_1 = m_2 = 1.5M_\odot = 2.98 \times 10^{30}$ kg
R = object separation = 20 km = 2×10^4 m
G = gravitational constant = $6.67 \times 10^{-11}\ m^3 kg^{-1} s^{-2}$
c = speed of light = $3 \times 10^8\ ms^{-1}$
T = orbital period = $2\pi (R^3 / GM)^{1/2} = 8.92 \times 10^{-4}$ s
f = GW frequency = $2/T = 2.24 \times 10^3\ s^{-1}$
λ = GW wavelength = $c/f = 1.34 \times 10^5$ m
h = observed GW amplitude = $(1/d)(G^2/c^4)(4m_1m_2/R)$
= 3.13×10^{-18} m

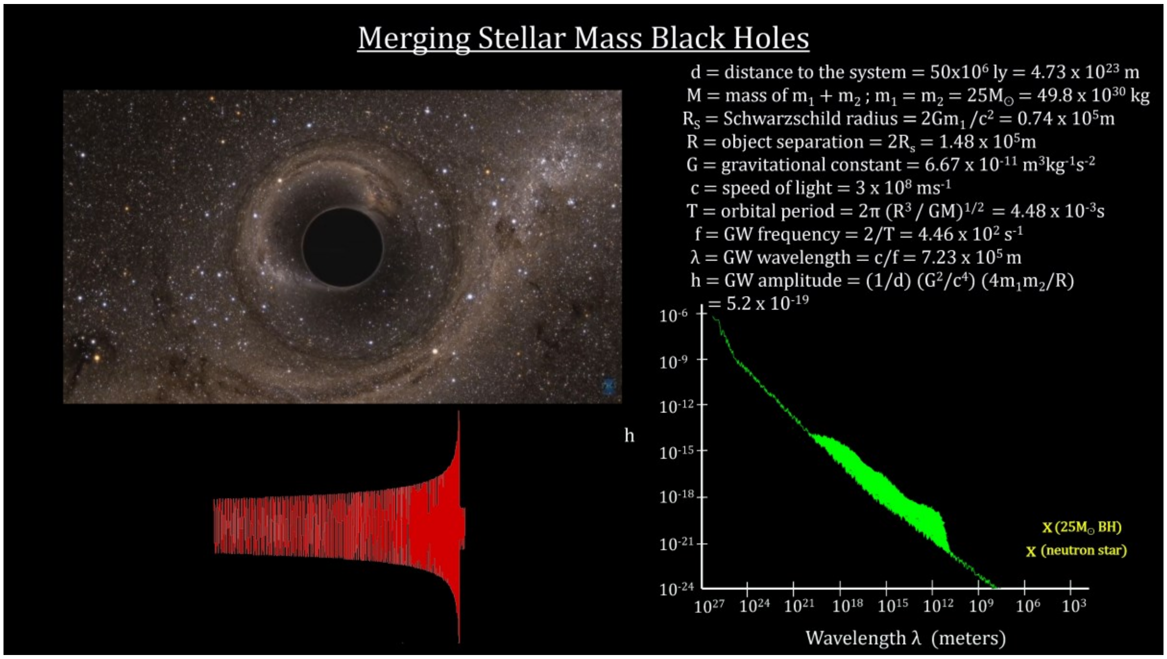
But coalescing neutron stars are not common events. [You'd have to have two massive stars both ending with supernovas that left them still in orbit around each other.]Astronomers estimate that there might be one of these neutron star mergers every 50 years inside the Milky Way. To get a higher rate, we have to move outside the galaxy into the Virgo Supercluster (our local supercluster that we covered in the 'How far away is it" video book. Within a 50 million light year radius we expect to have as many as 10 or more neutron star mergers per year because we're including thousands of galaxies. But as the distance increases, the waves' amplitude drops. Inside the supercluster, it goes into the 10^{-21} range. In fact, as we look out beyond 50 million lightyears, the neutron star merger amplitudes drop below 10^{-24} making them undetectable. If we're going to find detectable gravitational waves beyond this distance, we're going to need merging objects much more massive than neutron stars. For that, we have to involve black holes.



Merging Neutron Stars

d = distance to the system = 50×10^{6} ly = 4.73×10^{23} m
M = mass of $m_1 + m_2$; $m_1 = m_2 = 1.5M_\odot = 2.98 \times 10^{30}$ kg
R = object separation = 20 km = 2×10^{4} m
G = gravitational constant = $6.67 \times 10^{-11}\ m^3kg^{-1}s^{-2}$
c = speed of light = $3 \times 10^{8}\ ms^{-1}$
T = orbital period = $2\pi\,(R^3 / GM)^{1/2} = 8.92 \times 10^{-4}$ s
f = GW frequency = $2/T = 2.24 \times 10^{3}\ s^{-1}$
λ = GW wavelength = $c/f = 1.34 \times 10^{5}$ m
h = observed GW amplitude = $(1/d)\,(G^2/c^4)\,(4m_1m_2/R)$
= 2.07×10^{-21} m

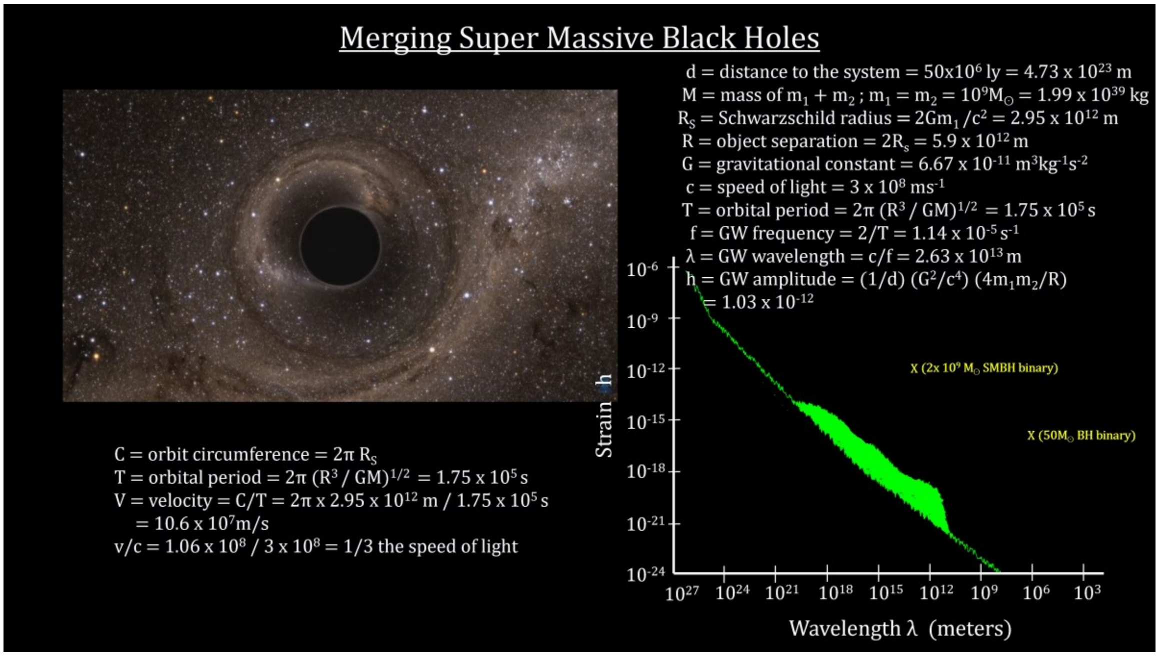
Here we have a black hole merger simulation. We'll start with stellar mass black holes that run from 3 to 50 time the mass of the Sun. In this example, each black hole has 25 times the mass of the sun and they are in orbit 50 mly away in the Virgo Supercluster. The minimum distance between them will be twice their Schwarzschild radius which is larger than the neutron star radius. This puts them further apart than the earlier neutron stars example. But the increase in mass more than compensates for that and produces a much larger amplitude.



Merging Stellar Mass Black Holes

d = distance to the system = 50×10^6 ly = 4.73×10^{23} m
M = mass of $m_1 + m_2$; $m_1 = m_2 = 25M_\odot$ = 49.8×10^{30} kg
R_S = Schwarzschild radius = $2Gm_1/c^2$ = 0.74×10^5m
R = object separation = $2R_s$ = 1.48×10^5m
G = gravitational constant = 6.67×10^{-11} m^{3}kg^{-1}s^{-2}
c = speed of light = 3×10^8 ms^{-1}
T = orbital period = $2\pi (R^3 / GM)^{1/2}$ = 4.48×10^{-3}s
f = GW frequency = 2/T = 4.46×10^2 s^{-1}
λ = GW wavelength = c/f = 7.23×10^5 m
h = GW amplitude = $(1/d)(G^2/c^4)(4m_1m_2/R)$
= 5.2×10^{-19}

Now suppose these were two SMBHs with each having 10 billion times the mass of the sun. The Schwarzschild radius would become 40 million times larger. The gravitational wave's wavelength would be 36 million times longer, and the wave's amplitude would be 2 million times greater. In addition, calculating the circumference of the orbit and dividing by the period, we see that these behemoths are traveling around each other at up to 1/3 the speed of light.

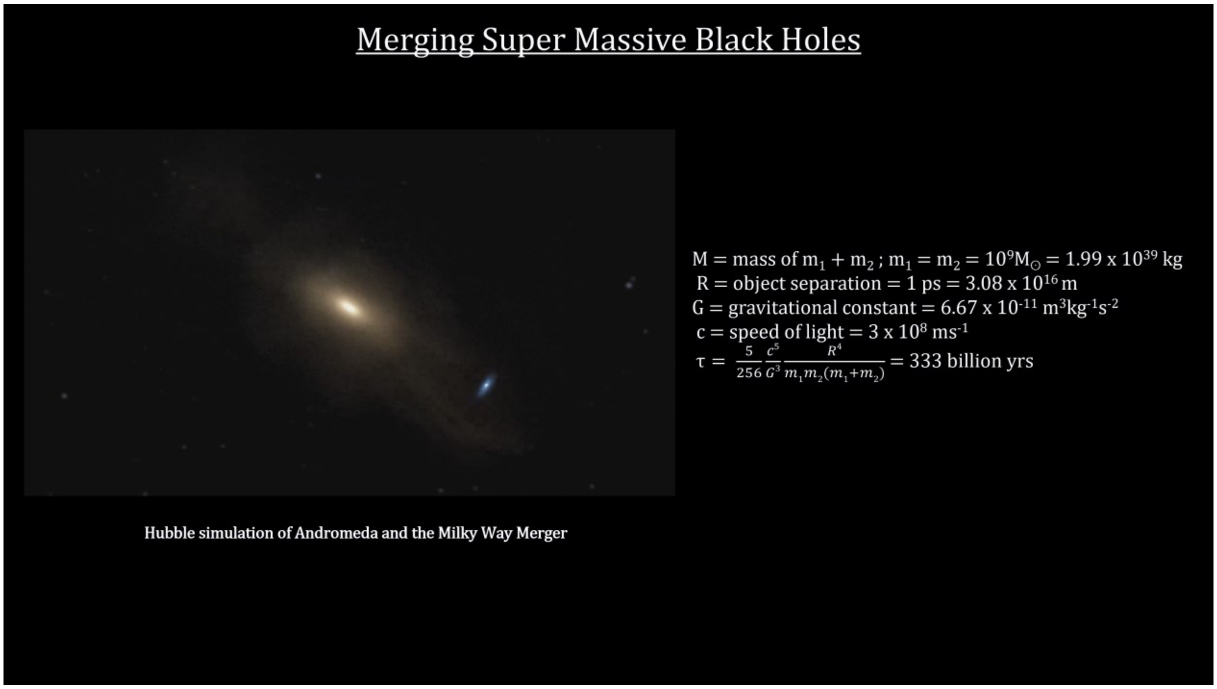


Merging Super Massive Black Holes

d = distance to the system = 50×10^6 ly = 4.73×10^{23} m
M = mass of $m_1 + m_2$; $m_1 = m_2 = 10^9M_\odot$ = 1.99×10^{39} kg
R_S = Schwarzschild radius = $2Gm_1/c^2$ = 2.95×10^{12} m
R = object separation = $2R_s$ = 5.9×10^{12} m
G = gravitational constant = 6.67×10^{-11} m^{3}kg^{-1}s^{-2}
c = speed of light = 3×10^8 ms^{-1}
T = orbital period = $2\pi (R^3 / GM)^{1/2}$ = 1.75×10^5 s
f = GW frequency = 2/T = 1.14×10^{-5} s^{-1}
λ = GW wavelength = c/f = 2.63×10^{13} m
h = GW amplitude = $(1/d)(G^2/c^4)(4m_1m_2/R)$
= 1.03×10^{-12}

C = orbit circumference = $2\pi R_S$
T = orbital period = $2\pi (R^3 / GM)^{1/2}$ = 1.75×10^5 s
V = velocity = C/T = $2\pi \times 2.95 \times 10^{12}$ m / 1.75×10^5 s
= 10.6×10^7m/s
v/c = $1.06 \times 10^8 / 3 \times 10^8$ = 1/3 the speed of light

But black holes this massive in circular orbits might never actually merge. Although merging galaxies with black holes at their centers will have their black holes sink to the center of the new merged galaxy, they may not get close enough to form a new single black hole. This is the case because, by the time the black holes reach a separation distance of around 1 parsec or 3.2 light years, the 'time to merger' equation shows that this one will take over 333 billion years. That's 24 times the age of the universe! Astronomers call this the 'Parsec problem'. Yet, the SMBHs Sagittarius A* and Andrameda A* are not binary systems.

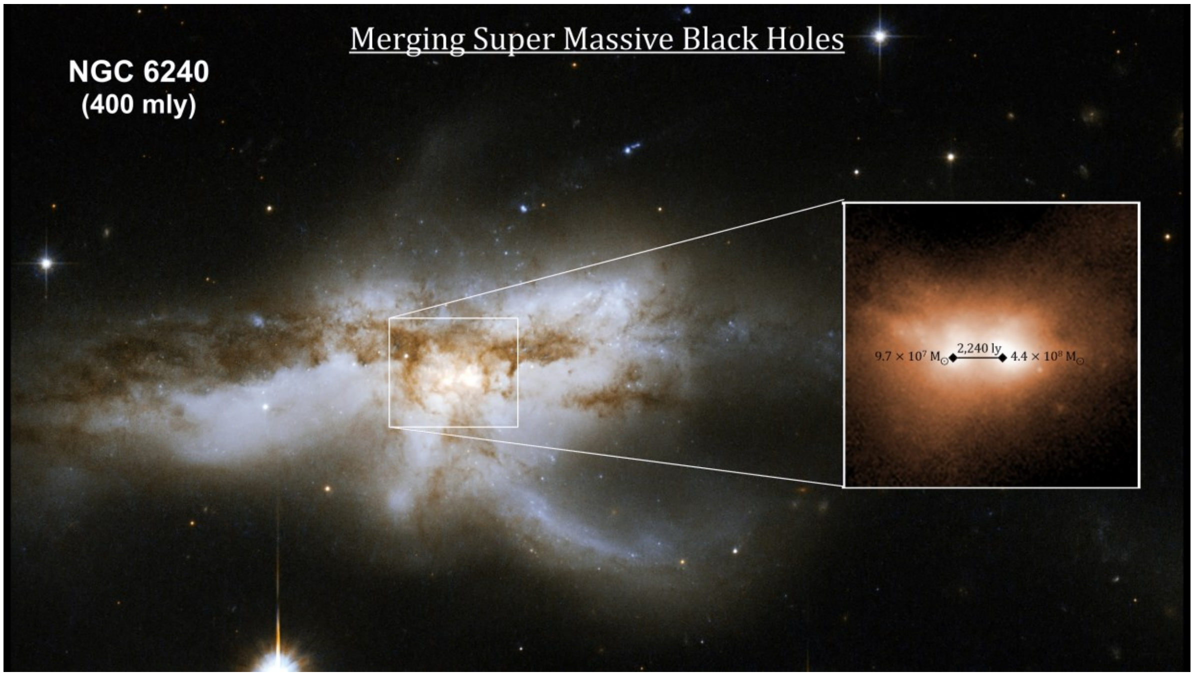


Merging Super Massive Black Holes

Hubble simulation of Andromeda and the Milky Way Merger

M = mass of $m_1 + m_2$; $m_1 = m_2 = 10^9 M_\odot = 1.99 \times 10^{39}$ kg
R = object separation = 1 ps = 3.08×10^{16} m
G = gravitational constant = 6.67×10^{-11} m^{3}kg^{-1}s^{-2}
c = speed of light = 3×10^8 ms^{-1}
$\tau = \frac{5}{256}\frac{c^5}{G^3}\frac{R^4}{m_1 m_2(m_1+m_2)}$ = 333 billion yrs

Putting things into perspective, equal mass SMBHs in a circular orbit would be rare. Elliptical orbits with different masses in varying environments are the norm. And this can dramatically change the time to merge. Here's the recently merged galaxy NGC 6240. It has two SMBHs orbiting each other 2,240 ly apart. (That's 750 parsecs). They are orbiting in a sea of molecular gas with a mass of around 9 billion times the mass of our Sun as measured by the Atacama Large Millimeter/submillimeter Array in northern Chile. Based on stellar dynamics, one is up to $9.7 \times 10^7 M_\odot$. The other is up to $4.4 \times 10^8 M_\odot$. Taking all this into account, researchers calculated that the SMBHs will merge around 55 million years from now.



Gravitational Wave Source Spectrum

Here's our Gravitational Wave Sensitivity Graph. I've included several of our equal-mass merger examples. And I've added the frequencies that correspond to the wavelengths in order to highlight the gravitational wave spectrum. Also, note the expanse covered by the two axes. The strain is always a small number, but the top value of 10 to the minus 6 is a billion trillion times larger than the lower value of 10 to the minus 24. And the wavelength goes from a thousand meters on the far right up to the diameter of the universe on the left. This large extreme covers the gravitational waves created at the time of the Big Bang that have been stretched by the expansion of the universe for over 13 billion years.

Here's the range for Super-Massive Black Hole Mergers. They produce a huge burst of gravitational waves at millihertz frequencies detectable throughout most of the known Universe.

Here's the range for Super-Massive Black Hole mergers with stellar mass black holes and neutron stars.

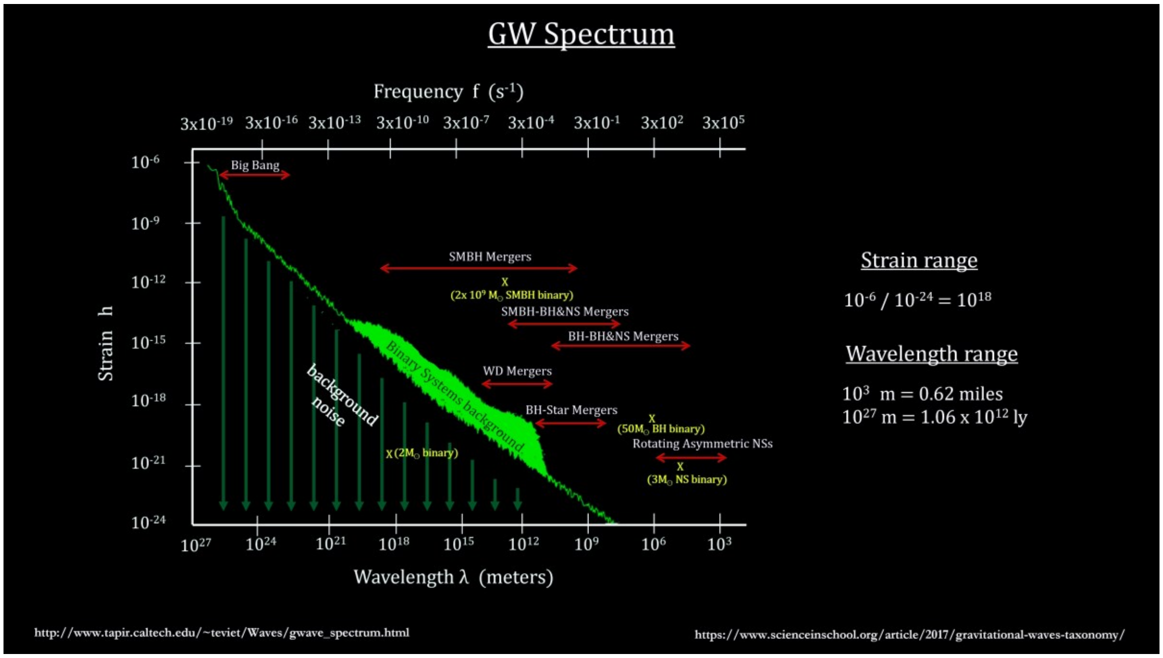
Here's the range for stellar mass black hole mergers with stellar mass black holes and neutron stars.

Here's the range for stellar mass black hole mergers with stars.

Here's the range for White Dwarf mergers. These would only be detectable if they happen in our galaxy.

And here's the range for rapidly spinning neutrons stars with an uneven mass distribution. It's the angular acceleration of uneven mass components (like a mountain on the surface) that produces the detectable gravitational waves.

The green area above the detectability line is for the millions of non-merging Super-Massive Black Hole binaries in galaxy centers. They create theoretically detectable gravitational waves, but there are too many of them to distinguish one wave from another. This makes their individual signals unresolvable. At the lower end, this would include non-merging binary White Dwarf stars in the Milky Way.



http://www.tapir.caltech.edu/~teviet/Waves/gwave_spectrum.html

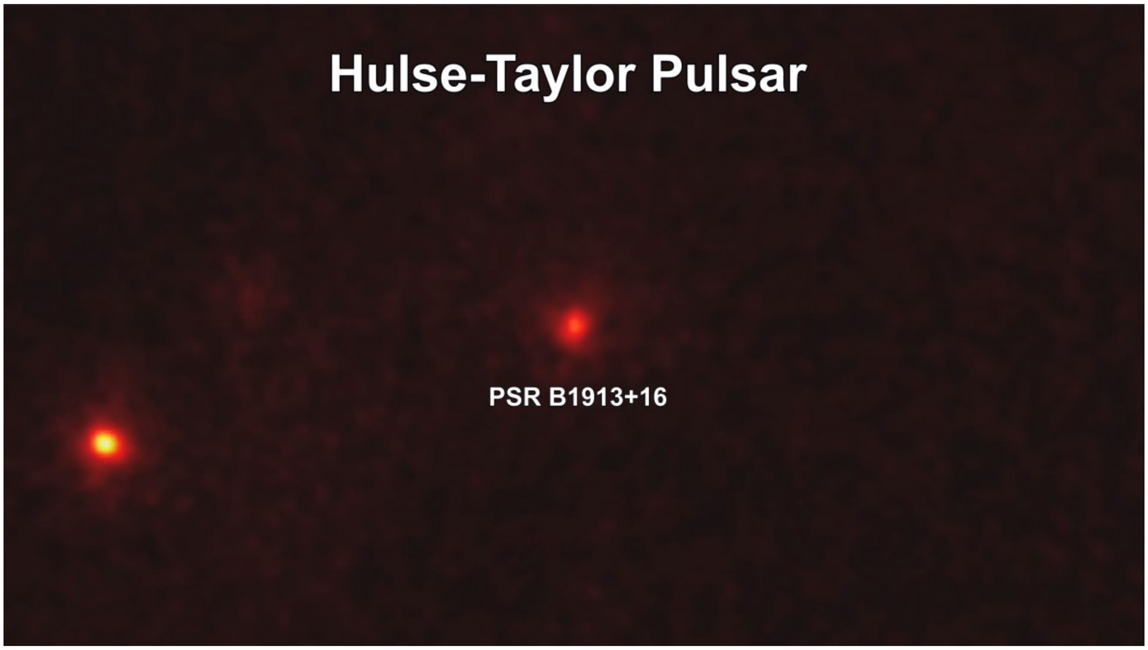
https://www.scienceinschool.org/article/2017/gravitational-waves-taxonomy/

We'll now turn our attention to how we can go about detecting these waves.

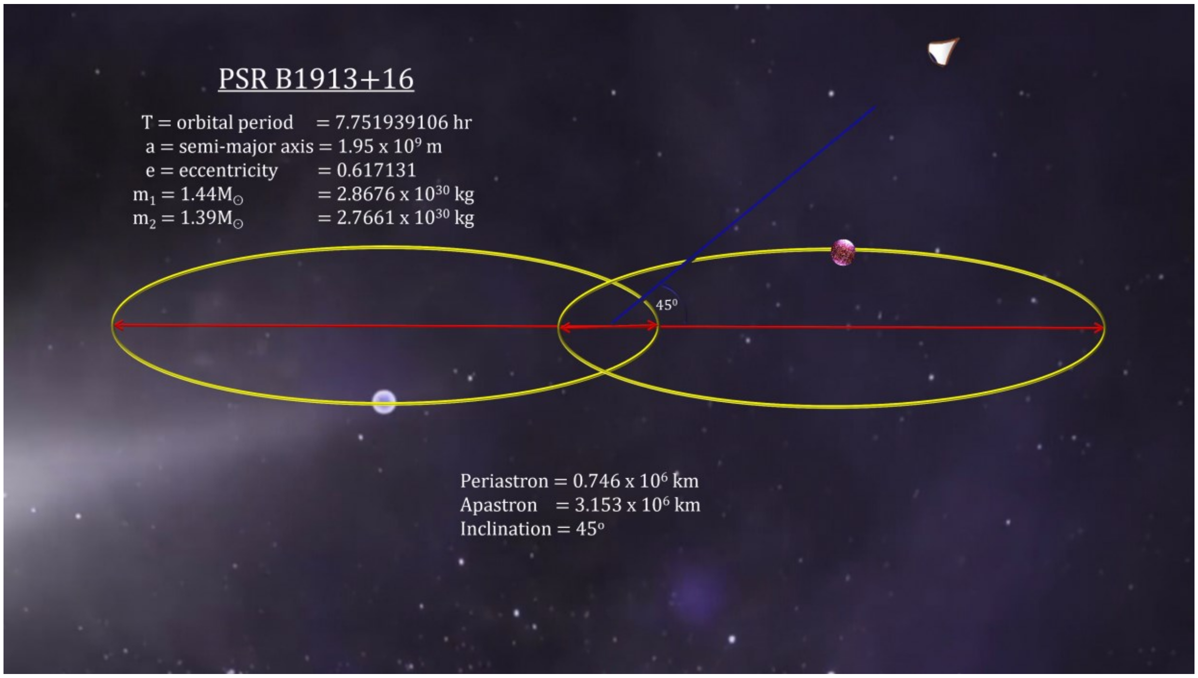
First Gravitational Wave Evidence

In 1974, 58 years after Einstein predicted the existence of gravitational waves, two radio astronomers (Joe Taylor and Russell Hulse) were looking for new pulsars using the 305-meter Arecibo Radio Telescope in Puerto Rico. They found one. It's named PSR B1913+16 and it led to the first indirect verification of Einstein's prediction.

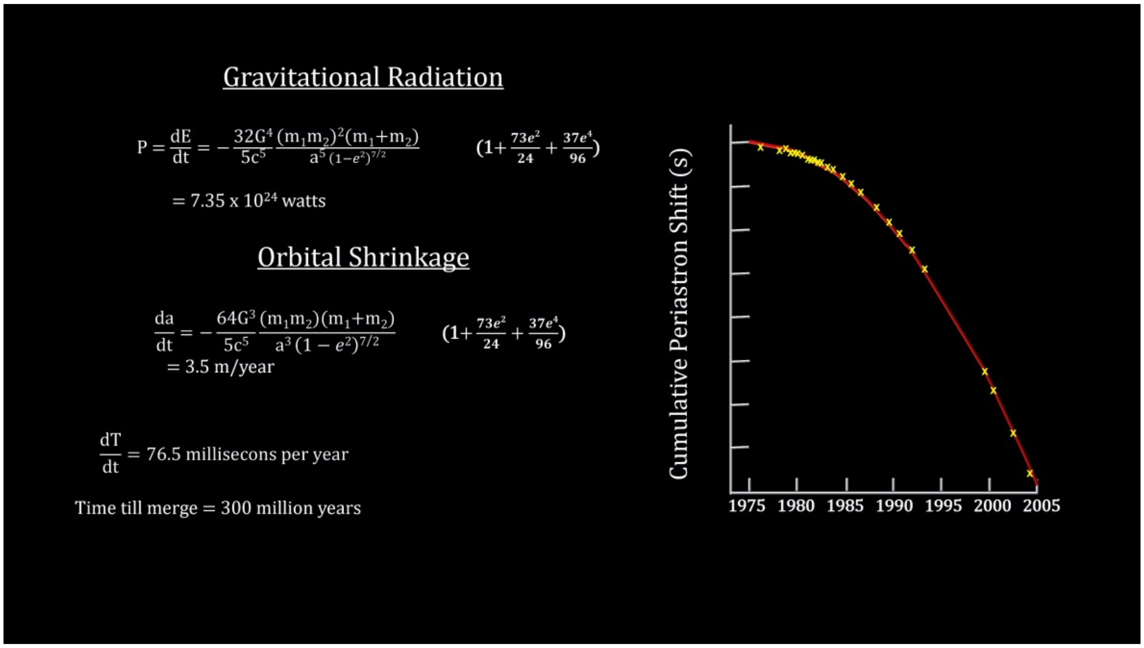
You'll recall from the "Globular Clusters and Supernova" chapter in the "How far away is it" video book, that a pulsar is a rapidly rotating neutron star with a powerful magnetic field that accelerates charged particles as it rotates. The accelerating charged particles produce electromagnetic radiation primarily at radio wavelengths. The result is a sort of magnetic lighthouse, which if aligned correctly, flashes in our direction twice each cycle. These signals are highly regular. In fact, pulsars are some of the best clocks in nature, and this allows extremely precise measurements of their motion.



This one was pulsing every 59 milliseconds indicating that the pulsar rotates 17 times per second. But Hulse and Taylor noticed that the pulses varied regularly every 7.75 hours with pulses arriving 3 seconds earlier at some times relative to others. This meat that the pulsar was in an elliptical orbit with another neutron star! This was the first binary neutron star system ever discovered. Detailed analysis identified the shape and size of the highly elliptical binary orbits. Using the orbital motion, they calculated the star masses; their closest approach (called the pariastron (**pare**-e-astrin)); and their furthest distance apart (called the Apastron) as well as the system's inclination.



With this information and the GW equations, they were able to calculate the amount of gravitational radiation; the expected decay of the orbit due to the lost gravitational energy; and the corresponding reduction in the time it takes per orbit. This graph maps the accumulated reduction in orbital periods against time assuming that Einstein's GW equations are correct. Hulse, Taylor and others have studied this binary system for 40 years now. This graph records their measurements. We see that the measurements fit the theory perfectly. **[**Joe Taylor and Russell Hulse received the Nobel Prize for this discovery in 1993.**]** This gave scientists confidence that Einstein's gravitational waves do indeed exist.



Gravitational Radiation

$$P = \frac{dE}{dt} = -\frac{32G^4}{5c^5}\frac{(m_1m_2)^2(m_1+m_2)}{a^5(1-e^2)^{7/2}} \qquad (1+\frac{73e^2}{24}+\frac{37e^4}{96})$$

$$= 7.35 \times 10^{24} \text{ watts}$$

Orbital Shrinkage

$$\frac{da}{dt} = -\frac{64G^3}{5c^5}\frac{(m_1m_2)(m_1+m_2)}{a^3(1-e^2)^{7/2}} \qquad (1+\frac{73e^2}{24}+\frac{37e^4}{96})$$

$$= 3.5 \text{ m/year}$$

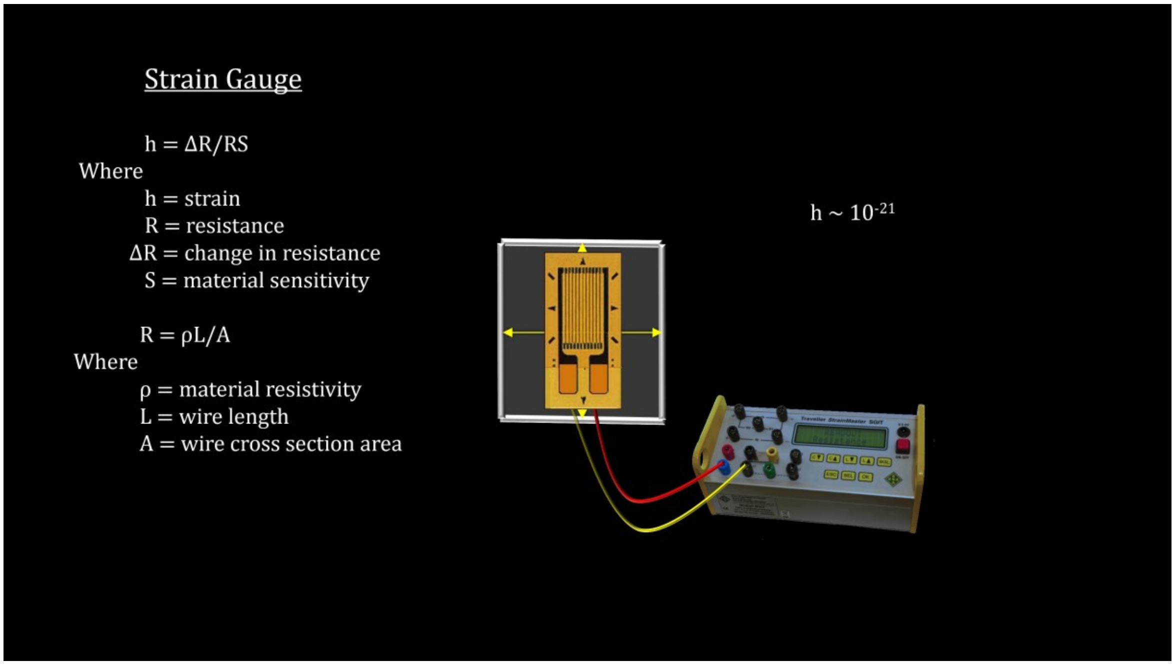
$$\frac{dT}{dt} = 76.5 \text{ millisecons per year}$$

Time till merge = 300 million years

[The system is expected to merge in around 300 million years. Because neutron stars are the end product of supernova explosions, it was thought that any binary partner would be ejected when the other explodes - making binary neutron stars unlikely. This discovery showed that they do exist and they will merge in a timeframe that should make the event quite common – say 3 or 4 per year somewhere in the Universe.]

Gravitational Wave Detectors – LIGO

Direct detection of gravitational waves is tricky for two main reasons. One is that the amplitudes of the waves are so small and the other is that the measuring sticks you might use to measure a change in length are changed themselves. In other words, the changed length will still read out as one meter. But the stretching and squeezing does put a strain on the plate that can be measured with an attached wire that acts as a resistor. It's called a strain gauge. If we attach wires along the plate instead of meter sticks, we can measure changes in the resistance of the wire as it is stretched and squeezed. A longer thinner wire will provide more resistance to an electric current, and a shorter fatter wire will provide less resistance to an electric current thus giving us a measure of the strain. Unfortunately, this technique is literally millions of times too insensitive to measure the tiny GW amplitudes **h**. But this technique is why we call **h** a measure of strain.



Strain Gauge

$h = \Delta R/RS$

Where

h = strain

R = resistance

ΔR = change in resistance

S = material sensitivity

$R = \rho L/A$

Where

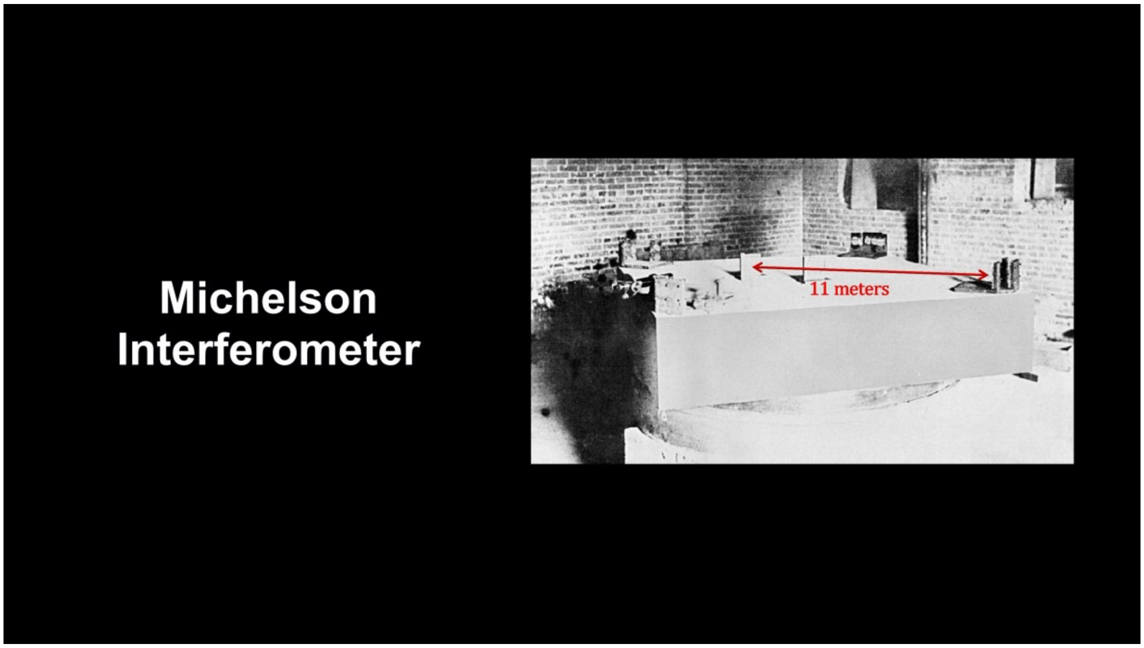
ρ = material resistivity

L = wire length

A = wire cross section area

$h \sim 10^{-21}$

Michelson Interferometers look like the best chance to detect these waves. You'll recall that we covered interferometers in the first chapter of this video book. The arms on that one where 11 meters long and its sensitivity was nowhere near what is needed for GWs.

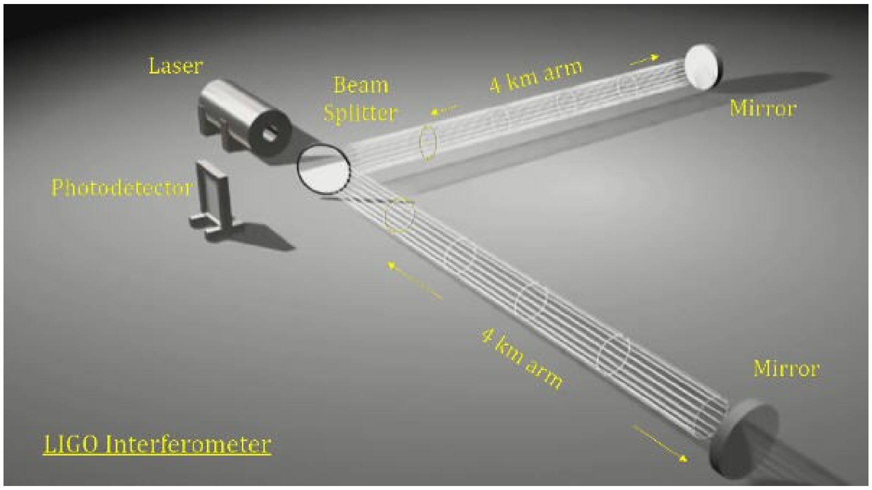


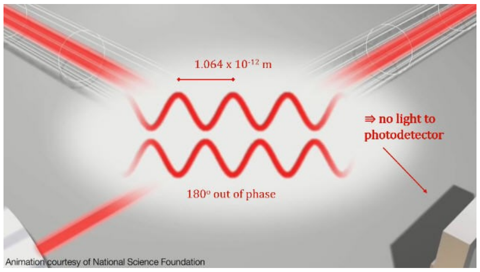
Michelson Interferometer

Today we have LIGO (the Laser Interferometer Gravitational-wave Observatory). In 2015, it started with two identical interferometers 3000 km apart (that's 1,864 miles): with one near Hartford, Washington and the other near Livingston, Louisiana.

Here are the L-shaped LIGO instrument components:

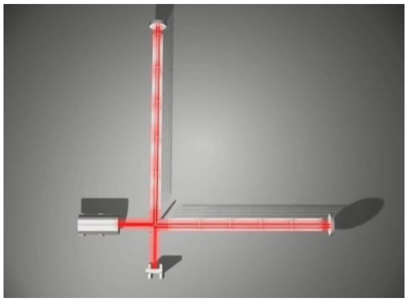
- It has a powerful near-infrared laser with an output, after amplification that reaches 200 watts of 1064 nanometer light.
- The beam splitter and mirrors (that act as test masses) are 40 kg objects suspended via fused silica glass fibers to minimize noise due to vibrations. Additional internal and external active vibration minimization technologies eliminate the effects of everything from nearby traffic to lunar tidal forces.
- The 4 km arms are 10,000 cubic cm of ultra-high vacuum equal to 1 trillionth of an atmosphere. In addition, each arm contains reflection mirrors that route the light back and forth inside the arms 280 times before it hits the exits for recombination.
- The photodetector is a state-of-the-art indium-gallium-arsenide photodiode array with a high quantum efficiency designed to detect extremely small amounts of light at a wavelength of 1064 nm.



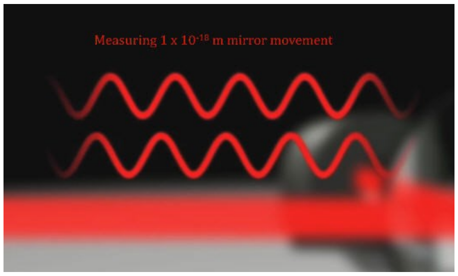


Animation courtesy of National Science Foundation

The laser light is split and sent to the two mirrors. On return, they are recombined and sent to the photodetector. The beams returning from the two arms are kept out of phase so that when the arms are both in synch (as when there is no gravitational wave passing through), their light waves subtract, and no light arrives at the photodetector.

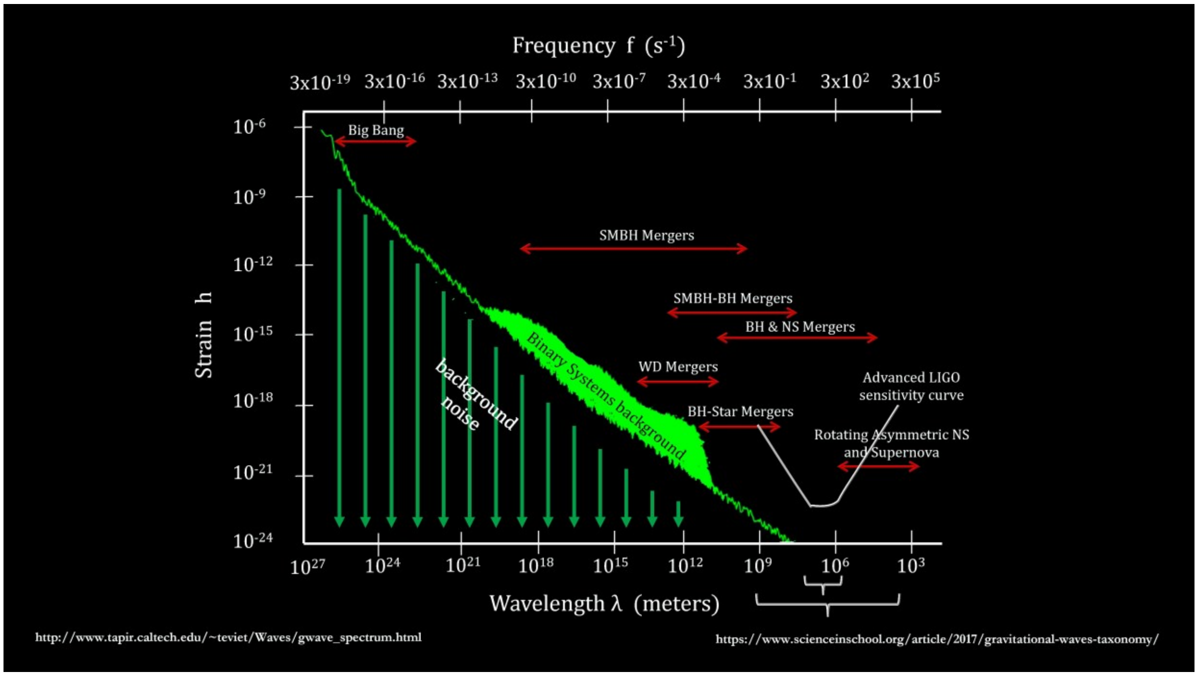


When a gravitational wave passes through the interferometer, the distances along the arms of the interferometer are shortened and lengthened, causing the beams to become slightly out of synch. Hence, some light arrives at the photodetector, indicating a signal.



Given LIGO's extra 280 passes through the tube, a GW strain amplitude of 10^{-21} would displace the mirrors 10^{-18} m (that's one thousandth the diameter of a proton).

Here's our sensitivity graph. And here's the sensitivity line for the current LIGO capabilities. All gravitational waves with strains below the curve are undetectable. The curve shows we should be able to detect gravitational waves with wavelengths between 10^3 and 10^9 meters, with the maximum sensitivity enabling detections with strains as small as 10^{-22} when the wavelengths are between 10^6 and 10^7 meters. This is a range where powerful binary system mergers of stellar mass black holes and neutron stars within the Virgo Supercluster, our local supercluster [see How Far Away Is It – Virgo Supercluster with annotation] should be detectable.

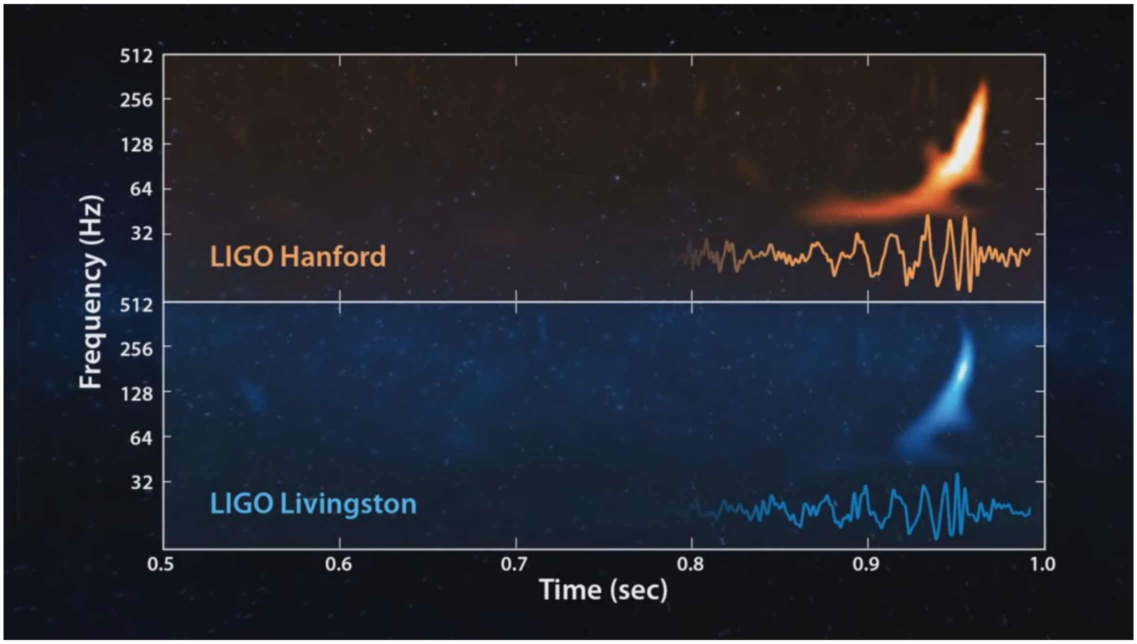


http://www.tapir.caltech.edu/~teviet/Waves/gwave_spectrum.html

https://www.scienceinschool.org/article/2017/gravitational-waves-taxonomy/

The 1st Gravitational Wave Ever Detected - GW150914

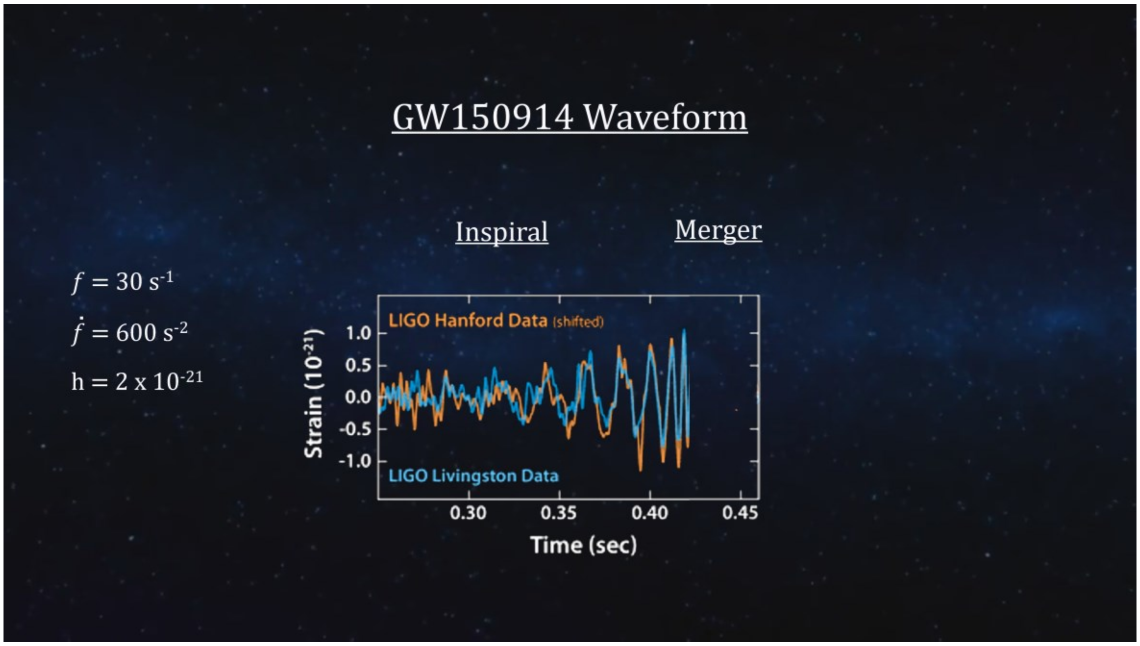
At 09:50:45 Coordinated Universal Time on the 14th of September 2015 a signal was detected by the LIGO detectors in Livingston and 6.9 milliseconds later in Hanford. It was a chirp signal that lasted just over 0.2 of a second. When we route the wave into a sound generator, here's what it sounds like.



This plot combines the data from both sites. The waveform is consistent with coalescing masses with:

- a 10 cycle, 200 ms inspiral that gives us the frequency, the rate that the frequency changes, and a peak wave amplitude,
- a merger that takes around 2 ms
- and a ringdown as the coalesced objects cease to radiate gravitational energy.

Detector noise introduces errors into all the calculations based on these figures. That's why we'll provide a range for each item.



GW150914 Waveform

Inspiral

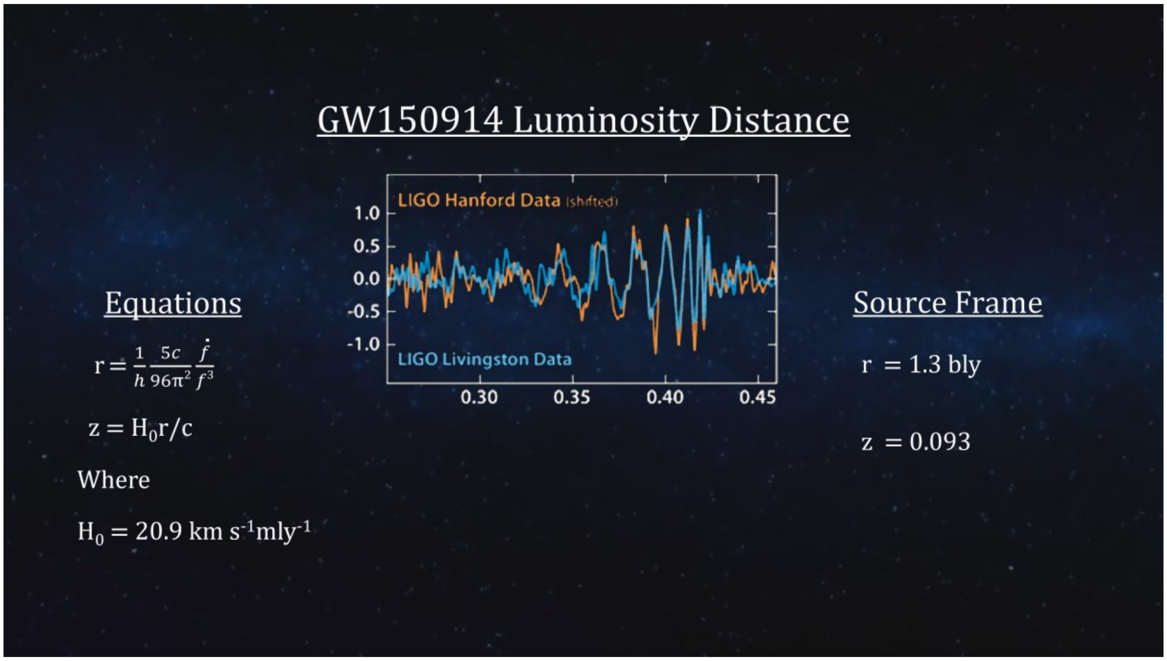
Merger

$f = 30\ s^{-1}$

$\dot{f} = 600\ s^{-2}$

$h = 2 \times 10^{-21}$

The amplitude and frequency data points give us the luminosity distance. It is important to note that gravitational waves experience redshifting as they travel across the cosmos just like light does. Having traveled around a billion light years, this wave would have experienced a redshift near .1. So, the frequency we see here is a bit smaller than the frequency at the start of the wave's journey here.



GW150914 Luminosity Distance

Equations

$$r = \frac{1}{h}\frac{5c}{96\pi^2}\frac{\dot{f}}{f^3}$$

$$z = H_0 r/c$$

Where

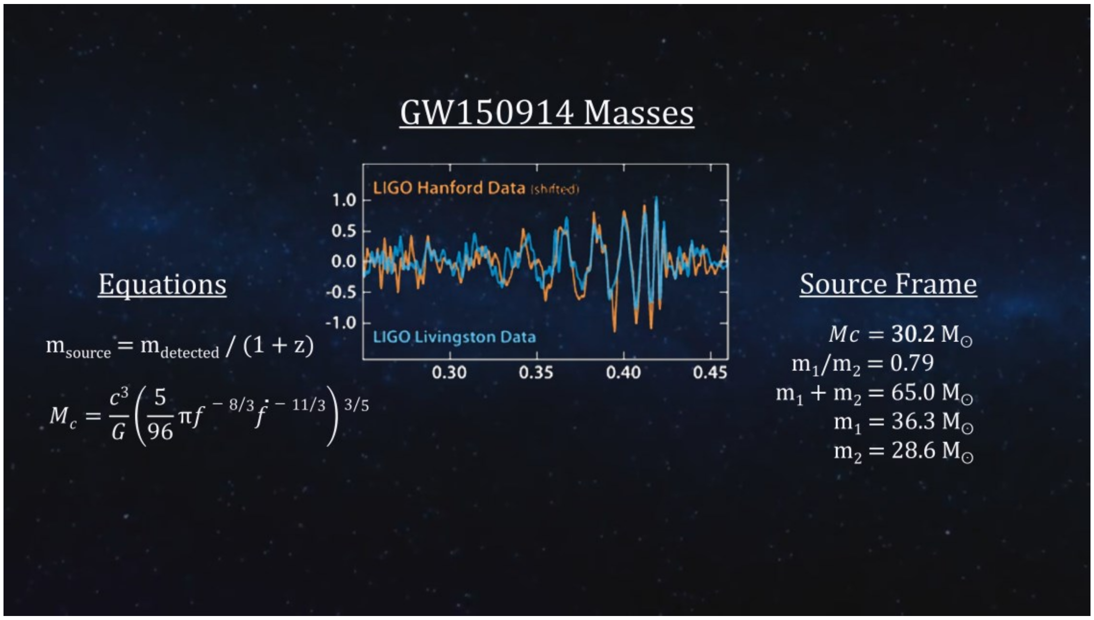
$H_0 = 20.9$ km s^{-1} mly^{-1}

Source Frame

r = 1.3 bly

z = 0.093

The frequency data also gives us the chirp mass. Taking the redshift and information gleamed from the merge and ringdown portions of the waveform, we get the binary system masses. These masses are too large for neutron stars that are only a few times the mass of the Sun. So, we must be witnessing the merger of two large stellar mass black holes!



GW150914 Masses

Equations

$$m_{source} = m_{detected} / (1 + z)$$

$$M_c = \frac{c^3}{G}\left(\frac{5}{96}\pi f^{-8/3}\dot{f}^{-11/3}\right)^{3/5}$$

Source Frame

$Mc = 30.2\ M_\odot$

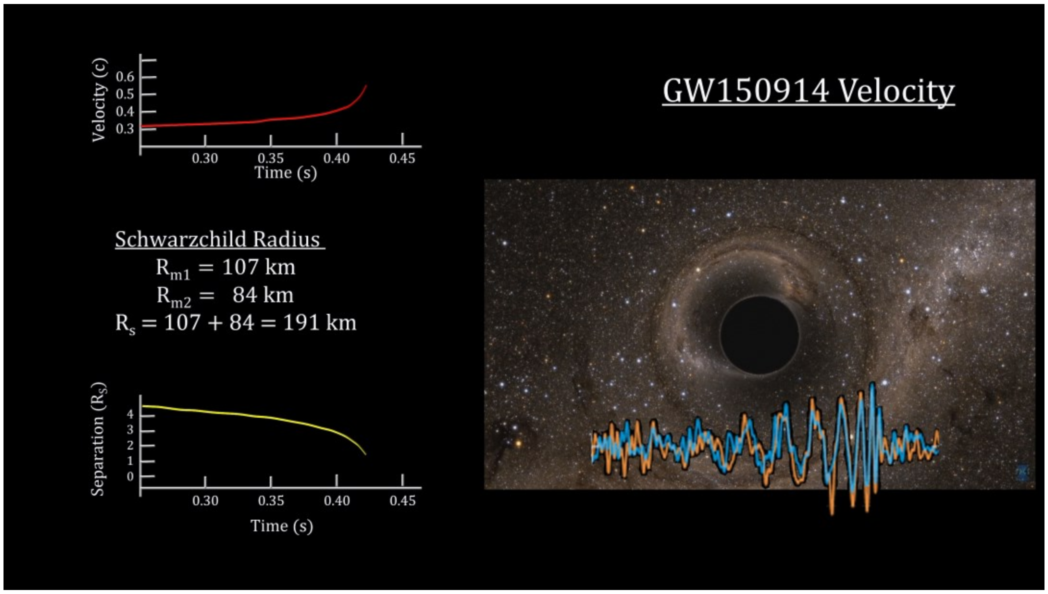
$m_1/m_2 = 0.79$

$m_1 + m_2 = 65.0\ M_\odot$

$m_1 = 36.3\ M_\odot$

$m_2 = 28.6\ M_\odot$

During the last 200 ms of their inspiral, the orbiting velocity of the black holes increased from 30% speed of light to 60% of the speed of light. Over the same period, the distance between the two black holes went from around a thousand km to just under 200 km when their event horizons made contact.



GW150914 Velocity

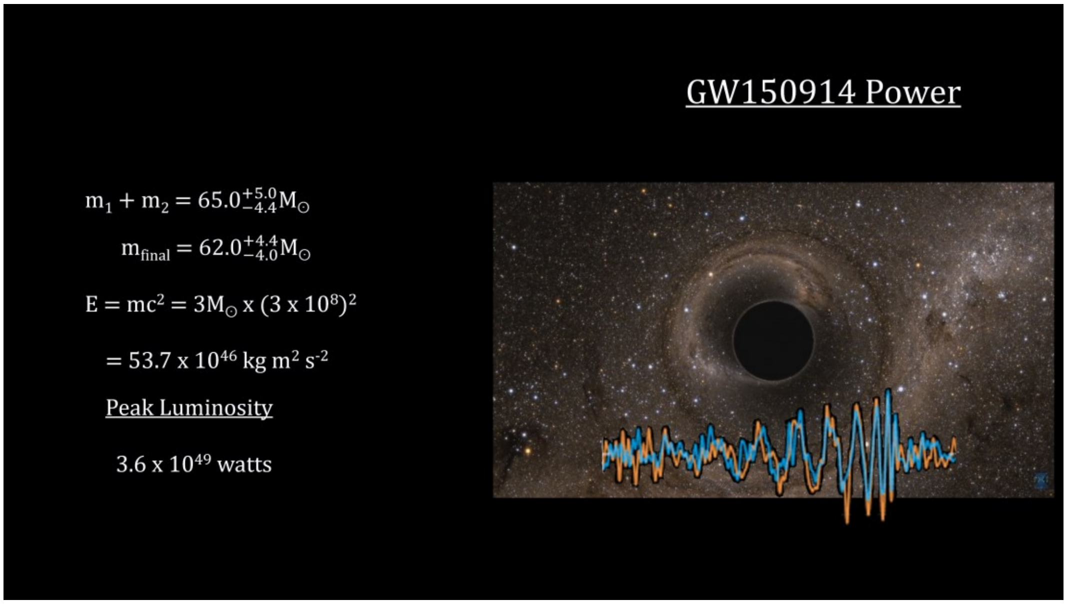
Schwarzchild Radius

$R_{m1} = 107$ km

$R_{m2} = 84$ km

$R_s = 107 + 84 = 191$ km

Modeling the final ringdown shows that the mass of the resulting Kerr Black Hole is around 62 solar masses. That's 3 solar masses less than the sum of the masses of the two inspiraling black holes. This mass was converted to the radiated gravitational energy. In other words, during the final 20 milliseconds of the merger, the power of the radiated gravitational waves peaked at about 3.6×10^{49} watts.



GW150914 Power

$m_1 + m_2 = 65.0^{+5.0}_{-4.4} M_\odot$

$m_{\text{final}} = 62.0^{+4.4}_{-4.0} M_\odot$

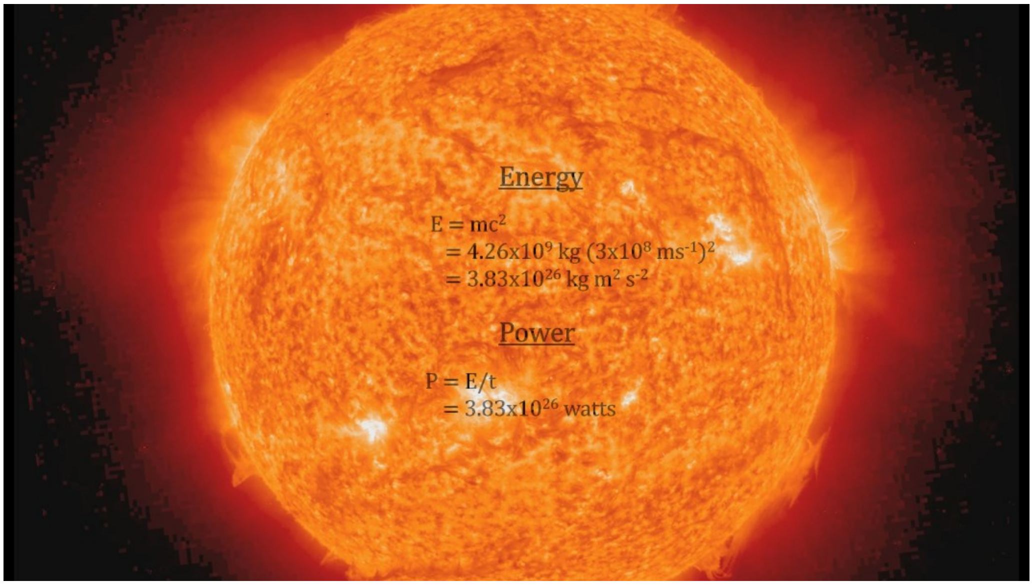
$E = mc^2 = 3M_\odot \times (3 \times 10^8)^2$

$= 53.7 \times 10^{46}$ kg m^2 s^{-2}

Peak Luminosity

3.6×10^{49} watts

Let's take a second to get a feel for how large this number is. In our 'How far away is it" segment on Nearby Stars, we found that the sun converts 4.26 million metric tons of matter into energy every second. The resulting power output is equal to 4 billion hydrogen bombs exploding every second! **[**3.85×10^{26} watts**]**. The Sun is an average star. So, we can use this as an average stellar power output.



Energy

$E = mc^2$

$= 4.26\text{x}10^9 \text{ kg } (3\text{x}10^8 \text{ ms}^{-1})^2$

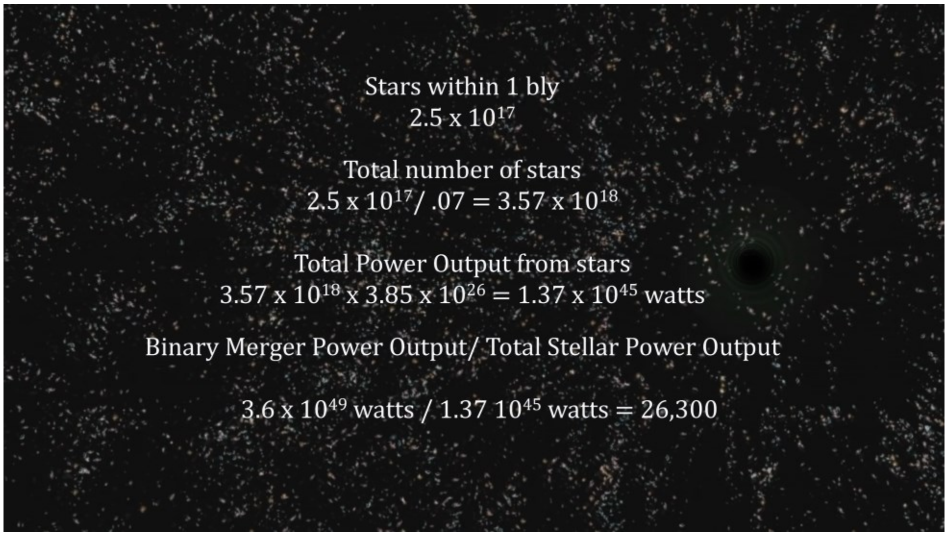
$= 3.83\text{x}10^{26} \text{ kg m}^2 \text{ s}^{-2}$

Power

$P = E/t$

$= 3.83\text{x}10^{26}$ watts

From our segment on Local Superclusters, we saw that there are 250 thousand trillion stars within one billion light years. This represented around 7% of the total number of stars in the Universe. We get the total power emitted by all stars in the visible Universe by multiply the average watts per star times the number of stars. The power generated over the last 20 milliseconds by this merger of two stellar mass black holes is 26 thousand times greater than the combined power of all the light radiated by all the stars in the Universe over that same period of time. That's the signal we saw in September, 2015 a billion years after it happened (sound chirp).



Stars within 1 bly
2.5×10^{17}

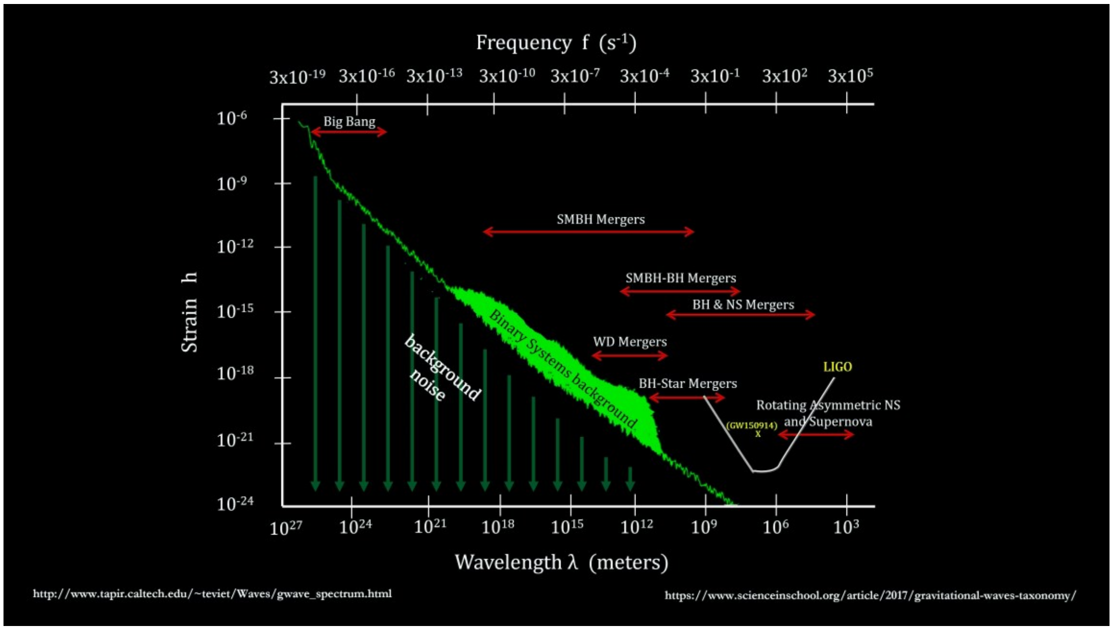
Total number of stars
$2.5 \times 10^{17} / .07 = 3.57 \times 10^{18}$

Total Power Output from stars
$3.57 \times 10^{18} \times 3.85 \times 10^{26} = 1.37 \times 10^{45}$ watts

Binary Merger Power Output/ Total Stellar Power Output

3.6×10^{49} watts / $1.37\ 10^{45}$ watts = 26,300

Here's this event's place on the sensitivity graph. You can see that it's well within the sensitivity area covered by LIGO.



http://www.tapir.caltech.edu/~teviet/Waves/gwave_spectrum.html

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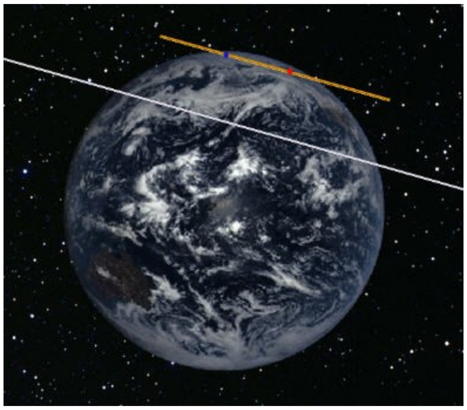
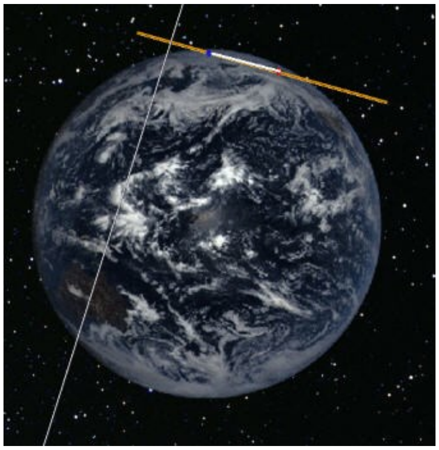
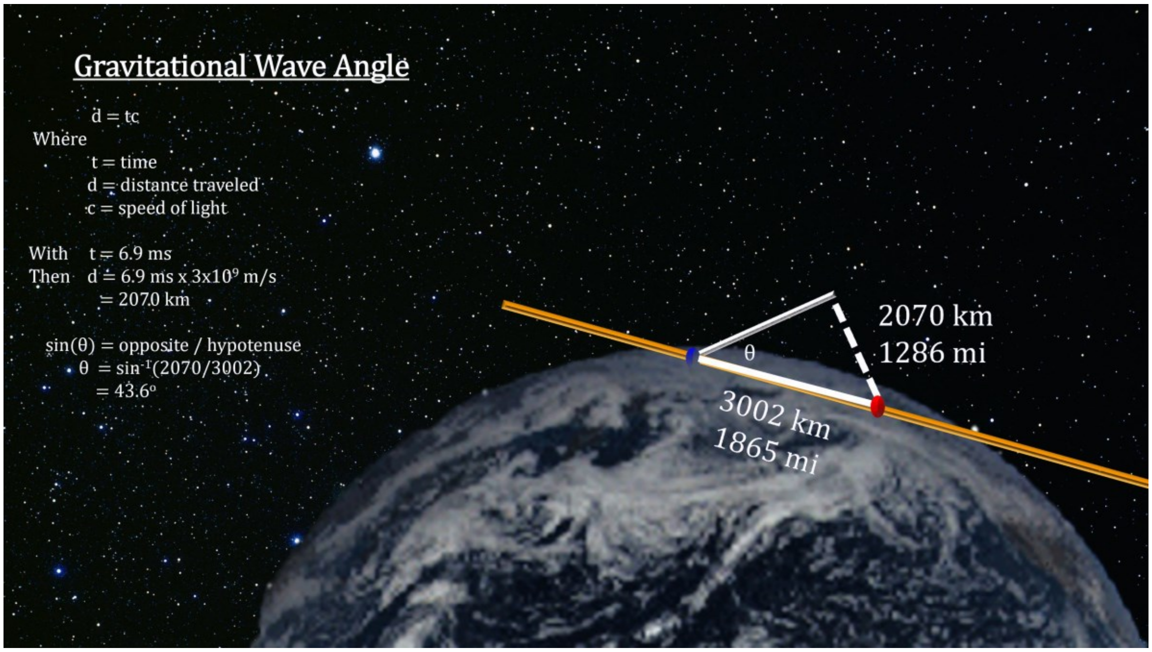


We have used the wave information to find the energy, wavelengths and masses involved as well as the distance to this event. But the wave information does not tell us in which direction it came from because each interferometer is a whole-sky monitor with very little directional information. But having two detectors does give us some directional data. For example, if the wave came in parallel to the line between the two sites, the signals would have registered at the exact same time.



If the wave was perpendicular to the line, we would have seen a time delay of 10 ms, because the wave travels the through the Earth 3002 km at the speed of light.

What we detected was a wave that came in at an angle that caused a delay of 6.9 ms. The dotted line represents the distance the wave had to travel for a piece if it to reach the Hanford interferometer. A little trigonometry gives us the angle.



Gravitational Wave Angle

$d = tc$

Where

t = time

d = distance traveled

c = speed of light

With $t = 6.9$ ms

Then $d = 6.9 \text{ ms} \times 3\text{x}10^9 \text{ m/s}$

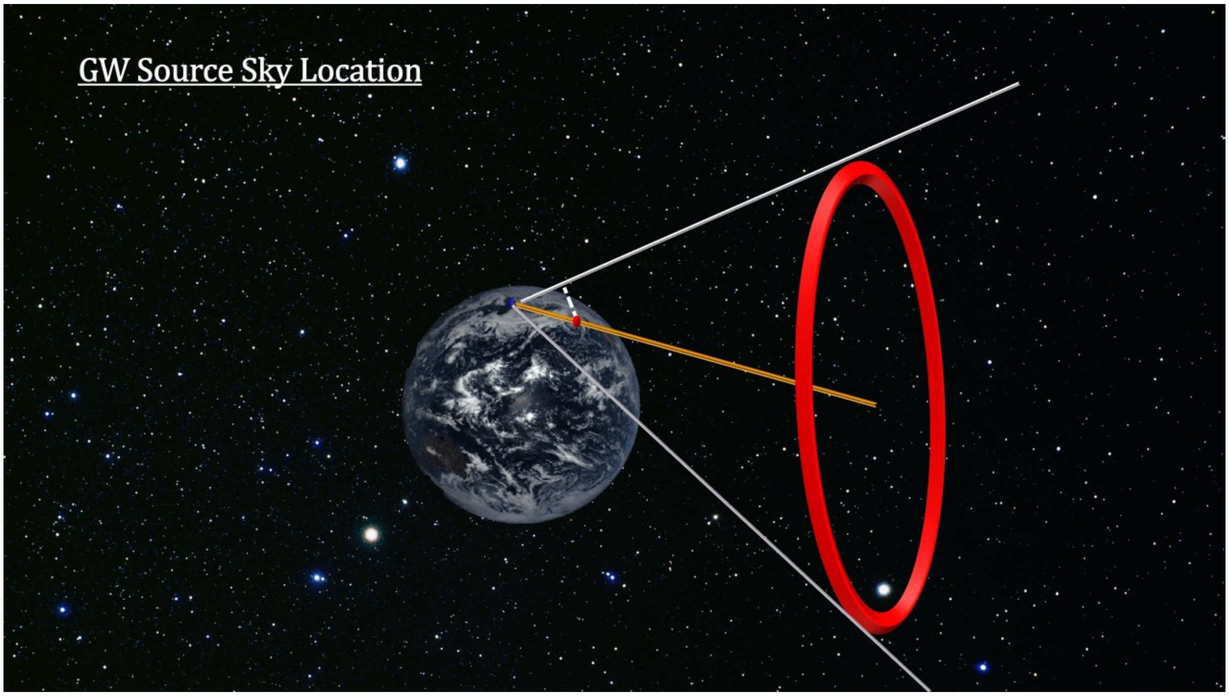
$= 2070$ km

$\sin(\theta)$ = opposite / hypotenuse

$\theta = \sin^{-1}(2070/3002)$

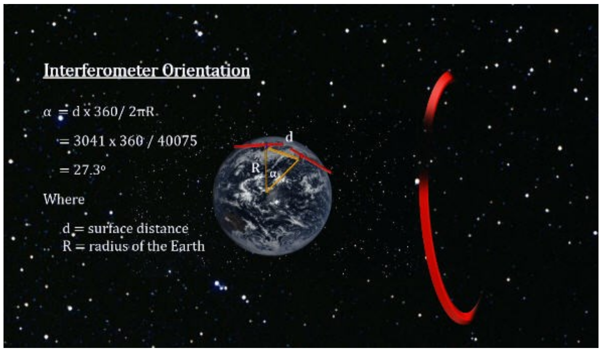
$= 43.6°$

But, with only two interferometers, this angle gives us a circle of possible directions – not the single direction that's needed.



GW Source Sky Location

[Interferometers are most sensitive to waves that come in perpendicular to their two arms. Sensitivity drops off as the incident direction departs from the perpendicular. The curvature of the Earth gives the two LIGO interferometers an angle difference around 27°. This creates slight amplitude and phase inconsistencies across the two detectors that enable a narrowing of the probabilities to a smaller portion of the sky.]



Interferometer Orientation

$\alpha = d \times 360 / 2\pi R$

$= 3041 \times 360 / 40075$

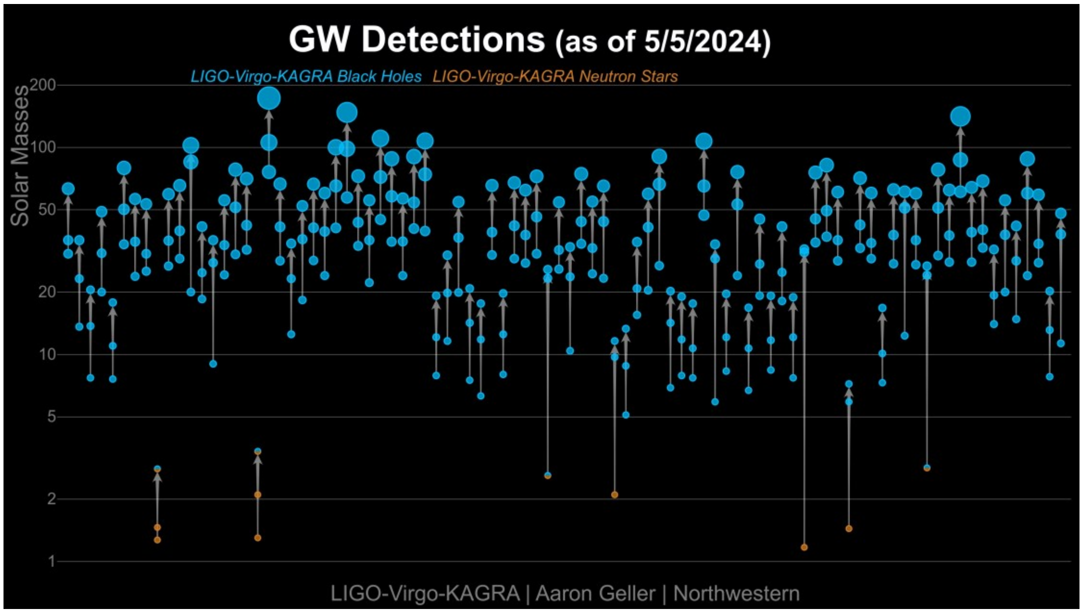
$= 27.3°$

Where

d = surface distance
R = radius of the Earth

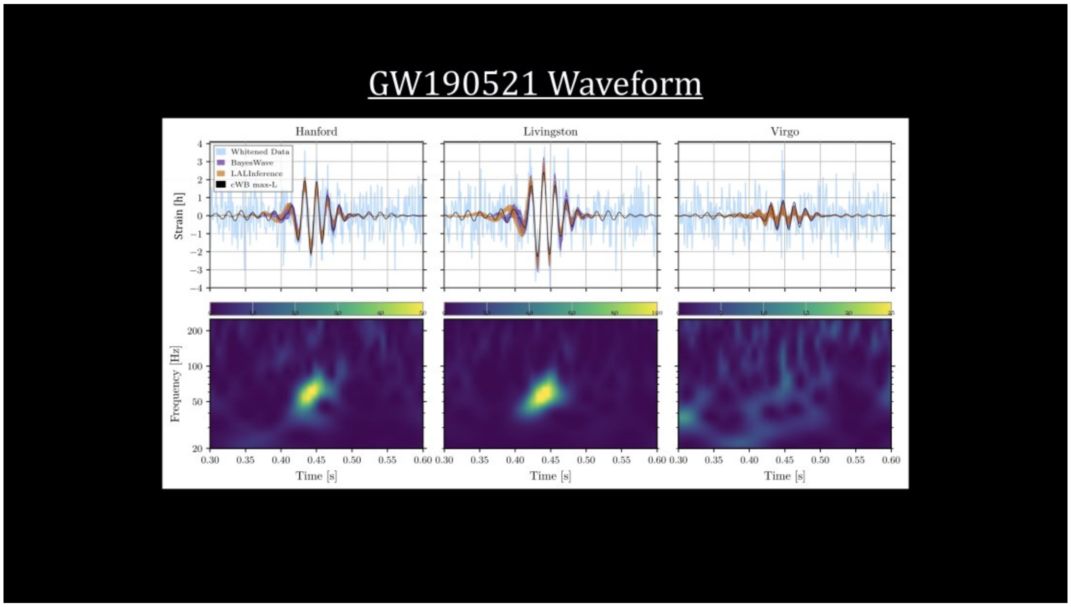
Now there are 4 such gravitational wave interferometers. In addition to LIGO's 2, there is VIRGO - located in Cascina, a small town near Pisa, Italy on the site of the European Gravitational Observatory, and KAGRA - located under Mount Ikenoyama in Japan. Its test-mass mirrors are cooled to cryogenic temperatures. All four work together to crosscheck each other and pinpoint gravitational wave source sky locations.

Combined, they have detected over 90 Gravitational Waves so far. In this graphic, each circle represents a different black hole (in blue) or neutron star (in orange). The half-blue/half-orange circles are compact objects whose classification is uncertain. The vertical scale indicates the mass as a multiple of the mass of our Sun. The arrows indicate which compact object merged and the remnant they produced. We'll cover this one – the most massive – in the next segment.

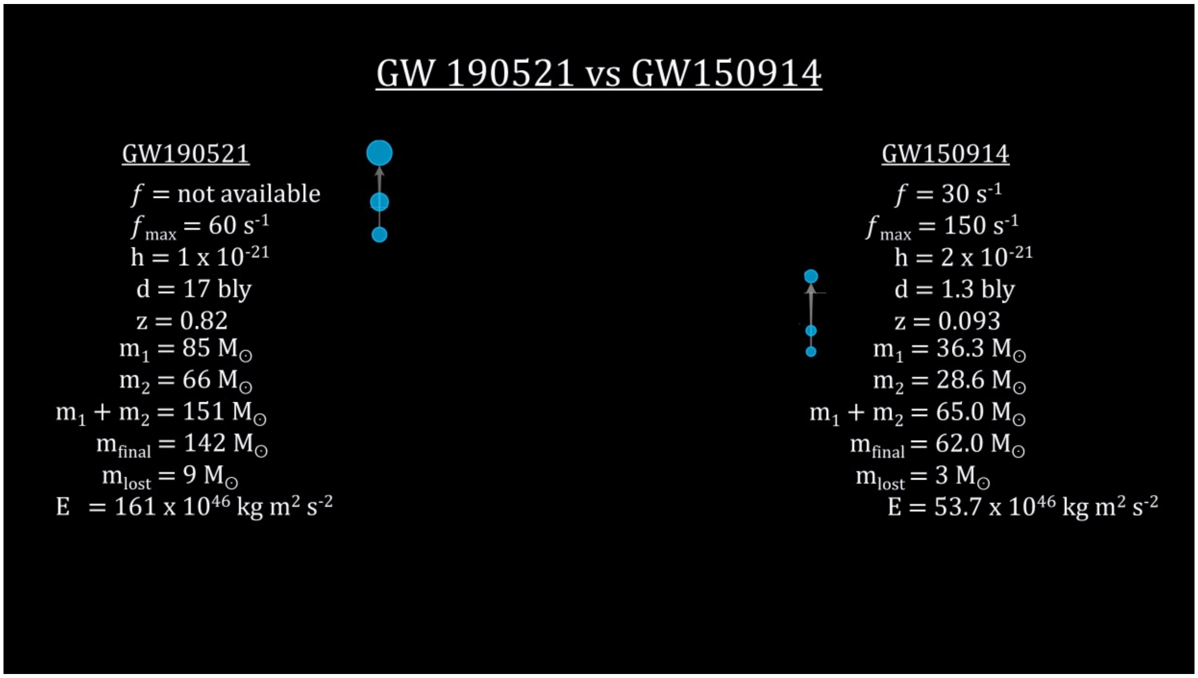


Largest Gravitational Wave – GW190521

On May 21, 2019 at 03:02:29 Coordinated Universal Time, Advanced LIGO and Advanced Virgo observed a short duration gravitational-wave signal, GW190521. The signal was of a shorter duration, and peaked at lower frequency, than any other binary black hole merger observed to date - indicating that it was going to be the most massive inspiral ever detected. The inspiral frequencies were so low that they were out of the sensitivity range for the LIGO detectors. Here's the 'chirp' sound. Without the lead-in wave and with a very low peak frequency, this sounds more like a thud than a chirp.


GW190521 Waveform

As we did earlier with the first Gravitational wave, the frequency gives us the masses of the merging black holes and is fed into numerical general relativity simulations to compute the remnant's mass. The analysis shows that the merger included two black holes with masses of 85 solar masses and 66 solar masses. The calculated mass of the remnant is 142 solar masses. This along with the strain amplitude gives us the luminosity distance to the source at 17.3 bly. Note that the remnant mass is around 9M⊙ less than the combined masses of the two merging black holes. This mass difference was converted into the energy of the gravitational-wave signal. That's almost triple the energy GW150914 created by our first gravitational wave.



GW 190521 vs GW150914

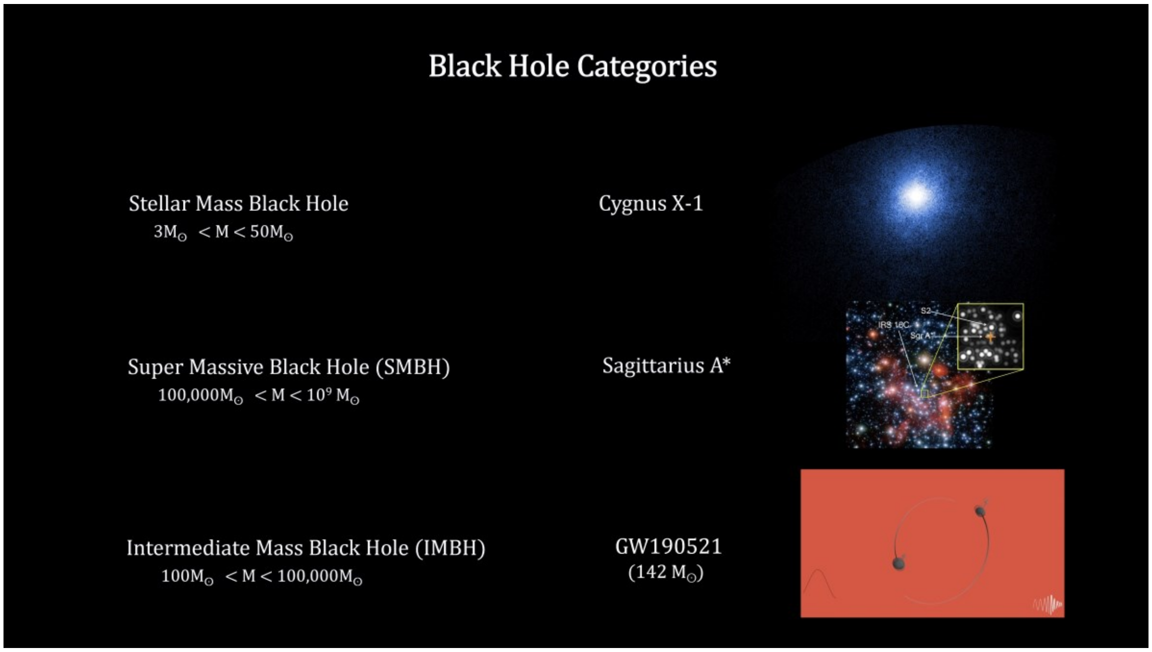
GW190521

f = not available
$f_{max} = 60\ s^{-1}$
$h = 1 \times 10^{-21}$
d = 17 bly
z = 0.82
$m_1 = 85\ M_\odot$
$m_2 = 66\ M_\odot$
$m_1 + m_2 = 151\ M_\odot$
$m_{final} = 142\ M_\odot$
$m_{lost} = 9\ M_\odot$
$E = 161 \times 10^{46}\ kg\ m^2\ s^{-2}$

GW150914

$f = 30\ s^{-1}$
$f_{max} = 150\ s^{-1}$
$h = 2 \times 10^{-21}$
d = 1.3 bly
z = 0.093
$m_1 = 36.3\ M_\odot$
$m_2 = 28.6\ M_\odot$
$m_1 + m_2 = 65.0\ M_\odot$
$m_{final} = 62.0\ M_\odot$
$m_{lost} = 3\ M_\odot$
$E = 53.7 \times 10^{46}\ kg\ m^2\ s^{-2}$

In our chapter on Black Holes, we identified 3 types: stellar mass (the smallest); supermassive (the largest); and intermediate mass black holes (IMBH for short) in-between the other two with a range from 100 to 100,000 times the mass of the Sun. We pointed out that, to date, there has not been a single confirmed discovery of a black hole in this range. So, with 142 solar masses, this event represents the first direct evidence of the existence of such objects.



Black Hole Categories

Stellar Mass Black Hole
$3M_\odot < M < 50M_\odot$

Cygnus X-1

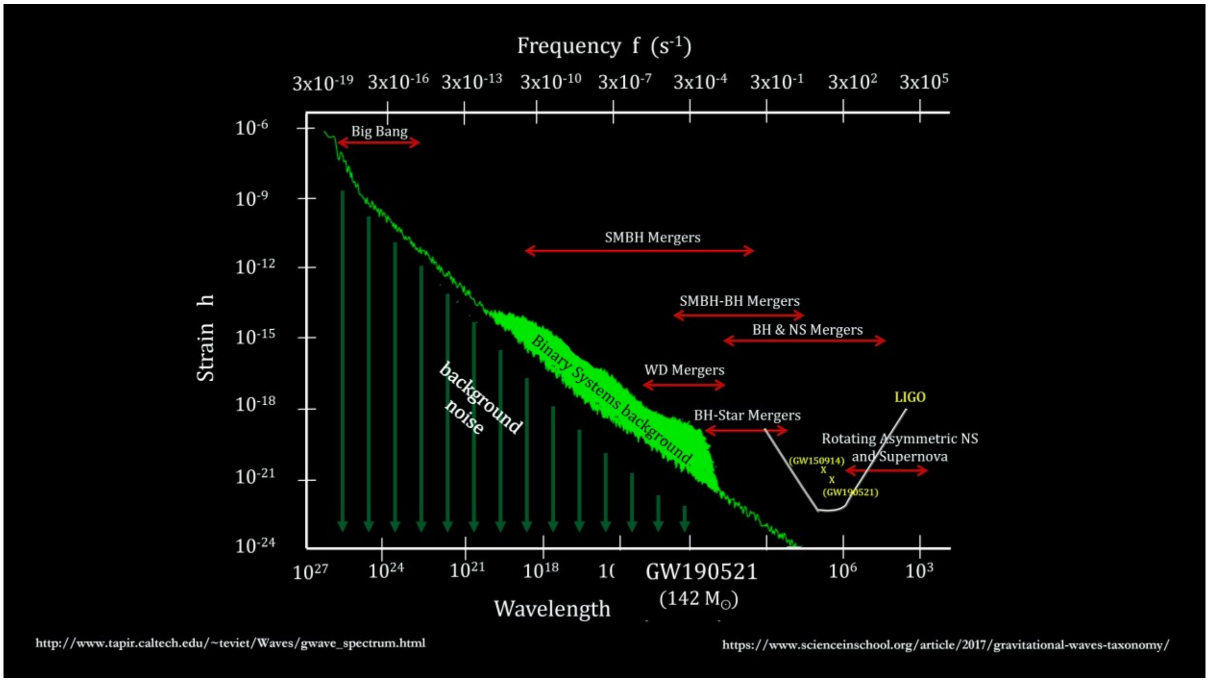
Super Massive Black Hole (SMBH)
$100{,}000M_\odot < M < 10^9 M_\odot$

Sagittarius A*

Intermediate Mass Black Hole (IMBH)
$100M_\odot < M < 100{,}000M_\odot$

GW190521
($142\ M_\odot$)

Here's where GW 190521 fits on the sensitivity graph.

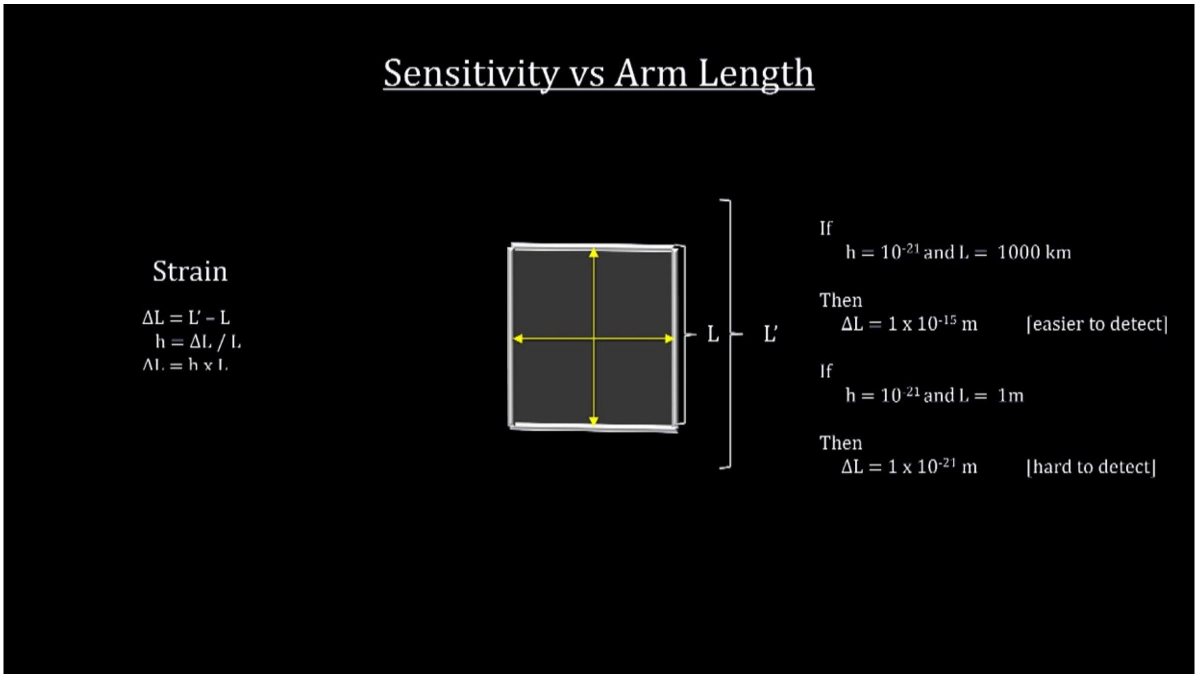


http://www.tapir.caltech.edu/~teviet/Waves/gwave_spectrum.html

https://www.scienceinschool.org/article/2017/gravitational-waves-taxonomy/

Next Gen GW Interferometers

All interferometers detect gravitational waves by measuring the change in distance between two arms as the wave passes through. The longer the arms, the greater the change. This makes the sensitivity of an instrument proportional to the wave's strain. With shorter arm lengths, we get smaller length changes. At some point, noise levels will prevent wave detection.



Sensitivity vs Arm Length

Strain

$\Delta L = L' - L$

$h = \Delta L / L$

$\Delta L = h \times L$

If

$h = 10^{-21}$ and $L =$ 1000 km

Then

$\Delta L = 1 \times 10^{-15}$ m [easier to detect]

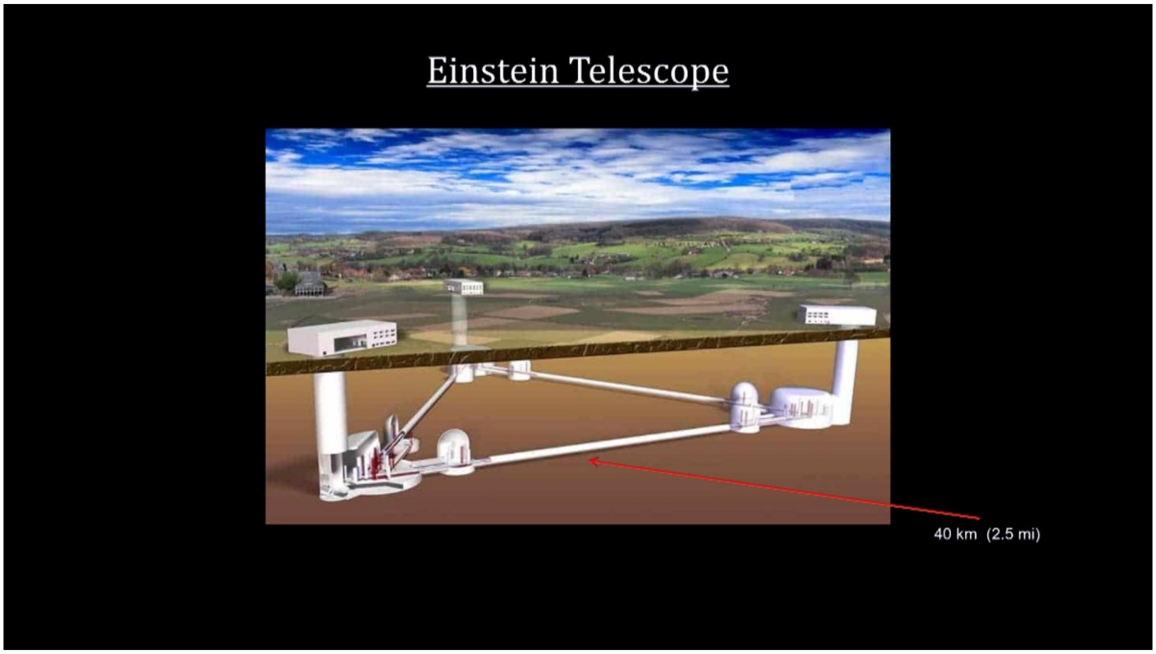
If

$h = 10^{-21}$ and $L =$ 1m

Then

$\Delta L = 1 \times 10^{-21}$ m [hard to detect]

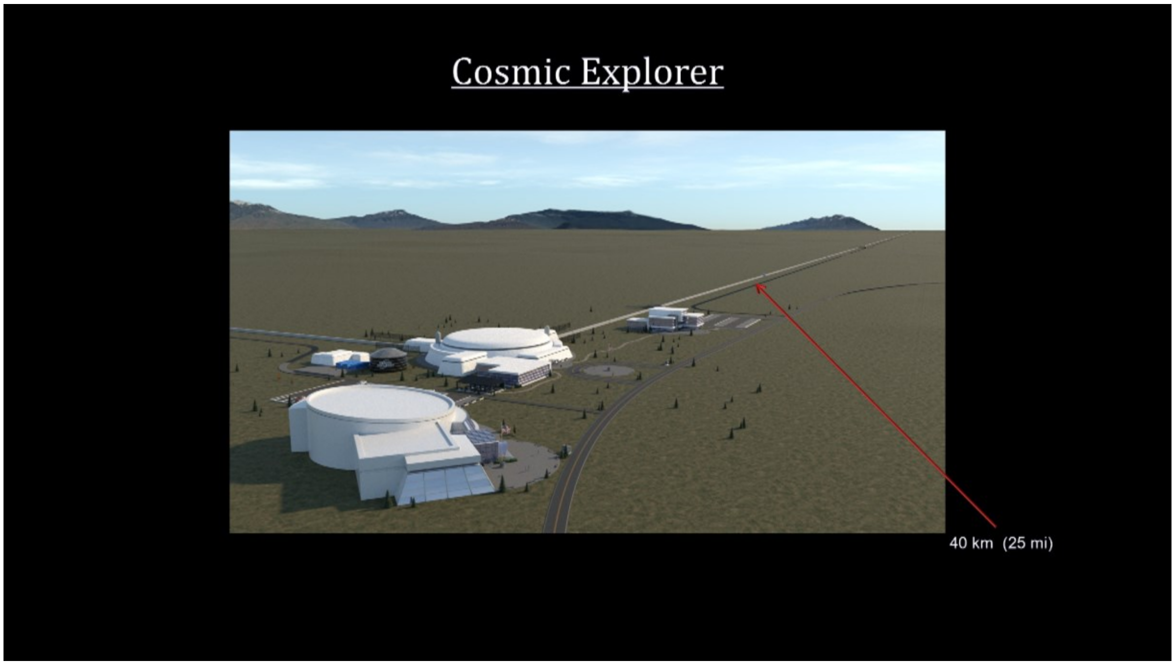
Current Earth based interferometers have arms around 4 kilometers long (that's 2.5 miles). Newer, larger Earth based interferometers are in design and development. One is the Einstein Telescope. It will be underground with arm length of 10km (that's 6.2 miles). Construction could begin as early as 2026 with the goal to start observations in 2035.



Einstein Telescope

40 km (2.5 mi)

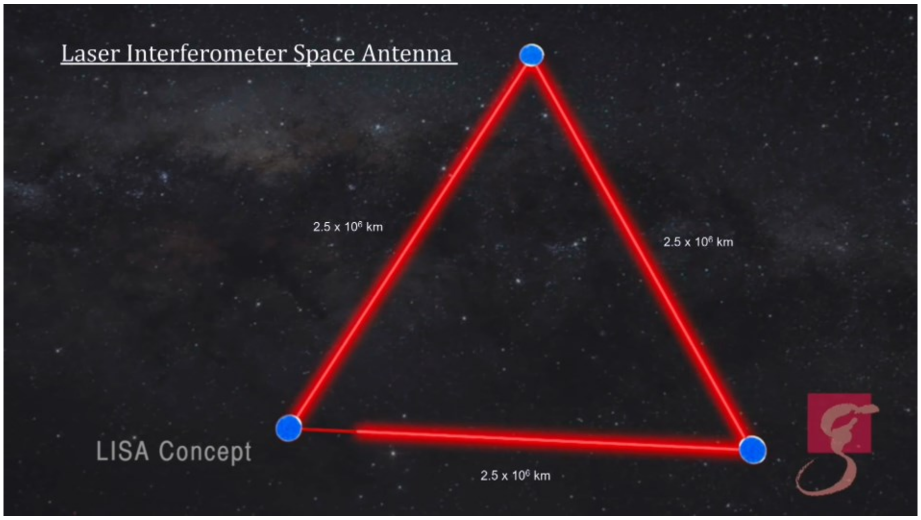
Another is the Cosmic Explorer. Its design features 40 km arms (that's 25 miles). This length will enable the detection of millions of gravitational waves per year! It is also planned for operation in the 2030s.



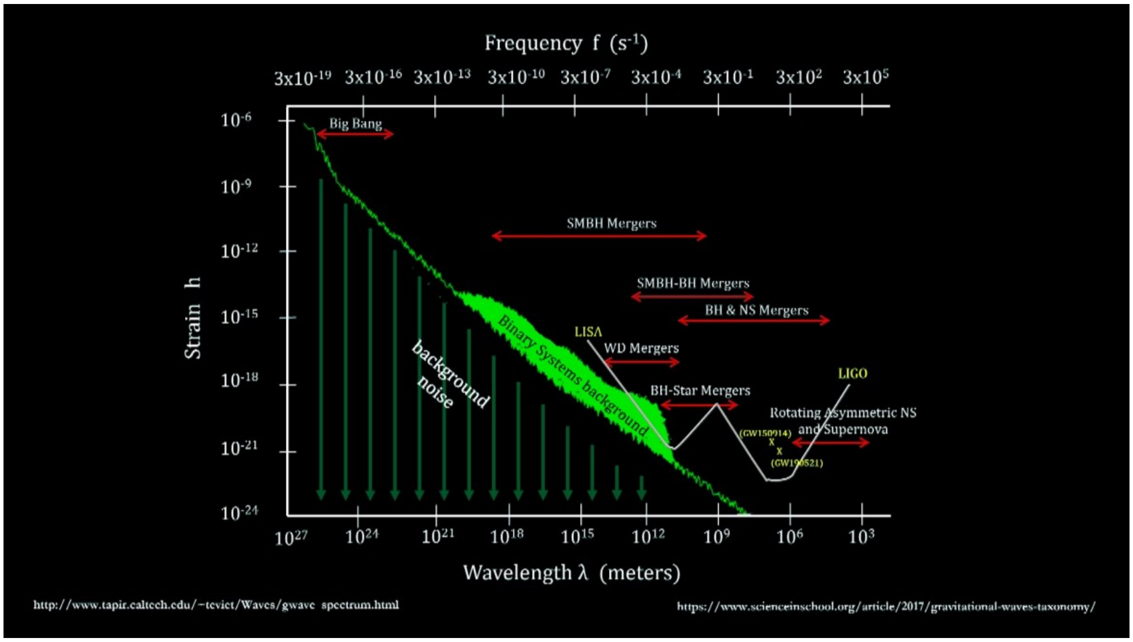
Cosmic Explorer

40 km (25 mi)

There is one additional planned project for the 2030s. This one will create an interferometer in space called the Laser Interferometer Space Antenna or LISA for short. It will consist of three spacecraft arranged in a triangular formation that follows the Earth from the L1 Lagrange Point. Each arm is 2.5 million km long. That's 1.6 million miles. When gravitational waves pass through the triangle, they will produce oscillations in the lengths of its arms and LISA will capture these changes.



This is similar to LIGO, only a million times more sensitive. Here's it's estimated coverage area on our Sensitivity graph. If all goes well, LISA will provide our first look at signals from super massive black hole mergers.

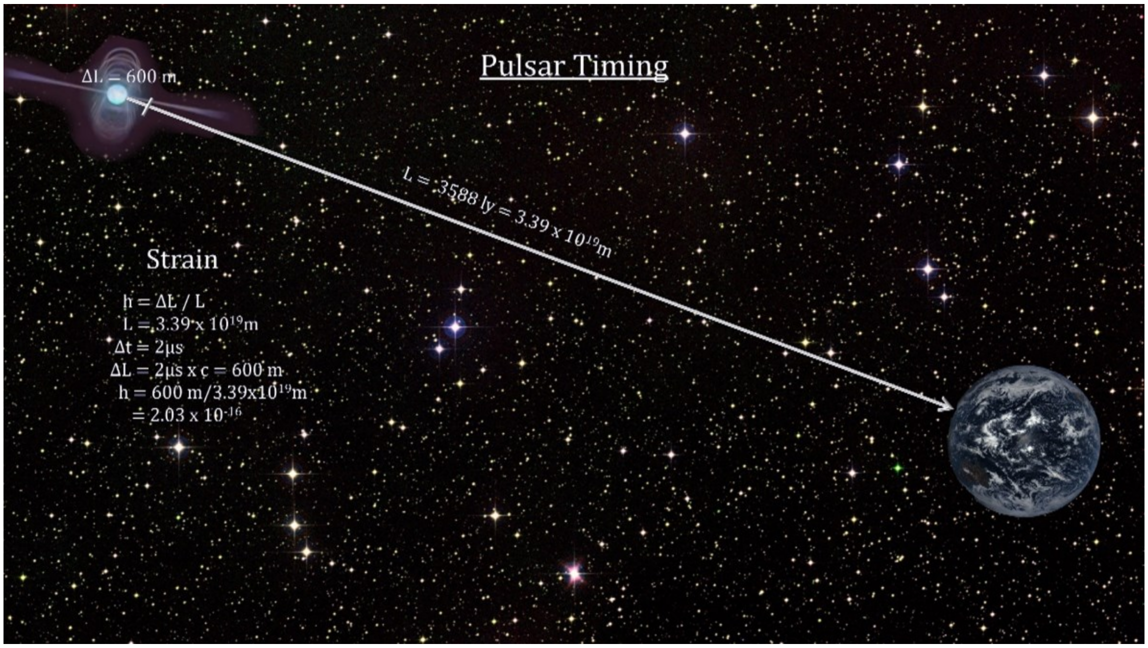


http://www.tapir.caltech.edu/~teviet/Waves/gwave_spectrum.html

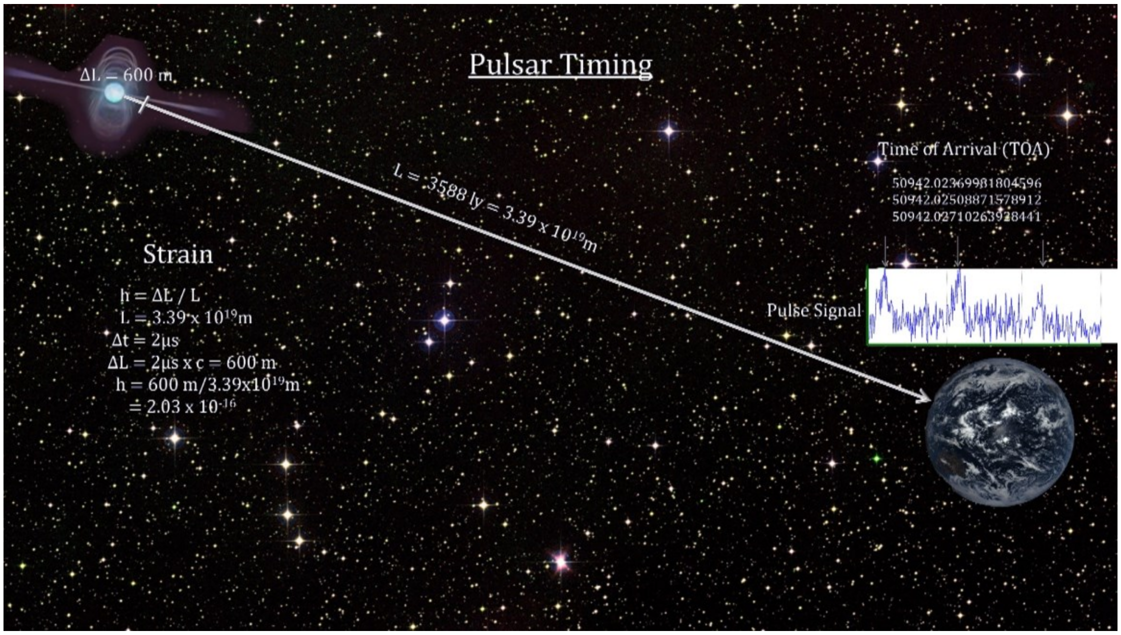
https://www.scienceinschool.org/article/2017/gravitational-waves-taxonomy/

Pulsar Timing Arrays

A totally different technique for detecting gravitational waves is to use pulsars with rotation rates greater than a hundred times per second - called millisecond pulsars. Instead of using interferometers to measure the tiny changes in the length of an arm, it uses the variation in the time it takes for pulsar light to reach Earth. For example, suppose there is a pulsar 3588 lightyears away with a pulse every 5 milliseconds. A passing gravitational wave would change the distance and therefore the time between pulses. In this example, a supermassive black hole inspiral creates a gravitational wave that increases the time between pulses by 2 microseconds over 6 years. That's 600 meters – giving us a strain of 2 x 10^{-16}.



One of the keys to the success of the method is the extremely accurate measurement of each pulse's arrival at the radio telescope – called the time of arrival or TOA for short. Given the arrival time of one pulse, we would just add pulsar rotation time to calculate the expected time of arrival for the next pulse with a few adjustments for things like the Earth's rotation; the orbital motion of the Earth and the pulsar if it's in a binary system; the dispersion delay caused by electrons in the Interstellar Medium; and a few additional relativistic effect items.



Pulsar Timing

$\Delta L = 600$ m

$L = 3588$ ly $= 3.39 \times 10^{19}$ m

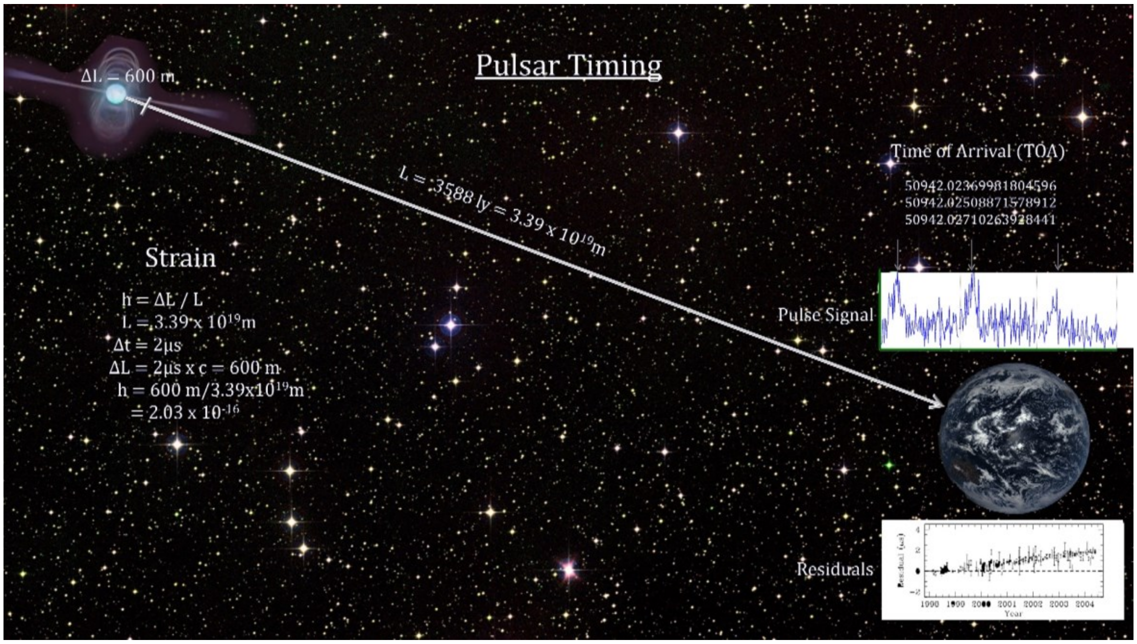
Strain

$h = \Delta L / L$
$L = 3.39 \times 10^{19}$ m
$\Delta t = 2\mu s$
$\Delta L = 2\mu s \times c = 600$ m
$h = 600 \text{ m}/3.39\times10^{19}$ m
$= 2.03 \times 10^{-16}$

Time of Arrival (TOA)
50942.02369981804596
50942.02508871578912
50942.02710263928441

Pulse Signal

Differences between the actual time and the expected time are called Residuals. For a steady state situation, we would see the average over time as a horizontal line. But a very long period gravitational wave would continuously increase the proper distance traveled by a pulse over a long-term observation program. This plot shows the effect of an increase in travel time of 2 microseconds over 6 years. That's the 600 meters in this example. Of course, through analysis is needed to rule out other causes for a graph like this. For example, we would see this 'residuals' pattern if instead of a GW, the pulsar rotation rate was actually slowing down.



Pulsar Timing

$\Delta L = 600$ m

$L = 3588$ ly $= 3.39 \times 10^{19}$ m

Strain

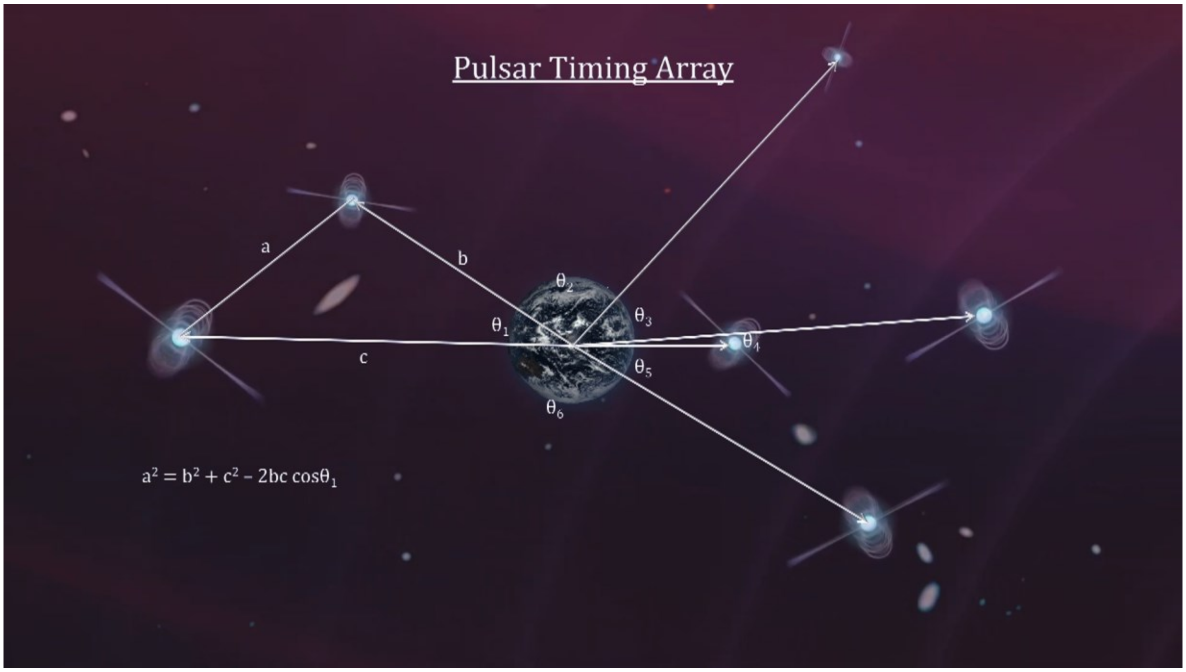
$h = \Delta L / L$
$L = 3.39 \times 10^{19}$ m
$\Delta t = 2\mu s$
$\Delta L = 2\mu s \times c = 600$ m
$h = 600 \text{ m}/3.39\times10^{19}$ m
$= 2.03 \times 10^{-16}$

Time of Arrival (TOA)
50942.02369981804596
50942.02508871578912
50942.02710263928441

Pulse Signal

Residuals

Another way to find gravitational waves with pulsars is to use an array of them and measure the distances and the viewing angles to all of them over time. That's enough to compute the distances between them as well. As gravitational waves pass through the array, we should see deviation patterns that are correlated across all the pulsars in the array.

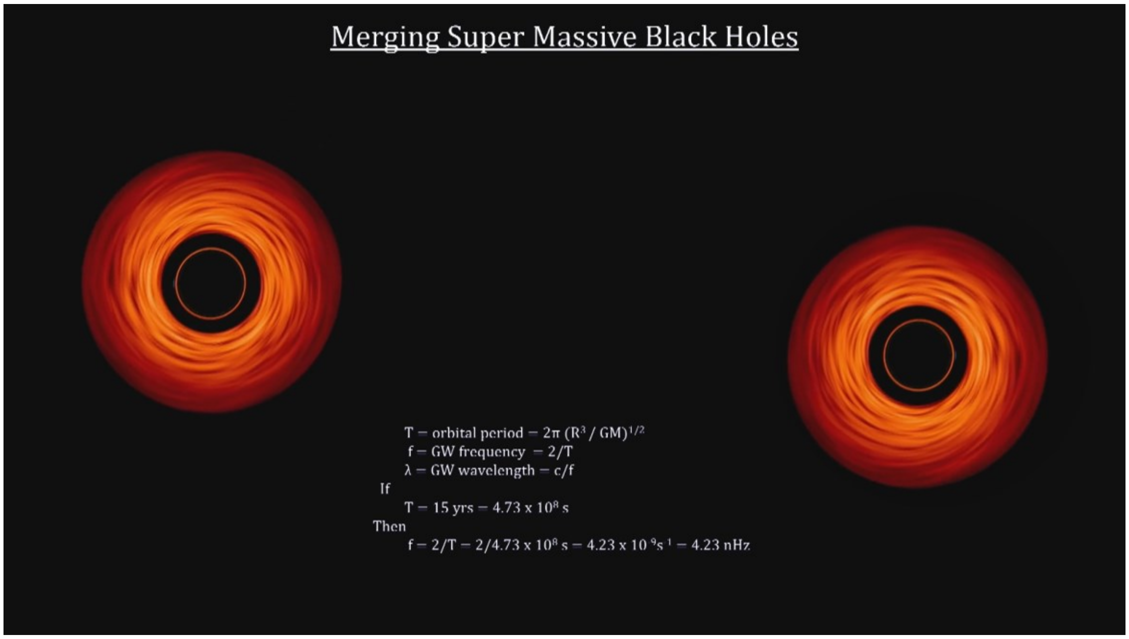


In recent years, a large number of radio wave observation teams have formed across the globe. They are cooperating as teams to find gravitational wave patterns. The frequencies of the GW detected by the pulsar timing array teams are at the lower end of the frequency scale called nanohertz (nHz).

One Nanohertz (nHz)

10^0	1 hertz (Hz)	One per second
10^{-1}	1 decihertz (dHz)	Acoustic – frequency of G
10^{-2}	1 centihertz (cHz)	One per minute
10^{-4}	100 μHz	Hourly
10^{-5}	10 μHz	Daily
10^{-6}	1 microhertz (μHz)	Weekly
10^{-7}	100 nHz	Monthly
10^{-8}	10 nHz	Yearly
10^{-9}	1 nanohertz (nHz)	Once per decade

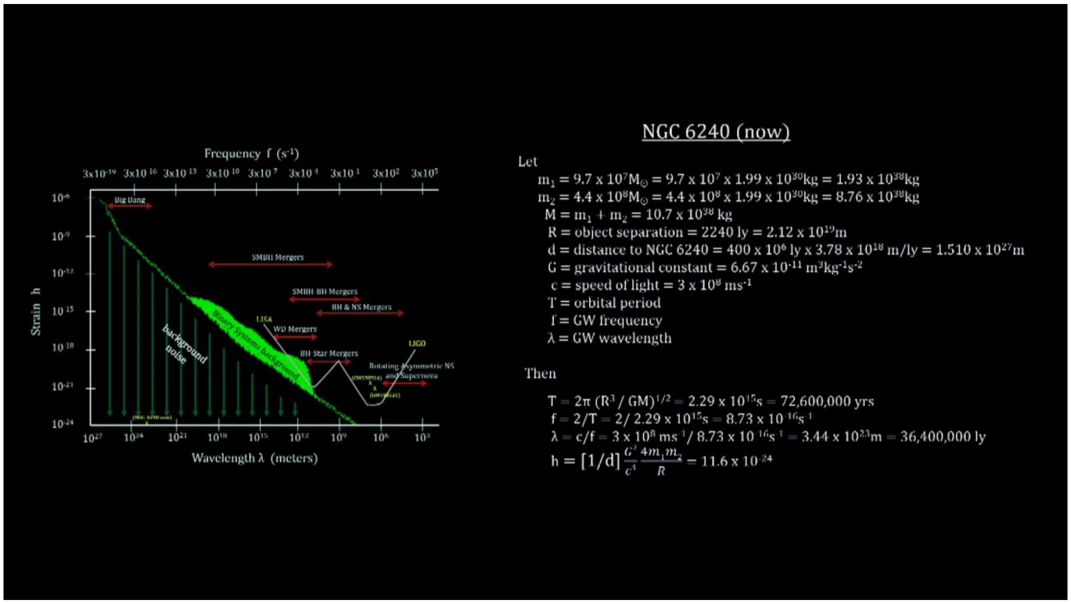
A nanohertz GW is generated by massive objects that are far enough apart to take 15 years or more to make one orbit. To create enough 'strain' to enable detection, these objects would have to be supermassive black holes like those residing at the center of most galaxies.



Merging Super Massive Black Holes

T = orbital period = $2\pi (R^3 / GM)^{1/2}$
f = GW frequency = $2/T$
λ = GW wavelength = c/f
If
T = 15 yrs = 4.73×10^8 s
Then
$f = 2/T = 2/4.73 \times 10^8 \text{ s} = 4.23 \times 10^{-9} \text{s}^{-1}$ = 4.23 nHz

For example, consider the 2 super massive black holes at the center of NGC 6240 that we covered earlier. It's 400 mly away, orbiting each other 2,240 ly apart. One is 97×10^6 M⊙. The other is 440×10^6 M⊙. Calculating the period, frequency, wavelength, and strain, we see that the gravitational waves created are deep in the noise and undetectable.

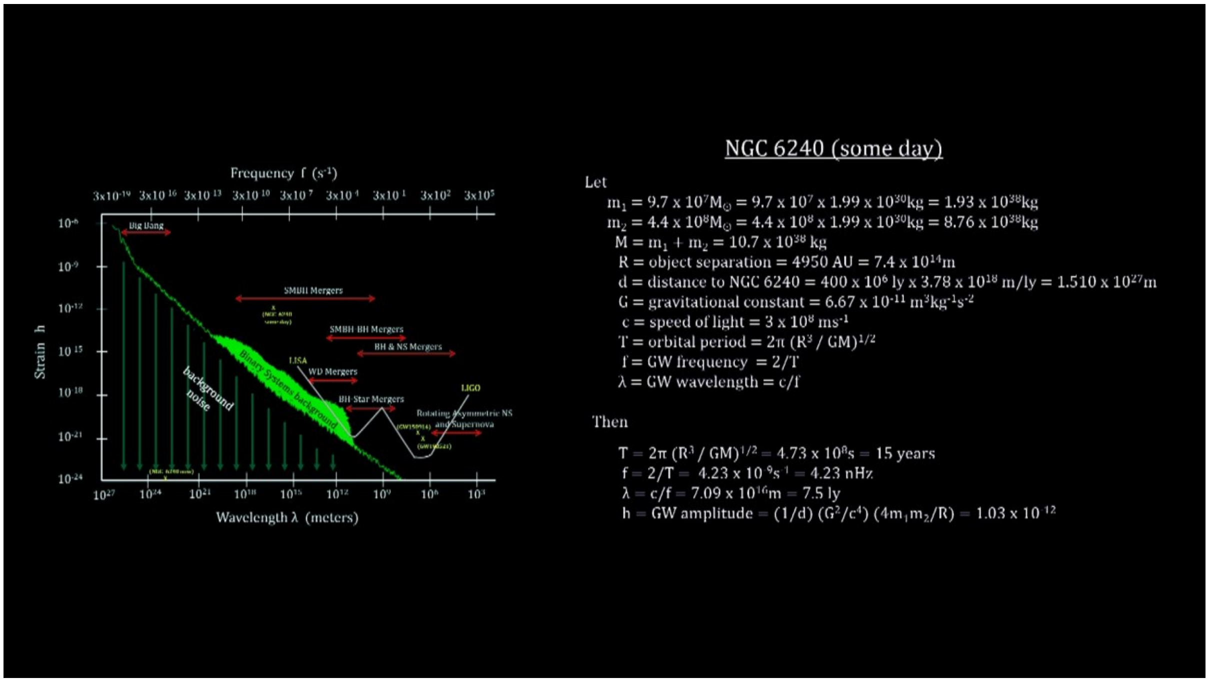


NGC 6240 (now)

Let
$m_1 = 9.7 \times 10^7 M_\odot = 9.7 \times 10^7 \times 1.99 \times 10^{30}\text{kg} = 1.93 \times 10^{38}\text{kg}$
$m_2 = 4.4 \times 10^8 M_\odot = 4.4 \times 10^8 \times 1.99 \times 10^{30}\text{kg} = 8.76 \times 10^{38}\text{kg}$
$M = m_1 + m_2 = 10.7 \times 10^{38}$ kg
R = object separation = 2240 ly = 2.12×10^{19}m
d = distance to NGC 6240 = $400 \times 10^6 \text{ ly} \times 3.78 \times 10^{18}$ m/ly = 1.510×10^{27}m
G = gravitational constant = $6.67 \times 10^{-11}\ \text{m}^3\text{kg}^{-1}\text{s}^{-2}$
c = speed of light = $3 \times 10^8\ \text{ms}^{-1}$
T = orbital period
f = GW frequency
λ = GW wavelength

Then
$T = 2\pi (R^3 / GM)^{1/2} = 2.29 \times 10^{15}\text{s}$ = 72,600,000 yrs
$f = 2/T = 2/\ 2.29 \times 10^{15}\text{s} = 8.73 \times 10^{-16}\text{s}^{-1}$
$\lambda = c/f = 3 \times 10^8\ \text{ms}^{-1} / 8.73 \times 10^{-16}\text{s}^{-1} = 3.44 \times 10^{23}\text{m}$ = 36,400,000 ly
$h = [1/d]\frac{G^2}{c^4}\frac{4m_1m_2}{R} = 11.6 \times 10^{-24}$

But by the time the distance between them is around 5 thousand times the distance between the Earth and the Sun, we get detectable waves. These are the kinds of waves that the world's pulsar timing array projects are designed to detect.



NGC 6240 (some day)

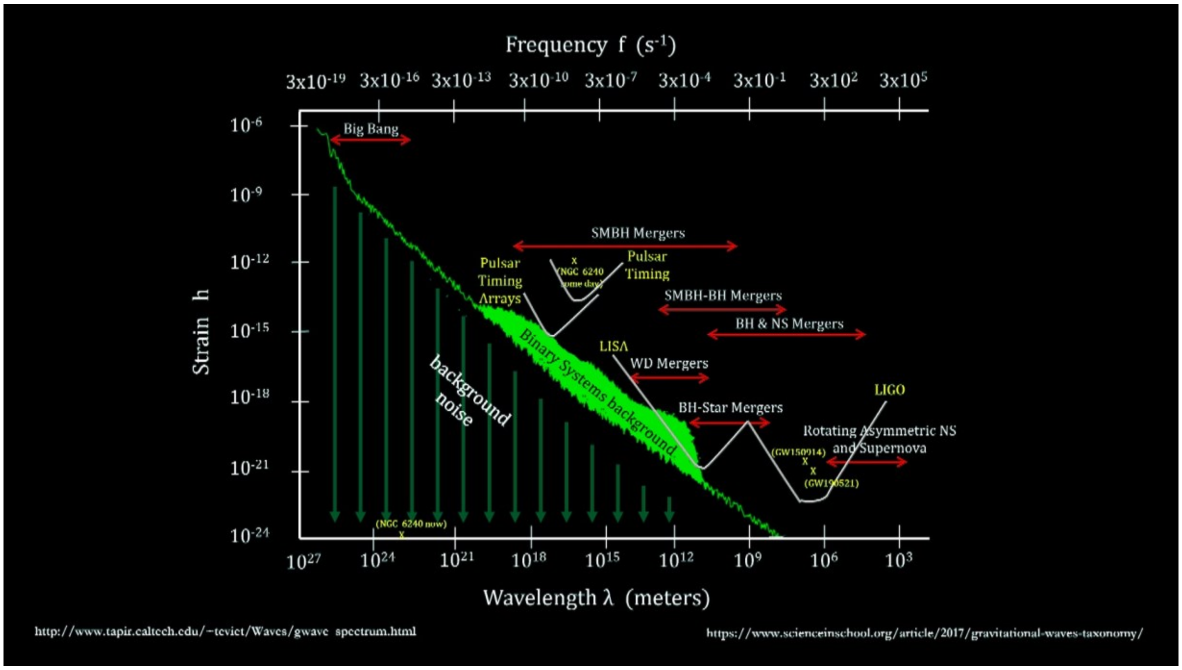
Let

$m_1 = 9.7 \times 10^7 M_\odot = 9.7 \times 10^7 \times 1.99 \times 10^{30}kg = 1.93 \times 10^{38}kg$
$m_2 = 4.4 \times 10^8 M_\odot = 4.4 \times 10^8 \times 1.99 \times 10^{30}kg = 8.76 \times 10^{38}kg$
$M = m_1 + m_2 = 10.7 \times 10^{38}$ kg
R = object separation = 4950 AU = 7.4×10^{14}m
d = distance to NGC 6240 = 400×10^6 ly x 3.78×10^{18} m/ly = 1.510×10^{27}m
G = gravitational constant = $6.67 \times 10^{-11}\ m^3kg^{-1}s^{-2}$
c = speed of light = $3 \times 10^8\ ms^{-1}$
T = orbital period = $2\pi\ (R^3 / GM)^{1/2}$
f = GW frequency = 2/T
λ = GW wavelength = c/f

Then

$T = 2\pi\ (R^3 / GM)^{1/2} = 4.73 \times 10^8 s = 15$ years
$f = 2/T = 4.23 \times 10^{-9}s^{-1} = 4.23$ nHz
$\lambda = c/f = 7.09 \times 10^{16}m = 7.5$ ly
h = GW amplitude = $(1/d)\ (G^2/c^4)\ (4m_1m_2/R) = 1.03 \times 10^{-12}$

Here are the regions covered by pulsar timing and pulsar timing arrays on our sensitivity graph.

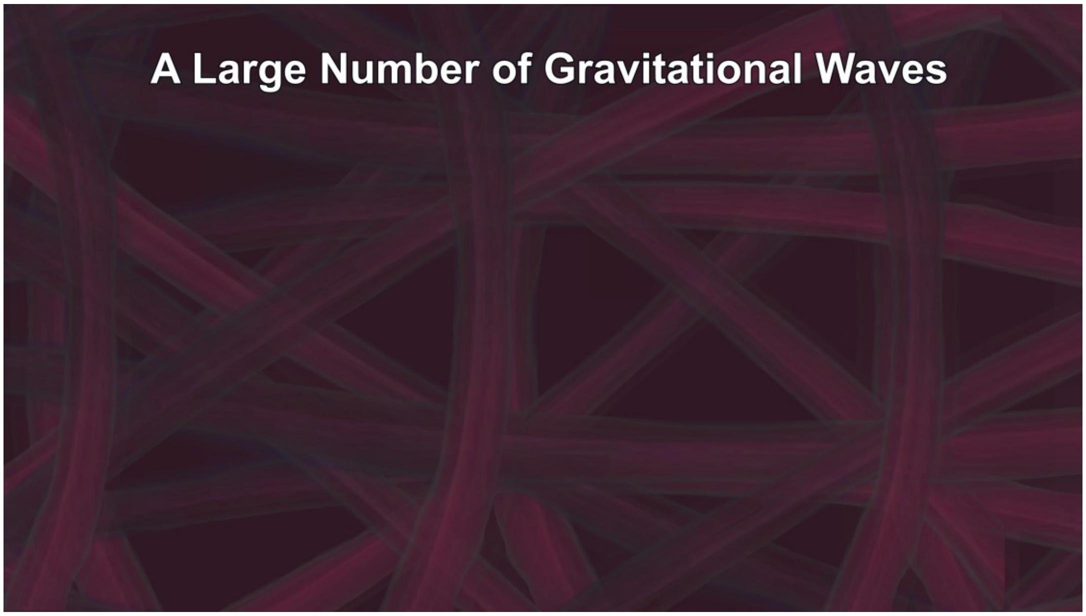


http://www.tapir.caltech.edu/~teviet/Waves/gwave_spectrum.html

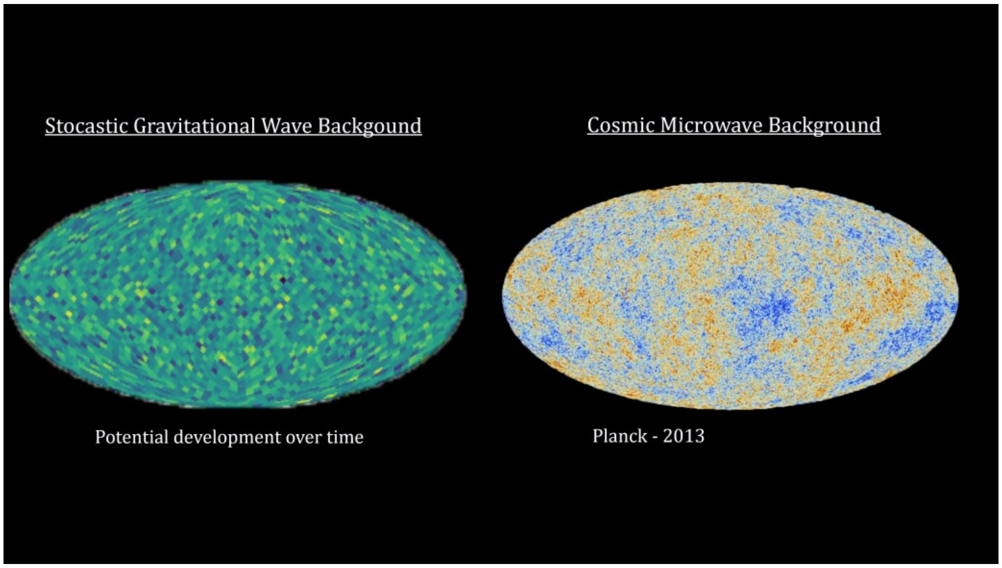
https://www.scienceinschool.org/article/2017/gravitational-waves-taxonomy/

Big Bang Gravitational Waves

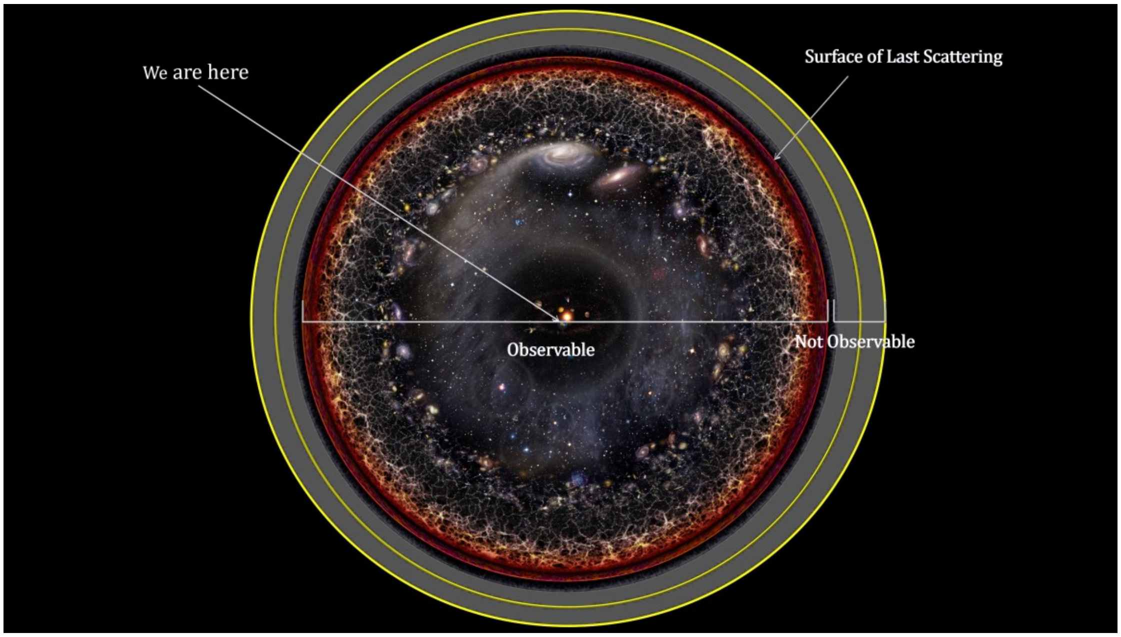
The large number of gravitational waves detected so far by interferometers around the world, implies that there could be a background of gravitational waves created by the superposition of numerous incoherent sources happening throughout the history of the universe.


A Large Number of Gravitational Waves

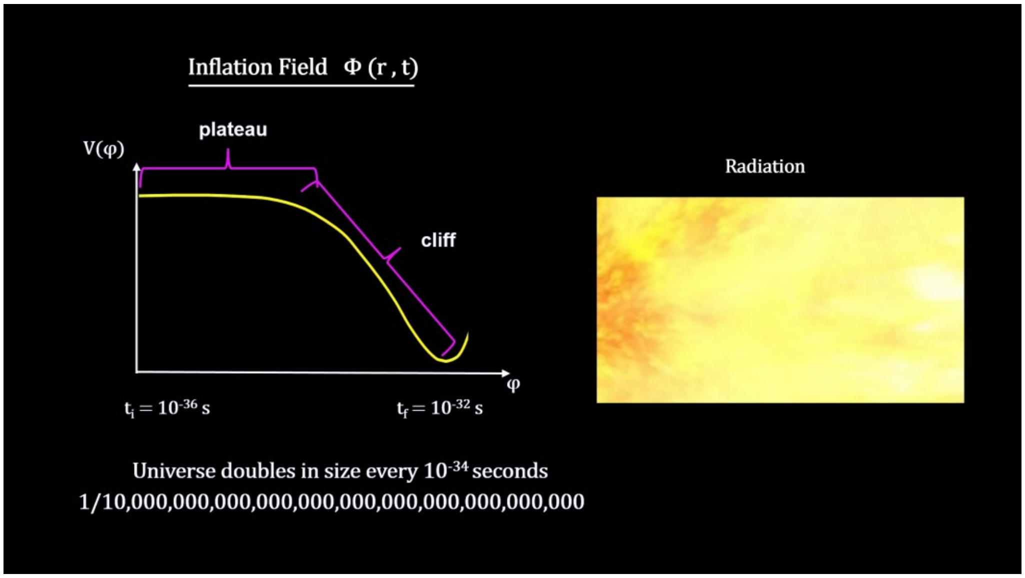
This would create a background coming from all directions with a pattern that can be analyzed statistically. Such a pattern is referred to as stochastic and the radiation is referred to as the Stochastic Gravitational Wave Background or SGWB for short. Here's a set of possible stochastic background maps that could happen as detection methods improve over time. It's set against what actually did happen with the Cosmic Microwave Background.



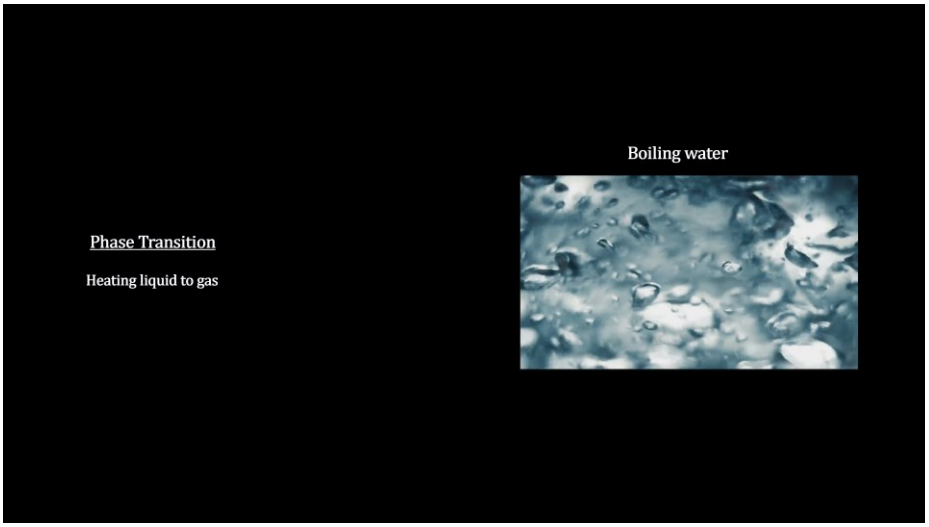
The universe is currently separated into the 'observable' where light travels across space relatively uninhibited, for us to collect and analyze; and the unobservable where light could not travel far. The demarcation line is the surface of last scattering that created the CMB. But any gravitational waves, created during the unobservable time, would have traveled across space and might still be with us today as part of the stochastic background.



For example, as we covered in the 'How Old Is It' video book on the 'Lambda Cold Dark Matter' big bang theory, the Universe began with an extremely short period of exponential expansion called Inflation (from 0 to 10^{-32} seconds) that produced a 'radiation-only' universe, that cooled into a quark plasma.



The phase transition from radiation to a quark plasma is similar to water's phase transition from liquid to gas in boiling water. Note that phase transitions do not create gravitational waves.



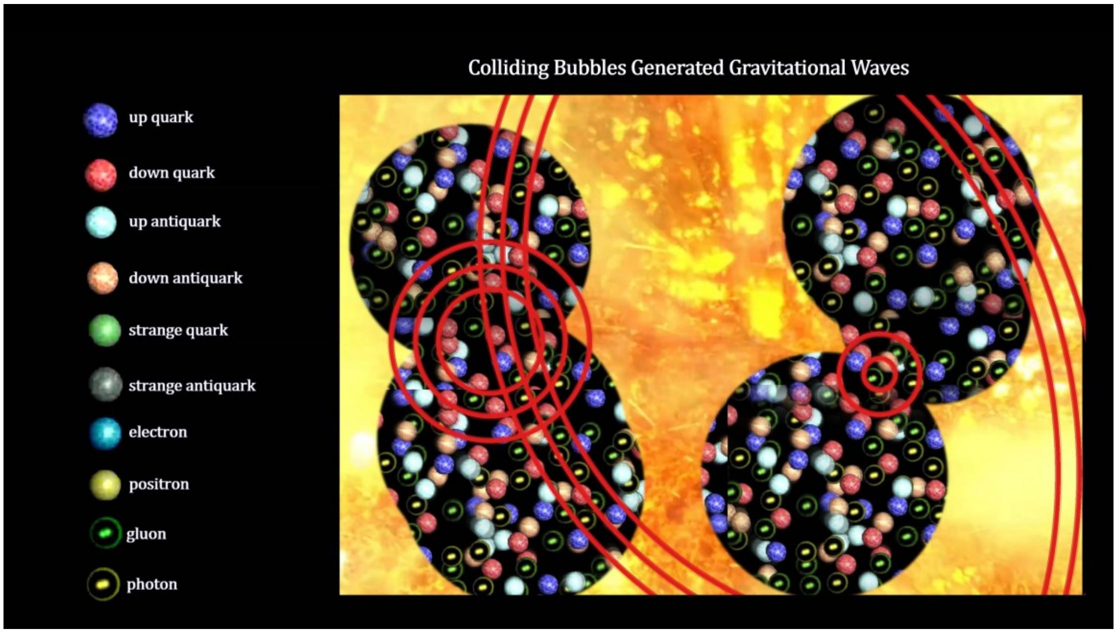
Phase Transition

Heating liquid to gas

Boiling water

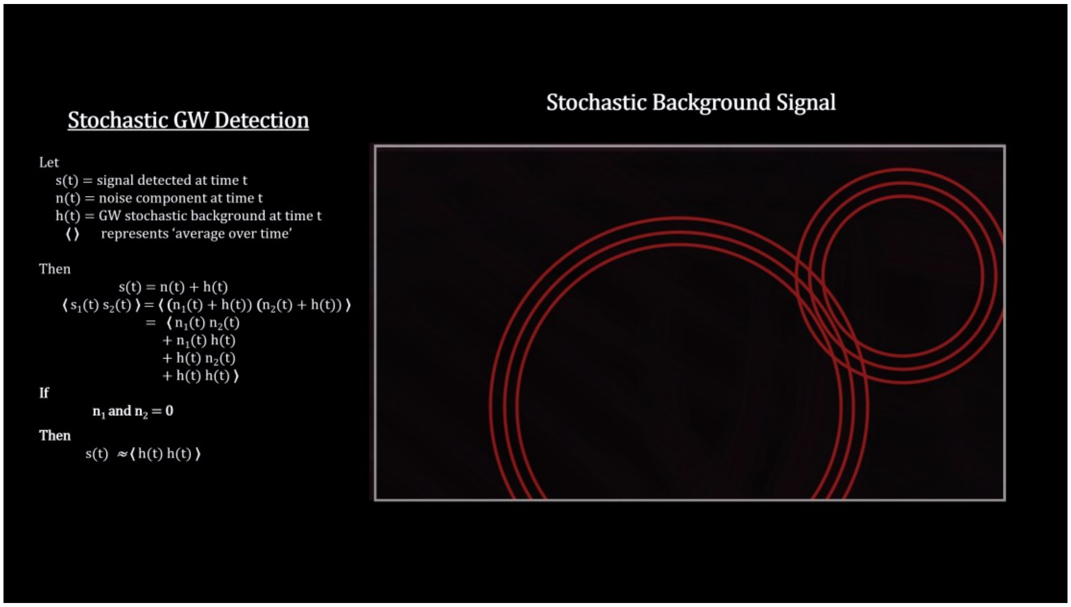
https://motionarray.com/

But transitions like Inflation, Baryogenesis and Nucleosynthesis would start out as growing bubbles of the new state material inside the old state material - like the gas bubbles in liquid water. Keeping in mind that even small bubbles could contain a percentage of all the mass in the universe, bubble collisions would generate massive gravitational waves.



Colliding Bubbles Generated Gravitational Waves

The search for the stochastic background is under way using very large numbers of pulsar timing arrays. [The magnitude of the stochastic gravitational wave background is usually reported in terms of its energy density per frequency interval. The background is assumed to be isotropic, so one could determine its statistical properties by observing any part of the sky.] Unlike the detectable gravitational wave signals, the stochastic-background would just appear as noise in a single gravitational wave detector. What's more, the magnitude of the stochastic-background will always be much smaller than the noise. Because the noise in each detector is statistically independent from one another, the time average for noise is zero. Multiple detectors can factor it out.



Stochastic GW Detection

Let

$s(t)$ = signal detected at time t

$n(t)$ = noise component at time t

$h(t)$ = GW stochastic background at time t

$\langle\,\rangle$ represents 'average over time'

Then

$$s(t) = n(t) + h(t)$$

$$\begin{aligned}\langle s_1(t)\, s_2(t) \rangle &= \langle (n_1(t) + h(t))\,(n_2(t) + h(t)) \rangle \\ &= \langle n_1(t)\, n_2(t) \\ &\quad + n_1(t)\, h(t) \\ &\quad + h(t)\, n_2(t) \\ &\quad + h(t)\, h(t) \rangle\end{aligned}$$

If

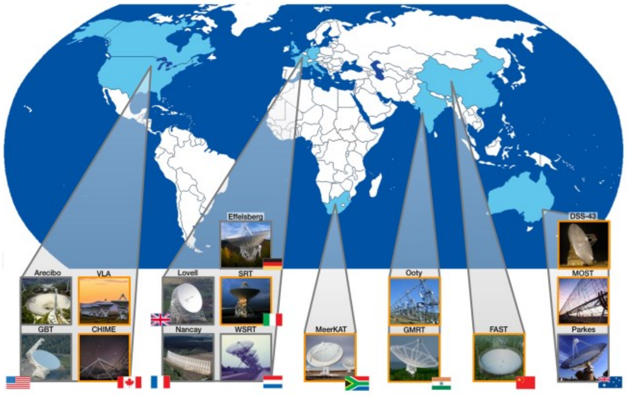
$$n_1 \text{ and } n_2 = 0$$

Then

$$s(t) \approx \langle h(t)\, h(t) \rangle$$

Stochastic Background Signal

In 2023, five pulsar timing array groups: the North American Nanohertz Observatory, along with the European, Indian, Chinses, and Parks pulsar timing array groups collectively announced that they have uncovered "the first evidence for low-frequency gravitational waves permeating the cosmos." The next era of gravitational waves and cosmology has begun!



Music

Puccini – Madama Butterfly

Handel - Concerto Grosso - Larghetto

Beethoven - The Creatures of Prometheus

Offenbach - Barcarolle - The Tales of Hoffman

Mozart - Symphony No 40 First Movement]

Puccini - Madame Butterfly - Un Bel Di Vedremo

Greek letters:
- α β γ δ ε ζ η θ ι κ λ μ ν ξ ο π ρ σ τ υ φ χ ψ ω
- Α Β Γ Δ Ε Ζ Η Θ Ι Κ Λ Μ Ν Ξ Ο Π Ρ Σ Τ Υ Φ Χ Ψ Ω

⇒ → ± ⊙ ∞ ↛ ∃ ∄ ∈ ∉ ∬ ∫ ≅ ≥ ≤ ≈ ≠ ≡ √ ∛ ~ ∝ ħ ÷ ∂ ⊥